International Workshop on Lot Sizing

IWLS'2010

August 25th - 27th, 2010

Gardanne - France

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PROGRAM OVERVIEW

Tuesday, August 24th, 2010

18:30-20:00	Get-together party (Les deux Garçons : 53 Cours Mirabeau, 13100 Aix-En-Provence).
20:30-23:00	Dinner (La Chimère Café : 15 Rue Bruyes, 13100 Aix en Provence)

Wednesday, August 25th, 2010

8:30-9:00	Welcome
9:00-9:30	Opening session
9:30-10:50	Session 1 (2 talks)
10:50-11:20	Coffee-break
11:20-12:40	Session 2 (2 talks)
	Lunch
14:20-16:20	Session 3 (3 talks)
16:20-16:50	Coffee break
16:50-18:10	Session 4 (2 talks)

Thursday, August 26th, 2010

8:30-9:50	Session 1 (2 talks)
9:50-10:20	Coffee-break
10:20-12:20	Session 2 (3 talks)
	Lunch
14:00-15:30	Discussion
15:30	Social Event : Departure
17:30-19:00	Visit of "Le château des Baux-de-Provence"
19:00-20:00	Visit of "Les Baux-de-Provence"
20:00	Departure to "Saint-Rémy-de-Provence"
20:30-23:00	Dinner at "Saint-Rémy-de-Provence" "Les Ateliers de l'Image"

Friday, August 27th, 2010

9:00-10:20	Session 1 (2 talks)
10:20-11:50	Coffee-break
11:50-12:10	Session 2 (2 talks)
	Lunch
14:00-15:20	Session 3 (2 talks)
15:20-16:00	Coffee-break

DETAILED PROGRAM

Wednesday, August 25th

8:30-9:00
Welcome

9:00-9:30
Opening Session

9:30-10:50
Economic lot-sizing with remanufacturing: complexity and effcient formulations
Mathijn J. Retel Helmrich   *Raf Jans*   *Wilco van den Heuvel*   *Albert P.M. Wagelmans*
Economic Lot-Sizing with Perishable Items
H. Edwin Romeijn   *Mehmet Onal*   *Amar Sapra*   *Wilco van den Heuvel*

10:50-11:20
Coffee-break

11:20-12:40
Coordination via Discounts and Rebates in a Two-level Supply Chain with Lot-sizing Cost Structures
Wilco van den Heuvel   *Joseph Geunes*   *H. Edwin Romeijn*
A two-level lot-sizing problem with bounded inventory
Nadjib Brahimi   *Nabil Absi*   *Stéphane Dauzère-Pérès*   *Safia Kedad-Sidhoum*

14:20-16:20

Extending and improving solution methods for the General Lot-sizing and Scheduling Problem for Parallel Production Lines (GLSPPL)

Herbert Meyr

Discrete Lot-Sizing and Scheduling on Identical Parallel Machines

Céline Gicquel *Laurence A. Wolsey* *Michel Minoux*

A solving procedure for a general integrated planning and scheduling problem

Riad Aggoune *Stéphane Dauzère-Pérè* *Cathy Wolosewicz*

16:20-16:50

Coffee-break

16:50-18:10

Formulations and a Branch-and-Cut Algorithm for a Production-Distribution-Routing Problem

Mirko Ruokokoski *Oğuz Solyalı* *Jean-François Cordeau* *Raf Jans* *Haldun Süral*

Lower bounds for the capacity constrained lot size problem with setup times

Silvio Alexandre de Araujo *Zeger Degraeve* *Raf Jans*

Thursday, August 26th

8:30-09:50
Fixed charge transportation on a path: the convex hull?
Mathieu Van Vyve
Lot-sizing by MIP: Some Open Problems and Recent Results
Laurence A. Wolsey

9:50-10:20
Coffee-break

10:20-12:20
Dynamic capacitated lot sizing with random demand and a service level constraint
Stefan Helber *Florian Sahling* *Katja Schimmelpfeng*
Robust Inventory Routing under Demand Uncertainty
Oguz Solyali *Jean-François Cordeau* *Gilbert Laporte*
A Column Generation Heuristic for Dynamic Capacitated Lot Sizing with Random Demand under a Fillrate Constraint
Horst Tempelmeier

14:00-15:30	Discussions
15:30	Social Event : Departure to *"Les Baux-de-Provence"*
17:30-19:00	Visit of *"Le château des Baux-de-Provence"*
	Visit of *"Les Baux-de-Provence"*
20:00	Departure to *"Saint-Rémy-de-Provence"*
20:30-23:00	Dinner at *"Saint-Rémy-de-Provence : Les Ateliers de l'Image"*

Friday, August 27th

9:00-10:20

Capacitated lot-sizing: Synchronization issues for multi-stage systems

Christian Almeder *Bernardo Almada-Lobo*

Tight Formulations for the Two and Three Level Serial Lot-Sizing Problems

Meltem Denizel *Oguz Solyali* *Haldun Sural*

10:20-10:50

Coffee-break

10:50-12:10

Performance Assessment of Production Planning Approaches in Semiconductor Manufacturing

Lars Mönch *Thomas Ponsignon*

Transportation lot sizing: Research topics and practical aspects

Atle Nordli

14:00-15:20

Lot-sizing with carbon emission constraints

Nabil Absi *Stéphane Dauzère-Pérès* *Safia Kedad-Sidhoum* *Bernard Penz* *Christophe Rapine*

Collaborative Planning in Supply Chains

Alf Kimms

15:20-16:00

Coffee-break

Book of Abstracts

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Contents

Economic lot-sizing with remanufacturing: complexity and efficient formulations

Mathijn J. Retel Helmrich, Raf Jans Wilco van den Heuvel, and Albert P.M. Wagelmans

1 Introduction

Reverse logistics (see [1]) is a field that has emerged during the last decades. It studies situations in which there is not only a product flow towards the customers, but products and materials are also returned to the manufacturer and these may be reused in production processes. Within the framework of reverse logistics, the classic economic lot-sizing problem has been extended with a remanufacturing option. As in the classic problem, we face a deterministic demand from customers in a number of discrete time periods. In each period, we must decide to set up a production process or not, and if so how much to produce. In order to find a production plan with minimal costs, we must find the optimal balance between set-up, holding and production costs. In the problem extended with a remanufacturing option, known quantities of used products are returned from customers in each period. There is no demand for these returned products themselves (or 'returns' in short), but they can be remanufactured, so that they are as good as new. Customer demand can then be fulfilled from two sources, namely newly produced and remanufactured items. Since both can be used to serve customers, they are referred to as 'serviceables'. We are to determine in which periods to set up a production process to remanufacture returned products and in which to set up a production process to manufacture new items. This setting is similar to the one described by [6], although we do not assume zero production and remanufacturing costs. As in [6], we consider two variants of lot-sizing with remanufacturing. In the first variant, manufacturing new products and remanufacturing used products take place in two separate processes, each with its own set-up costs. We call this problem ELSRs (Economic Lot-Sizing with Remanufacturing and Separate set-ups). In the second variant, the manufacturing and remanufacturing process have one joint set-up cost, for instance because manufacturing and remanufacturing operations are performed on the same production line. We call this problem ELSRj (Economic Lot-Sizing with Remanufacturing and Joint set-ups).

2 Complexity results

We can show that ELSRs is $\mathcal{NP}$-hard even if all costs are time-invariant, by using a reduction from the well-known $\mathcal{NP}$-complete PARTITION problem. Furthermore, we know that ELSRj with time-invariant costs can be solved in $\mathcal{O}(T^4)$ time with the dynamic programming algorithm in [6]. However, we prove that ELSRj is $\mathcal{NP}$-hard in general, by showing that ELSRs is a special case of ELSRj.

3 Formulations

Because of their complexity, it makes sense to look at good mixed integer programming (MIP) formulations of both problems. A first formulation with a 'natural' choice of variables was presented in [6]. We call this formulation Original and it serves as our benchmark. We see, however, that such a formulation contains so-called 'big M' constraints. It is generally known ([4]) that these big M constraints in the natural lot-sizing formulation often lead to a bad LP-relaxation and hence high running times. Consequently, we propose several new, alternative formulations of the lot-sizing problem with remanufacturing. The first reformulation is based on a shortest path type formulation. Such a reformulation was first proposed by [2] for the capacitated lot-sizing problem (without remanufacturing). The problem with remanufacturing can be viewed as having two products: serviceables and returns. We apply a shortest path type reformulation (SP) to both. The second reformulation is a partial shortest path reformulation (PSP). This reformulation has fewer variables than the full shortest path reformulation, while preserving the quality of the LP-relaxation as much as possible. This idea was used by [7] for the classic lot-sizing problem. A first partial shortest path formulation was proposed by [5]. A different approach to improve the MIP formulation is to add valid inequalities to the Original formulation. A well-known set of strong valid inequalities for the classic (single-item uncapacitated) lot-sizing problem consists of the (l, S, WW) inequalities, as introduced by [3]. We adapt them for both the returns and serviceables layer of lot-sizing with remanufacturing.

4 Computational results & conclusions

In order to gain insight into the performance of the different formulations, we randomly generated 360 problem instances, both for ELSRs and ELSRj. All problems were solved by CPLEX 10.1 with a one hour time limit.

When we compare the results for ELSRj with those for ELSRs, we see that ELSRj is easier to solve than ELSRs. This was to be expected, because the problem with separate set-ups has twice as many integer variables as the problem with separate set-ups. When we look at the differences between formulations, we see roughly the same pattern with ELSRs and ELSRj. We found that, for both problem variants, SP (our shortest path formulation) performs better than the Original and (l, S, WW) formulations, especially in terms of the quality of the LP relaxation. The computation times and MIP gaps are also smaller in the vast majority of test instances. When the return rate is high though, faster results may be obtained by (l, S, WW) (for a large horizon) or Original (for a shorter horizon). It should be noted though, that the performance of the Original formulation can go down quite dramatically when the set-up costs are higher. The partial shortest path formulation (PSP) exhibits many features of SP, such as the quality of the LP relaxation, while having fewer variables and needing less computer memory.

A possible extension involves extending the shortest path reformulations with production capacities, which should be quite straightforward, since [2] introduced their shortest path reformulation of the lot-sizing problem without remanufacturing in the context of production capacities. Other avenues for further research include changing the assumption that remanufactured products are as good as new to a situation with a separate demand for new and remanufactured products, where new products can serve as substitutes for remanufactured ones (but not vice versa). Another track worth exploring is using the solution of the LP relaxation of SP in a heuristic, e.g. a rounding or relax-and-fix heuristic. Since this formulation has a good LP relaxation, we would expect such a heuristic to give good feasible solutions in a short amount of time.

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Economic Lot-Sizing with Perishable Items

H. Edwin Romeijn, Mehmet Önal, Amar Sapra and Wilco van den Heuvel

The goal of the classical Economic Lot-Sizing (ELS) problem is to find a minimum-cost procurement plan for a single item with deterministic, nonstationary demand over a finite time-horizon in discrete-time. Typically procurement and inventory holding costs are assumed to be concave functions of the quantity procured and the quantity held in inventory at the end of a period, respectively. Most variants of the ELS problem implicitly assume that the items do not deteriorate between procurement and demand satisfaction. However, in some settings this assumption is unrealistic; consider, for example, agricultural and dairy products, fashion items, and items subject to rapid technological change. Research in this area has focused on extensions of the EOQ model, the ELS model, as well as models that allow for stochastic demands. Nahmias [3] provides an extensive review on production and inventory models that deal with such *perishable* items.

For this talk, we will focus on ELS models with perishable items. Friedman and Hoch [1] consider an extension of the ELS where, at the end of each period, a certain age-dependent fraction of all inventories of a given age spoil. They show that the optimal solution is not necessarily found among procurement plans that satisfy the so-called *zero inventory ordering* (ZIO) property. However, they do demonstrate that, if the deterioration rate increases as a function of the age of the item then there exists an optimal solution where procurement in each period satisfies some consecutive set of demand periods. Hsu [2] proposes an extension of this model in which the deterioration rate can depend on both on the age of the item as well as the period in which the item was produced. Similar to Friedman and Hoch, Hsu assumes that, in any period, older items cannot deteriorate at a slower rate than newer items. (Interestingly, they do not assume that items procured in a given period deteriorate at a faster rate as they get older.) While he allows for general concave procurement and inventory holding cost functions he does assume that carrying any number of units of items of a given age at the end of a given period is larger when the age of these items is larger. Under these assumptions, the optimal solution structure is the same as the one in Friedman and Hoch: without loss of optimality we can restrict ourselves to procurement plans in which procurement in a period satisfies a set of consecutive demand periods.

In this talk, we study a new variant of the economic lot sizing problem with perishable items (ELS-PI) where each item is assumed to have an *expiration date* that depends on the procurement period of the item. In particular, we assume that no items perish up to the expiration date, and that all units become obsolete after the expiration date. As is common in ELS problems, we assume that procurement costs consist of a fixed component (incurred if a strictly positive quantity is procured in a given period) and a variable component (linear in the number of items procured). Inventory holding costs are assumed to be linear in the quantity in inventory at the end of a period and independent of the age of an item. If we assume that the expiration date is a nondecreasing function of the procurement period, i.e., items that are procured later cannot perish before items that are procured earlier, our model becomes a special case of the model by Hsu discussed above. Although this assumption seems reasonable at first sight, it may be violated if, for example, a supplier may use different raw materials in different periods, not impacting the immediate functionality of the product but potentially impacting the expiration date. Even if, for a given supplier, the assumption holds it may also be effectively violated for the products actually procured if there are several potential suppliers for the product.

If we are interested in minimizing total procurement and inventory holding costs, we cannot guarantee the existence of an optimal procurement plan in which products procured in a given period satisfy the demand in a consecutive number of periods, which significantly complicates the problem. Interestingly, note that in this model we implicitly assume that both retailer and customer are indifferent between items of different ages and expiration dates (as long as the expiration date has not passed) that may be available in a given period. This is not necessarily the case. For example, the retailer may have a preference for selling items in the order that minimizes total cost. Alternatively, the retailer may face practical (e.g., storage or presentation) restrictions. On the other hand, the customer may prefer items that have a longer remaining lifetime to items with a shorter one. In other words, how the available items are matched with customer demands depends on (i) the relative power of the two players (retailer and customer), and (ii) practical constraints. We therefore consider different *allocation mechanisms.* In particular, we focus on mechanisms that do *not* distinguish between items procured in the same period, and that are *consistent* between periods. That is, an allocation mechanism specifies a preference ordering of the potential procurement periods: we write $t_1 \succ t_2$ if items procured in period t_1 are preferred over items procured in period t_2. A given preference ordering leads to constraints that have to be added to the optimization problem.

We pay particular attention to four special cases. Two are based on the procurement period: First-In First-Out (FIFO) and Last-In First-Out (LIFO). The other two are based on the expiration date: First-Expiration First-Out (FEFO) and Last-Expiration First-Out (LEFO). We study conditions under which some of these allocation mechanisms are equivalent and derive properties of optimal solutions in each of these cases. We then present polynomial-time algorithms for finding a minimum-cost procurement plan under each of these mechanisms. We end the talk with some topics for future research.

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Coordination via Discounts and Rebates in a Two-level Supply Chain with Lot-sizing Cost Structures

Wilco van den Heuvel, Joseph Geunes, and H. Edwin Romeijn

Systems often perform suboptimally because of the selfish behavior of the players in the system. A decentralized supply chain, where the players have different objectives, is an example of such a system. The cost of a supply chain (the system cost) can be measured as the sum of the costs incurred at each stage of the supply chain satisfying the demand for a customer good.

If a supply chain is controlled by a single player, a so-called centralized supply chain, then the system cost can be optimized. This is in contrast with a decentralized supply chain, where each player optimizes her cost (or profit) given the actions of the other players. We call the difference between the system cost under a decentralized system and a centralized system the cost of anarchy. If the cost of anarchy can be reduced in a supply chain and the savings are allocated fairly among the players, then all players benefit. In practice, cooperation between the players is often not possible, because it may be too costly or the players are just not willing to cooperate.

In this talk, we try to determine coordination mechanisms that mitigate the cost of anarchy without explicit cooperation between the players. We have the following setting. There is a two level supply chain with a single manufacturer and a single retailer. The retailer faces a deterministic demand over a discrete and finite planning horizon. Furthermore, both the supplier and the retailer have a lot-sizing cost structure, i.e., a fixed plus linear procurement cost and linear holding cost. Finally, we assume that explicit collaboration is not possible.

In this setting, the following promotion activities of the supplier are examples of coordination mechanisms: price discounts, rebates, and compensation for holding inventory. By means of the promotion, the supplier is able to influence the procurement plan of the retailer, because it changes the cost structure of the retailer. The promotion scheme ensures that the retailer is better off than in the situation without coordination. Furthermore, if the promotion costs are less than the savings in procurement and holding costs, then the supplier is also better off. This means that the supply chain as a whole is better off (compared to the decentralized situation without promotion), and hence a promotion mechanism is able to reduce the cost of anarchy. The percentage by which the gap between the cost under anarchy and the system optimal is closed, is used as an efficiency measure of the coordination mechanism. Note that the efficiency depends on the coordination mechanism that is used.

The main research questions that we will address in the talk are the following. How high can the cost of anarchy be? Do there exist (simple) mechanisms that can reduce the cost of anarchy? And if yes, by what amount can it be reduced? Can we develop efficient algorithms to find the optimal parameters of a coordination mechanism? Past research has answered some of these questions, but only for simple models, like a supply chain satisfying the EOQ assumptions (i.e., an infinite time horizon model with a constant demand rate and time-independent cost parameters). For an overview of coordination in an EOQ environment we refer to Sarmah et al. [3].

To the best of our knowledge, the only two papers that try to achieve a system optimal solution and deal with a lot-sizing environment are Sahin and Robinson [2] and Dudek and Stadtler [1]. Sahin and Robinson [2] consider different degrees of cooperation in a supply chain model and analyze the effects. Their work is different from our work as we will proceed from the assumption that explicit collaboration is not possible. Dudek and Stadtler [1] propose a negotiation-based planning between the players in the supply chain. An advantage of this approach is that it can be applied to quite general supply chain structures, and players do not need to reveal all their private information. However, a disadvantage is that the negotiation scheme is quite complicated.

In this talk, we will only consider simple coordination mechanisms. In particular, the coordination mechanisms are specified by just one or two parameters. An example of a simple one-parameter mechanism is an $\alpha\%$ price discount given by the supplier to the retailer in every period (where the supplier optimizes over α). The reasons to consider simple mechanisms are twofold: (i) their implementability in practice, and (ii) their mathematical tractability. The main findings of our research are that the simple mechanisms can reduce the cost of anarchy significantly. Moreover, we show that the optimal parameter in case of a single-parameter mechanism can be found in subexponential time.

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Solving a two-level lot sizing problem with bounded inventory

Nadjib Brahimi, Nabil Absi, Stéphane Dauzère-Pérès and Safia Kedad-Sidhoum

Abstract. We consider a two-level lot-sizing problem where the first level consists of N end products competing for a single type of raw material (second level), which is supposed to be critical. In particular, the storage capacity of raw materials is limited and must be carefully managed. The goal is to simultaneously determine an optimal replenishment plan for the raw material and optimal production plans for the end products on a horizon of T periods. The problem is modeled as an integer linear program and solved using both a Lagrangian relaxation-based heuristic and a commercial optimization software. The results obtained using the Lagrangian heuristic are promising and new ideas are generated to further improve the quality of the solution.

1 Introduction

In this paper, we address the two-level multi-item lot-sizing problem with bounded inventory (2LLSP-BI). It is a production planning problem where a time varying demand of N end products has to be produced over a planning horizon of length T. The end products require a critical raw material. Indeed, the storage capacity for the raw material can be limited or the raw material is shipped over long distances. Both situations imply that the consumption by the end products of the raw material must be carefully managed. In this work, we focus on both determining an optimal replenishment plan for the raw material and optimal production plans for the end products over the planning horizon. The problem is close to the disassembly planning problem with the objective of minimizing the amount of waste by means of recycling and remanufacturing (see Lambert and Gupta [5]). Crowston and Wagner [1] and Kim *et al.* [3] present a literature review for the planning problem in disassembly systems.

For the two-level disassembly problem, Kim *et al.* [4] present a polynomial algorithm where a product (usually mechanical or electronic equipment) is to be disassembled while satisfying the demand of leaf items over a given planning horizon with the objective of minimizing the sum of setup and inventory holding costs. The differences between our problem and the one suggested in [4] are presented in the following assumptions of their paper. There is no shortage for the used product (to be disassembled). Whenever the disassembly process is set up, there are always enough used product to process. Moreover, no partial extraction of components is allowed, when a certain amount of the used product is processed all its components are extracted in a proportional way (based on the gozinto factor).

In the problem addressed in this paper, the equivalent of the used product to disassemble is a Raw Material (RM) which can be involved in the production of different types of derived products. The whole quantity of processed RM can be used to make one single type of derived product. We note that the RM can be purchased and kept in stock. This is different from the two-level disassembly problem presented by Kim *et al.* [4] where they assume that the raw material (the product to be processed) is always available. In this paper, we must decide on when to order (produce) the RM. Additionally, they do not consider the cost related to purchasing and/or holding the product, the only costs considered in their model are the setup related to the disassembly process and the holding cost of the components. We consider (at least initially) production (purchasing), holding and setup cost of the RM and all derived products. Moreover, the production variables in their model must be integer (they usually represent equipment to be disassembled).

2 Mathematical programming models

We propose two mathematical formulations: a straightforward aggregate model and a disaggregate formulation. The objective is to minimize the total production, purchasing, setup and inventory holding costs. The constraints are related to the inventory balance equations for the end products and raw material, to the inventory capacity of the raw material, and to the setups. To our surprise, the standard solver used in our numerical experiments solves the aggregate formulation faster than the disaggregate formulation presented here.

3 Lagrangian heuristic

Lagrangian relaxation approaches are often used to solve multi-item lot-sizing problems with coupling constraints. The main idea of this method is to decompose a multi-item lot-sizing problem into several easy to solve single item lot-sizing problems by relaxing the complicating constraints. When considering our two-level lot-sizing problem, by relaxing some constraints, we obtain a problem that decomposes into N classical single-item Uncapacitated Lot-Sizing problem (ULS) and a particular single item problem that can be solved analytically. Each ULS sub-problem is solved using an $O(T \log T)$ dynamic programming algorithm (see Wagelmans *et al.* [7]). We also propose some valid inequalities to improve the Lagrangian based lower bound.

Three construction heuristics are developed and evaluated to generate initial solutions, that will be improved in the iterations of the Lagrangian heuristic. Because it dominates the two others, only one construction heuristic, named WHK-BI, was used in our extensive numerical experiments. In this heuristic, the production plan of the end products is calculated using the algorithm of [7] (WHK). The raw material demands correspond to the aggregation of the resulting production levels of end products. The raw material production plan is determined by solving a single item problem with bounded inventory (explicit consideration of the inventory capacity for raw material), recently solved in $O(T^2)$ by Liu [6]. On the other hand, it was shown by Wolsey [8] that the single item problem with bounded inventory is equivalent to the single item problem with non-customer specific time windows (introduced by Dauzère-Pérès *et al.* [2]), which was first solved in $O(T^4)$ in [2], and then in $O(T^2)$ in [8]. It is easy to convert the data of the bounded inventory problem into a non-customer specific time window problem using a simple heuristic with linear time complexity. This approach has been used in the second stage of the WHK-BI heuristic. The WHK-BI heuristic is also used in a smoothing heuristic that takes the solution of the Lagrangian problem and modifies it to build a good *feasible* solution.

4 Conclusions

A two-level lot-sizing problem with bounded storage capacity for raw material has been analyzed and solved. A Lagrangian relaxation heuristic has been proposed. Numerical experiments, presented in the workshop, show it is effective compared to a standard solver. However, we believe that better results can be obtained by improving the smoothing heuristics.

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Extending and improving solution methods for the General Lot-sizing and Scheduling Problem for Parallel Production Lines (GLSPPL)

Herbert MEYR

1 Introduction

When producing standard products like consumer goods, usually a large number of final items has to be produced on several parallel, highly automated production lines, which are organized in a flow shop consisting of one to three production stages. Deterministic dynamic demand (usually demand forecasts because of a make-to-stock production) is to be met without back–logging. The production lines offer — at least partially — the same services and thus can be used alternatively. However, they do not have to be technically identical. Since commonly such production lines are expensive and highly utilized, they represent potential bottlenecks. Quite often, even in a multi-stage production system, only a single production stage (again consisting of several parallel lines) is a stationary bottleneck. This situation is assumed in the following.

The final items can be assigned to a few setup families. Changeovers between two items of the same family are not a problem and thus can be disregarded. Changeovers between two items of different setup families, however, may incur significant setup costs and setup times that are sequence dependent, in general. Because of the strong interdependencies between lot-sizes, -frequencies and -sequences and because of the limited capacities of the production lines, decisions about the sizes and the scheduling of production lots have to be taken simultaneously.

Meyr [4] proposed the General Lotsizing and Scheduling Problem for Parallel production Lines (GLSPPL) as an adequate model to represent such a situation. He uses the local search meta-heuristic "threshold accepting" to generate setup sequences and solves embedded generalized network flow problems (with losses and gains) by means of "dual reoptimization" in order to evaluate the setup patterns' corresponding inventory holding and production costs. The principle of dual re-optimization was introduced by [3] for heuristically solving the single line special case GLSPST of the GLSPPL. The idea is to take advantage of already known solutions by applying re-optimization and to use a dual optimization algorithm to recognize and refuse bad candidate solutions very early. In the single line case, ordinary network flow problems are solved using the dual network flow algorithm of [1], in the parallel line case the embedded generalized network flow problems with losses and gains are solved by an algorithm described in [2]. In the following, Meyr's single line heuristic combining Threshold Accepting with Dual Reoptimization will be called TADR, his Parallel Line heuristic will be called TAPL.

A recent case study with a company of health care industry (producing hygiene products like incontinence pads) has revealed two drawbacks of TAPL:

1. The company needs to solve the GLSPPL as part of its mid-term master planning. Due to the mid-term character of the problem and due to the productivity targets of the company production speeds may vary over time and thus the production coefficients of the GLSPPL get an additional time index. When trying to adapt the principle of dual re-optimization to this new situation, ordinary linear programs (LPs) have to be solved as embedded sub-problems instead of network flow problems by means of a dual LP solver.
2. Even when ignoring the dynamic character of the production speeds, the computation times of TAPL are quite high given the intended planning horizon and problem size. Thus a means to speed up computation times has to be found.

The presentation will show how these drawbacks have been tackled. To deal with (1.), the algorithm of [2] has been substituted by state-of-the-art LP solvers. Different solvers have been compared to gather experiences what price has to be paid (in terms of computation times) to replace specialized network flow algorithms by more general solvers.

Furthermore, because of (2.), an aggregation/decomposition (A/D)-approach has been developed to improve the scalability of TAPL. The basic idea is to aggregate the original GLSPPL problem instance and to solve the then smaller aggregate instance by TAPL in order to derive line-specific demands. By doing this, the original problem is decomposed into independent single line GLSPSTs, which can be solved by TADR. In case of infeasibilities, capacity-related data of the original problem are adapted and the overall procedure is iterated. As opposite to TAPL, the TADR implementation is very efficient. Thus the A/D approach appears very promising. Preliminary computational results, which demonstrate this, will be presented. As a next step it is planned to apply the A/D-approach to the models and methods developed for (1.) in order to completely cover the practical case.

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Discrete Lot-Sizing and Scheduling on Identical Parallel Machines

C. Gicquel, L. A. Wolsey and M. Minoux

1 Introduction

We consider the multi-item discrete lot-sizing and scheduling problem on identical parallel machines.

The case with one machine, known as the discrete lot-sizing and scheduling problem (DLSP), has the particularity that at most one item is produced per period, that production of that item (if any) is at full capacity and that start-up or changeover costs have to be incurred when switching production from one item to another.

We study here a variant of the DLSP in which there is a number of identical parallel resources available for production and propose an exact solution approach for this optimization problem based on the use of tight mixed-integer linear programming formulations.

2 Problem Formulations and Equivalence Results

We introduce two mixed-integer linear programming formulations of the problem:

- a "disaggregate" formulation where machine-specific binary variables are used to provide a detailed production plan for each production resource,
- an "aggregate" formulation where integer variables are used to describe a global production plan at the production system level.

We show that these two formulations are equivalent in terms of solution sets and optimal values both as integer and as linear programs. However, the aggregate formulation leads to a better computational efficiency thanks to its smaller size and the elimination of symmetry problems. Hence we focus in the sequel on two ways of strengthening the "aggregate" formulation.

3 Strengthening the Aggregate Formulation

First, we develop an approximate extended formulation based on the extended formulation proposed in [1]. The extended formulation is obtained by considering the production planning problem for each item as a shortest-path problem in a directed graph $G = (V, A)$ and by formulating this SPP as an integer program. Each node $v \in V$ represents a possible state of the production system and is described by a triple providing the planning period, the cumulative production of the item until this period and the current number of machines setup for the item. Arcs between nodes represent decisions on the number of machines to be used for the item during the following period. The proposed approximate extended formulation is obtained by aggregating nodes and arcs in G whenever the corresponding stock level exceeds some user specified value Δ.

Second we derive a new family of valid inequalities, that can be seen as an extension to the parallel-machine case of the valid inequalities proposed in [4] for the single-machine problem. The idea underlying these valid inequalities is to compute a lower bound on the inventory level of an item at the end of a planning period according to the forthcoming demands and the available production periods for this item.

4 Numerical results

We present the results of some computational experiments carried out to compare the effectiveness of the various mixed-integer programming formulations discussed in sections 2 and 3.

We randomly generated instances of the problem using a procedure adapted from that described in [2] for the multi-item DLSP with a single machine and used a standard MILP software (CPLEX 11.1) with the solver default settings to solve the problems with one of the following formulations:

- AGG: Aggregate formulation.
- AGG1: Aggregate formulation strengthened by the valid inequalities proposed in [3].

– AGG2: Aggregate formulation strengthened by the valid inequalities proposed in section 3.
– EXT: Extended formulation proposed by Eppen and Martin in [1].
– APP: Approximate extended formulation proposed in section 3.

Our results indicate that it is possible to solve instances with 10 or more items, up to 10 machines and up to 50 periods to optimality, and that for instances with up to 150 periods, solutions within 5% of optimality can be obtained in less than 10 minutes.

Moreover, they show that:

– Formulation EXT is the most efficient at involving instances involving a small to medium number of production resources.
– Formulation AGG2 is the most efficient at solving instances involving a large number of production resources.

5 Perspectives

Among the possible directions for further research, the question of whether the proposed solution procedures could be extended to the case of heterogenous resources would be worth investigating.

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A solving procedure for a general integrated lot-sizing and scheduling problem

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Abstract. We present a novel approach for solving an integrated lot-sizing and scheduling problem. These two decision levels are most of the time treated sequentially. Scheduling largely depends on the production quantities (lot sizes) computed at the production planning level, and ignoring scheduling constraints in planning leads to inconsistent decisions. A new mathematical formulation is proposed to determine a feasible optimal production plan for a fixed sequence of operations on the machines when setup costs and times are taken into account. Capacity constraints correspond to paths of the conjunctive graph associated to the sequence. An original Lagrangian relaxation approach is proposed to solve this problem in the single-level case. A lower bound is derived and an upper bound is calculated using a novel constructive heuristic. To solve the global lot-sizing and scheduling problem, two approaches were developed, based on Simulated Annealing and Tabu Search, and compared to an existing iterative approach. Numerical experiments are presented.

1 Introduction

At the lot-sizing level, the objective is to determine a production plan, i.e. production quantities for every period of the horizon, that satisfies demands and minimizes various costs (production costs, setup costs, holding costs, ...). These production quantities correspond to the sizes of the lots processed in the shop floor. At the scheduling level, these lots are sequenced on the resources. However, in practice and in theory, planning and scheduling decisions are still most often taken sequentially. Mathematical planning models take into account aggregate capacity constraints, and thus do not guarantee that the proposed production plan is feasible when it is forwarded to the scheduling level, i.e. that there exists a schedule which allows lots of the production plan to be produced on time (see [2]).

Lot-sizing problems have been treated extensively in the literature. However, most lot-sizing problems consider aggregate capacity constraints. Dauzère-Pérès and Lasserre [2] and Ouenniche [8] study the impact of sequencing decisions on a multi-item lot-sizing and scheduling problem. Small time-bucket lot-sizing problems consider short time periods and the sequencing of lots. A basic small-bucket problem is the Discrete Lot Sizing and scheduling Problem (DLSP) [5]. The main drawback of the DLSP is that only one item may be produced per period and, in that case, the production quantity is equivalent to using full capacity. This drawback is overcome in the Continuous Setup Lot Sizing Problem (CSLP), but still only one item per period may be produced. In the Proportional Lot Sizing and scheduling Problem (PLSP) [3], the remaining capacity in a given period is used for scheduling a second item. These models allow simultaneous lot sizing and scheduling but limit the number of products to be manufactured in one period. The General Lot Sizing and scheduling Problem (GLSP) [6] takes into account multiple products but features a single machine. An extension of these problems to multiple machines is proposed by Kimms [7] for the PLSP and Fandel and Stammen-Hegene [4] for the GLSP. Another class of problems considers lot sizing and scheduling with sequence-dependent setup costs and/or setup times.

2 Mathematical model and Lagrangian heuristic for a fixed sequence

We propose a new model where all paths of the conjunctive graph associated to a fixed sequence of operations of lots on machines are modeled. A conjunctive graph is a representation of a scheduling problem, where nodes correspond to operations and arcs to precedence constraints between two operations. Precedence constraints are between two successive operations in the routing of an item, or between two successive operations on a resource in the sequence. The makespan of a feasible schedule is equal to the length of the longest paths in the graph. In order to meet deadlines, the last operation of each path must be completed before its due date. Hence, the sum of processing and setup times of all operations in a path must not exceed the due date of the last operation of this path. And this must be true for all paths of the graph. These constraints can be seen as capacity constraints.

Lagrangian relaxation aims at decomposing an optimization problem into a number of easy-to-solve subproblems by dualizing some complicating constraints. Many lot-sizing problems have been solved using Lagrangian relaxation. Applying Lagrangian relaxation to solve our problem is justified by the fact that our mathematical model has an exponential number of capacity constraints, since these constraints correspond to paths of a conjunctive graph. These constraints are relaxed but, as their number is very large, it is not possible to consider them simultaneously. We thus initialize all multipliers to zero and choose among the set of the most violated constraints a set of multipliers to increase. This method allows us to only handle the most relevant constraints, and the number of these constraints is never very large in our numerical experiments.

Since there is no guarantee that the Lagrangian solution satisfies the relaxed capacity constraints, we designed a procedure which modifies the solution to satisfy the violated constraints. In classical lot-sizing problems, increasing or decreasing lot sizes in one period has no impact on the feasibility in other periods. Thus, most smoothing heuristics in Lagrangian relaxation approaches proposed in the literature successively transfer product quantities between lots in two different periods until a feasible solution is found. For each transfer, only the impacts on the two periods need to be measured. In our integrated problem, increasing or decreasing lot sizes in one period may influence the feasibility in all successive periods since the schedule considers lots on the entire horizon. Hence, we propose different and more complex smoothing heuristics.

3 Solving the integrated lot-sizing and scheduling problem

Using the Lagrangian heuristic developed for the lot-sizing problem with a fixed sequence of lots on the machines, several heuristics have been proposed to solve the integrated lot-sizing and scheduling problem. The first one is based on the iterative approach introduced in [1]. Two other heuristics are developed, based on Simulated Annealing and Tabu Search, that aim at improving both the sequence and the lot sizes. These two heuristics are better integrated to the Lagrangian heuristic, in particular information from the Lagrangian solution is used to improve the sequence.

4 Conclusions

An effective approach is proposed to solve a general integrated lot-sizing and scheduling problem. The numerical experiments, presented in the workshop, show that the proposed algorithms are effective. We are working on improving these heuristics, but also on extending the approach to consider multi-level lot sizing and additional constraints.

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Formulations and a Branch-and-Cut Algorithm for a Production-Distribution-Routing Problem

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Lotsizing problems have been extended in many directions to either include more operational details or to include other upstream (inbound logistics) or downstream (distribution) decisions in the supply chain. This latter direction results in more comprehensive formulations integrating various related problems in supply chain decision making. In a typical supply chain, we see that multiple products from multiple plants are shipped to different other plants, warehouses, shops and end customers. In many cases, research on production planning and transportation planning remained two totally separate areas and there was very little research on the integration of the two. This has changed and researchers started to recognize the value but also the challenges of integrating production and transportation planning.

We want to look at the integration of lotsizing with a specific form of transportation planning, namely the routing decisions. Here we have to determine in which order clients are visited when a vehicle plans its delivery route. The resulting problem is called the production-distribution-routing problem (PDRP), and is basically and integration of the classical lot sizing and inventory routing. We assume that both the plant and the vehicle have an unlimited capacity. Consequently, we have to design a production schedule for the uncapacitated plant, customer replenishment schedules, and a set of routes for a single uncapacitated vehicle starting and ending at the plant. The aim of the problem is to fulfill the demand of the customers over a finite horizon such that the total cost of distribution, setups and inventories is minimized. This paper introduces a basic mixed integer linear programming formulation and we propose several strong reformulations for the problem. Next, a branch-and-cut algorithm is developed to solve the formulations. Two families of valid inequalities are introduced to strengthen these formulations. In addition, an uncoordinated approach is considered to demonstrate the impact of the simultaneous optimization of production and distribution planning. Finally, a heuristic algorithm adapted from an inventory routing problem context is investigated. Computational results on a large set of randomly generated instances are presented. Instances with up to 40 customers and 15 time periods have been solved to optimality within a two-hour limit. The heuristic algorithm is able to find an excellent solution for each instance considered within a short amount of time.

LOWER BOUNDS FOR THE CAPACITY CONSTRAINED LOT SIZE PROBLEM WITH SETUP TIMES

Silvio Alexandre de Araujo, Zeger Degraeve and Raf Jans

Abstract. In [1] a branch-and-price algorithm for the Capacity Constrained Lot Size Problem with Setup Times is developed. The capacity constraints are the linking constraints and the problem decomposes into subproblems per item containing the demand and setup constraints. Column generation is accelerated by a combination of simplex and subgradient optimization for finding the dual prices. The computational results shown that branch-and-price is computationally tractable and competitive with other approaches found in the literature. They proved optimality to twenty eight percent of the instances from [4]. [3] present stronger lower bounds for the problem, applying decomposition to the network reformulation ([2]). The demand constraints are the linking constraints and the problem decomposes into subproblems per period containing the capacity and setup constraints. Computational results and a comparison to other lower bounds are presented and demonstrate that the lower bounds have good quality but the computational times are larger compared to the item decomposition. Due to the high computational times the method proposed by [3] has not been used in a branch-and-price algorithm. In this paper we study other methods to obtain high quality lower bounds in a small computational time. We investigate the simultaneous item and period decomposition for the original formulation and present some computational results; For the network reformulation, we improve a customized branch-and-bound algorithm proposed by [3] for solving the subproblems per period; We present a transformation of the network reformulation which simplifies and speeds up the updating of the Langrange multipliers and propose a new hybrid method to solve the master problem. Computational results are presented and show that the proposed methods are competitive with other state-of-the-art approaches found in the literature. Finally, we discuss some initial results on a branch-and-price algorithm.

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Fixed charge transportation on a path: the convex hull?

Mathieu Van Vyve

Abstract. The fixed-charge transportation problem is a basic problem in supply chain management. It is also both a special case and a strong relaxation of the big-bucket multi-item lot-sizing problem and a special case of the more general fixed charge network flow problem. This paper is a polyhedral study of the polynomially solvable case where depots and clients are arranged in an alternating manner, and can only serve/be served by their one or two immediate neighbours (FCTP). ¿From a graph perspective, the associated bipartite graph is a path. We give a $O(n^3)$-time optimization algorithm and a $O(n^2)$-size linear programming extended formulation. We describe a new class of inequalities that we call "path-modular" inequalities. We prove their validity and conjecture that they are sufficient to describe the convex hull of the feasible solutions to FCTP. We give a separation algorithm that runs in $O(n^3)$ time. We also present computational experiments tu support our conjecture and show the effectiveness of the path-modular inequalities.

1 Introduction

In the fixed charge transportation problem (FCT), we are given a set of depots $i \in I$, each with a quantity of available items C_i, and a set of clients $jinJ$, each with a maximum demand D_j. For each depot-client pair (i, j), both the unit profit $q_{i,j}$ of transporting one unit from t the depot to the client is known, together with the fixed charge $g_{i,j}$ of transportation along that arc. The goal is to find a profit-maximizing transportation program. Problem FCT can therefore be expressed as the following mixed-integer linear program:

$$
\begin{aligned}
\max \quad & \sum_{i \in I} \sum_{j \in J} (q_{i,j} w_{i,j} - g_{i,j} v_{i,j}), \\
& \sum_{j \in J} w_{i,j} \leq C_i, \qquad \forall i \in I \\
& \sum_{i \in I} w_{i,j} \leq D_j, \qquad \forall j \in J \\
& 0 \leq w_{i,j} \leq \min(C_i, D_j) v_{i,j}, \qquad \forall i \in I, j \in J, \\
& v \in \{0,1\}^{I \times J},
\end{aligned}
$$

where $w_{i,j}$ is a variable representing the amount transported from depot i to client j and $v_{i,j}$ is the associated binary setup variable.

In this description, the role of clients and depots are interchangeable. Indeed, this problem can be modelled as a bipartite graph where nodes are either depots or clients and edges between a depot and a client exist if the client can be served from that depot. A standard variant (and indeed, a special case) is when the demand of each client must be satisfied, in which case the unit profit is usually replaced by a unit cost.

Problem FCT is a relatively basic problem in supply chain management, and a special fixed-charge network flow problem. However surprisingly few polyhedral results are known for FCT. When there is only one client or one depot, FCT reduces to a single node flow set for which the (lifted) cover and reverse cover inequalities have been described and shown to be effective [PvRW85, vRW87, GNS99]. Note that this also implies that FCT is NP-Hard.

The flow structure of FCT is similar to the one of the Capacitated Facility Location (CFL) problem , but this last problem has fixed cost for transportation (edges) instead of for opening depots (nodes). Known valid inequalities are essentially flow cover type inequalities [Aar98, CFLP00].

FCT also appears to have stong ties to the multi-item big-bucket lot-sizing problem (BBLS). This actually constitutes the initial motivation of this paper. BBLS appears as a substructure in many production planning problems described in the literature, and has been tackeld as such by several authors [TTM89, MNS03, AM09]. We show in Section 2 that FCT is a special case of BBLS, but also that FCT is a strong relaxation of BBLS modelled using the facility location reformulation [KB77]. .

In this paper, we study the polynomially solvable special case of FCT where depots and clients are arranged in an alternating manner, and can only serve/be served by their one or two immediate neighbours (FCTP). ¿From a graph perspective, the associated bipartite graph is a path. This special case FCTP is also a relaxation to FCT, and it can therefore be hoped that polyhedral results for FCTP will be helpful in solving the general case.

For FCTP, we assume we have $n+1$ depots and clients (or nodes) in total, grouped in the set $N_0 = \{0, 1, \ldots, n\}$. We will index depot-client pairs $(i-1, i)$ (or edges) by $i \in N = N_0 \setminus \{0\}$. Depots might be seen as represented by even nodes while clients are represented by odd nodes.

The problem FCTP can be formulated as the following mixed-integer program:

$$
\begin{aligned}
\max \qquad & \sum_{i=1}^{n} p_j x_j - \sum_{i=1}^{n} f_j y_j, \\
& x_1 \le a_0, \\
& x_i + x_{i+1} \le a_i, \qquad \forall i \in N \setminus \{n\}, \\
& x_n \le a_n, \\
& 0 \le x_i \le \min(a_{i-1}, a_i) y_i, \qquad \forall i \in N, \\
& y \in \{0,1\}^n,
\end{aligned}
$$

where x_i is the amount transported between $i-1$ and i, y_i is the setup variable associated with x_i, and p_i and f_i are respectively the unit profit and the fixed cost of transportation between $i-1$ and i.

In Section 3 we study the max-flow variant of FCTP (no fixed cost and a profit of 1 for all depot-item pairs). In particular, we give a simple algorithm and prove basic properties of its optimal solution that will be useful in the next sections.

In Section 4 we characterize extreme points of X^{FCTP}. This directly leads to

- a $O(n^3)$ dynamic program for solving FCTP, together with
- a linear-programming extended formulation of size $O(n^2)$ nonnegative variables, $O(n^2)$ constraints and $O(n^2)$ non-zero coefficients (Section 5).

In Section 6, we introduce a new class of valid inequalities that we call the "path-modular inequalities" and conjecture that they are sufficient to describe the convex hull of FCTP. We also give an $O(n^3)$ separation algorithm.

We finally report on computational experiments in Section 7 before discussing future research on the topic.

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Lot-sizing by MIP: Some Open Problems and Recent Results

Laurence Wolsey

Abstract. We discuss what we consider to be some of the important and/or intriguing open problems both from a practical and theoretical level, and point out what results are available to date.

From a computational viewpoint "big bucket" capacity constraints, sequence-dependent changeovers and multi-level supply chains pose major challenges. Among others, we discuss some recent work (with R. Melo) on a basic two-level production/transportation model.

On the theoretical and modeling side, most variants of constant-capacity single-item lot-sizing are polynomially solvable, so the hope is to find tight convex hull descriptions in either the original or an extended space. We consider among others the variants with a) start-ups b) lost sales, c) stock bounds and fixed costs and d) multiple machines. (work with M. Di Summa and M. Conforti)

1 Introduction

Below we indicate briefly a couple of the problems, involving both "easy" and "hard" models that we plan to discuss.

2 Hard Problems

2.1 Big Bucket Constraints

One major choice in building a multi-item lot-sizing model is the choice of "big" or "small" time buckets. What is well-known, but rarely discussed is the fact that most models with "big bucket" capacity constraints can be made very difficult just by appropriately tightening the capacity constraints. The only inequalities treating these constraints are due to Miller [6] and Akartunali and Miller [1], but computationally they are relatively ineffective. It seems that big bucket constraints act globally across the time horizon creating highly combinatorial problems.

2.2 Sequence-Dependent Changeovers

For small bucket problems in which a machine just produces one or two items per period, it is natural to model the changeovers as paths in a simple network. As shown in [2], this allows one to use results on reformulations with start-up variables. See Gicquel [4] for some recent extensions. However there are no valid inequalities that explicitly deal with the sequence-dependent changeovers.

3 Single-Item Polynomially-Solvable Problems

3.1 Start-Up Variables

Constantino [3] developed a variety of valid inequalities and separation algorithms for the constant- and varying capacity Wagner-Whitin relaxation with start-up variables, $X^{WW-CC-SC} =$

$$\{(s,y,z) \in R^n_+ \times \{0,1\}^n \times \{0,1\}^n : s_{k-1} + \sum_{u=k}^{t} C_u y_u \geq \sum_{u=k}^{t} d_u \;\; 1 \leq k \leq t \leq n; \;\; y_t \geq z_t \geq y_t - y_{t-1} \;\; 1 \leq t \leq n\}$$

For fixed k, one has the discrete lot-sizing set with start-up variables, denoted $X_k^{DLSI-CC-SC}$. Clearly $X^{WW-CC-SC} = \bigcap_{k=1}^n X_k^{DLSI-CC-SC}$.

Immediate question are:
i) Does $\text{conv}(X^{WW-CC-SC}) = \bigcap_{k=1}^n \text{conv}(X_k^{DLSI-CC-SC})$?
ii) The inequalities of Constantino are very effective, but they do not give a complete description of the convex hull. However in the constant capacity case, the missing inequalities closely resemble those of Constantino, so can a complete description be found?

3.2 Lost Sales

Consider the single-item constant capacity discrete lot-sizing problem with sales

$$\{(s_0, v, y) \in R^1_+ \times R^n \times \{0,1\}^n : s_0 + C\sum_{j=1}^{t} y_j \geq \sum_{j=1}^{t}(d_j + v_j).\ v_t \leq u_t \quad 1 \ \leq t \leq n, \}$$

For the uncapacitated case, Loparic et al. [5] provide both an inequality description of conv($X^{WW-U-SL}$) and an extended v when $d \in R^n_+$. Recently Conforti et al. have shown that in the constant capacity case, an extended formulation for conv($X^{DLSI-CC-SL}$) is obtained as an intersection of an exponential number of mixing sets, and optimization can be solved by solving a linear program over the intersection of $n+1$ of these sets.

Here there are at least two open questions:
i) Does conv($X^{WW-CC-SL}$) $= \bigcap_{k=1}^{n}$ conv($X_k^{DLSI-CC-SL}$)?
ii) Is there a combinatorial polynomial algorithm for separation over conv($X_k^{DLSI-CC-SL}$)? This question is even open for the uncapacitated case when $d \notin R^n_+$.

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Dynamic capacitated lot sizing with random demand and a service level constraint

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Abstract. We present a stochastic version of the single-level, multi-product dynamic lot sizing problem subject to a capacity constraint. A production schedule has to be determined based on demand forecasts so that expected costs are minimized and a given gamma service level is met. This leads to a non-linear model that is approximated by a second linear model. In the latter model a scenario approach is used and expected values of random variables are replaced by sample averages, thus yielding a mixed-integer linear program. This program can be solved to determine an efficient, robust and stable production schedule in the presence of uncertain and dynamic demand. A numerical analysis of synthetic problem instances shows under which conditions precise demand forecasts are particularly useful from a production scheduling perspective.

1 Introduction

From a practical point of view, all deterministic lot sizing models and algorithms may be of questionable value for a real-world decision maker who faces an uncertain demand. It may be tempting to build a production schedule based on point forecasts, i. e., to ignore all the uncertainty and to determine lot sizes using a deterministic lot sizing model, e.g., the CLSP, see [2]. It is our impression that very often practitioners treat point forecasts as if they represented deterministic quantities. In addition, the overwhelming majority of the research papers on dynamic lot sizing under capacity constraints ignores demand randomness. However, if demand is uncertain, this approach leaves the management without any control of the service this system provides to the customers. In order to gain this control, demand randomness must be reflected explicitly within the lot sizing model and the service level associated with a production schedule needs to be quantified.

2 Problem statement

In a single-level production environment K product types are manufactured all requiring the same capacitated production system. The capacity can be extended by overtime. The production of a product type leads to setup times and costs in each period the resource is set up for the respective product. The processing time per unit is given and independent of the production quantity. The holding costs are proportional to the inventory at the end of a period. The expected demand has to be fulfilled at the end of the planning horizon while temporary backlogging is allowed. The objective is to minimize the total sum of setup, inventory, and overtime costs.

We consider a service level constraint over the complete planning horizon. We use a gamma-service level to reflect not only the size of the back-orders, but also the waiting time of the customer. However, we modify the standard definition of the gamma-service level, see [3],

$$\gamma = 1 - \frac{\text{expected backlog}}{\text{expected demand}} \tag{1}$$

using the following modified measure

$$\gamma = 1 - \frac{\text{total expected backlog}}{\text{total maximum expected backlog}}. \tag{2}$$

This measure is defined for the complete planning horizon. The backlog is related to the maximum possible backlog, if nothing is produced, so that using this measure we always find $0 \leq \gamma \leq 1$.

3 Formulation of a non-linear model based on random variables

The Stochastic Capacitated Lot Sizing Problem (SCLSP) based on random variables can be stated as a non-linear model using the notation in Table 1.

Model SCLSP

Table 1 Notation used for model SCLSP

Indices and index sets:	
$\mathcal{K}$	set of products ($k \in \{1, \ldots, K\}$)
$\mathcal{T}$	set of periods ($t \in \{1, \ldots, T\}$)
Deterministic parameters:	
b_t	available capacity in period t
oc	overtime cost per unit of overtime
hc_k	holding cost of product k per unit and period
sc_k	setup cost of product k
tb_k	production time per unit of product k
ts_k	setup time of product k
M	big number
γ_k	minimum gamma service level of product k
Random variables:	
BL_{kt}	backlog of product k in period t
D_{kt}	demand of product k in period t
Y_{kt}	net inventory of product k at the end of period t
YP_{kt}	physical inventory of product k at the end of period t
Decision variables:	
o_t	overtime at resource j in period t
q_{kt}	production quantity (lot size) of product k in period t
x_{kt}	binary setup variable of product k in period t

$$\min Z = \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} hc_k \cdot \mathrm{E}[YP_{kt}] + \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} sc_k \cdot x_{kt} + \sum_{t\in\mathcal{T}} oc \cdot o_t \tag{3}$$

subject to:

$$\sum_{k\in\mathcal{K}} (ts_k \cdot x_{kt} + tb_k \cdot q_{kt}) \leq b_t + o_t, \qquad \forall t \tag{4}$$

$$q_{kt} - M \cdot x_{kt} \leq 0, \qquad \forall k,\, t \tag{5}$$

$$Y_{k,t-1} + q_{kt} - Y_{kt} = D_{kt}, \qquad \forall k,\, t \tag{6}$$

$$YP_{k,t} = \max(0, Y_{k,t}), \qquad \forall k,\, t \tag{7}$$

$$BL_{k,t} = \max(0, -Y_{k,t}), \qquad \forall k,\, t \tag{8}$$

$$\sum_{t\in\mathcal{T}} q_{kt} \geq \sum_{t\in\mathcal{T}} \mathrm{E}[D_{kt}], \qquad \forall k \tag{9}$$

$$\sum_{t\in\mathcal{T}} \mathrm{E}[BL_{kt}] \leq (1-\gamma_k) \sum_{t\in\mathcal{T}} (T - t + 1) \cdot E[D_{kt}], \qquad \forall k \tag{10}$$

The objective function (3) minimizes the total sum of the setup, holding, and overtime costs. Inequalities (4) ensure that production quantities and setups meet the capacity constraints in each period. Inequalities (5) link the production variables to the setup variables, i. e., if product k is produced in period t, the machine has to be set up for this product. Constraints (6) are the inventory balance equations. Equations (7) determine the physical inventory. In the case of negative net inventory, a backlog occurs, see equations (8). The equations (9) guarantee that the expected total demand is satisfied at the end of the planning horizon. Inequalities (10) are the service level constraints. Note that the SCLSP is a non-linear problem as the expected backlog is a non-linear function of the production quantities.

4 Outline of the algorithmic approach

As no solution approach is known to solve the SCLSP, we approximate the non-linear problem by a deterministic linear mixed-integer problem using scenarios. The idea is to approximate the expected values of the demand $\mathrm{E}[D_{kt}]$ by sample averages. Each scenario s contains a independent realization d_{kts} of the random variable D_{kt}. The randomness in the problem can hence be approximated by a very large number of independent scenarios.

Our solution approach consists of two parts. In the first part, we determine a robust setup pattern. To derive this setup pattern, three different possibilities exist:

- Solving the standard CLSP without backlogging
- Solving the standard CLSP with backlogging extended and service level constraints
- Solving the scenario-based model formulation of the CLSP with a small number of scenarios

The respective problem instance can be solved exactly by CPLEX or by an efficient heuristic, e. g. the Fix&Optimize-heuristic, see [1]. In the second part, the setup-pattern is fixed and only the production quantities are adjusted to aim at the desired service level. In this part of the solution approach, we only solve the scenario-based CLSP with a now larger number of scenarios than in the first part. This can be done efficiently as the fixed setup-pattern yields a linear program with respect to the remaining decision variables. A larger simulation with 10.000 scenarios is finally used to evaluate the determined production plan. If necessary, the production plan is adjusted with a marginally changed service level.

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Robust Inventory Routing under Demand Uncertainty

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1 Extended Abstract

The inventory routing problem (IRP), an important problem arising in vendor-managed inventory systems where the supplier (vendor) is responsible for the management of inventories at the customers, can be defined as the problem of determining the delivery times, delivery quantities and delivery routes to a set of geographically dispersed customers served by a supplier. The IRP is basically an integration of lot sizing and vehicle routing problems.

There exist several variants of the IRP depending mainly on the nature of the demand at customers (deterministic vs. stochastic) and on the length of the planning horizon (finite vs. infinite). For a recent review on the IRP, one can refer to Andersson et al. [4].

Demand is widely accepted to be dynamic and stochastic in real life inventory routing problems [8]. Many studies consider the IRP with dynamic deterministic demand, which leads to more tractable yet less realistic models compared to those with stochastic demand. On the other hand, stochastic IRP models are intractable in that only very small instances can be solved to optimality [9] and therefore several heuristics have been proposed for their resolution. Studies on stochastic IRPs also assume full knowledge of the probability distribution of demand, which may be unavailable or difficult to obtain. There is clearly a need to consider the IRP with dynamic stochastic demand in a tractable way, where no information for the probability distribution of demand is required.

In this study, we introduce a single product multi-period finite horizon IRP with dynamic stochastic demands at customers, where a polyhedral (interval) demand uncertainty structure with no specific probability distribution is considered. The supplier holds an unlimited amount of the product to replenish the customers, and backlogging (i.e. not meeting demand on time) of demand at customers is allowed. Although most distribution management problems involve multiple vehicles, for simplicity and because this study is the first to address such a complex problem, we consider a single vehicle for the distribution of the product. The vehicle can visit all customers in a period, provided that the total amount shipped to the visited customers does not exceed its capacity. We make use of recently developed robust optimization approaches in addressing demand uncertainty in a tractable way. The problem, referred to as the robust inventory routing problem (RIRP), is to decide on the delivery times and quantities to customers as well as delivery routes such that whatever value the demands take within their supports, the solution remains feasible and the total cost composed of transportation, inventory holding and backlogging costs is minimized. The RIRP is obviously NP-hard since it includes the classical traveling salesman problem as a special case.

Robust optimization has emerged as a powerful methodology for problems involving uncertain parameters with no information on their probability distributions. This is achieved by finding the best solution (often called minimax solution) which ensures feasibility regardless of the realized values of uncertain parameters. We use the approach developed by [6, 7], called the "budget of uncertainty" approach, which controls the level of conservativeness by allowing only some of the uncertain parameters to deviate from their nominal values simultaneously. This approach has the desirable property that the robust counterpart preserves the complexity of its nominal (i.e. without uncertainty) problem (e.g. if the nominal problem is an LP problem, the robust counterpart is also an LP problem), and thus this approach naturally extends to discrete optimization problems unlike other robust optimization approaches.

To the best of our knowledge, the paper by Aghezzaf [3] is the only study that incorporates robustness into an IRP. In contrast to the dynamic demands with an ambiguous probability distribution considered in this paper, Aghezzaf [3] assumes normally distributed stationary demands at the customers, and travel times with constant averages and bounded standard deviations. He only considers a cyclic distribution strategy, which is theoretically not optimal but claimed to be a good approximation, and he develops a nonlinear mixed integer programming (MIP) formulation to find a minimum cost solution that is feasible for all possible realizations of demands and travel times within their supports.

The RIRP is also closely related to the IRP studied by Abdelmaguid and Dessouky [1], and Abdelmaguid et al. [2], where deterministic demands, storage capacity limits at the customers and multiple vehicles are considered unlike what is done in the RIRP. Abdelmaguid and Dessouky [1] propose a genetic algorithm, while Abdelmaguid et al. [2] develop construction and improvement heuristics for the problem. Both studies present an MIP formulation of the problem, which is a combination of standard inventory balance equations for the lot sizing decisions of the customers and a multi-commodity flow based formulation for the routing decisions. The upper and lower bounds obtained by solving the MIP formulation within a given time limit are used as benchmarks to measure the quality of the heuristics. The MIP formulation proposed could only solve very small instances to optimality using an off-the-shelf optimization solver. The first exact IRP algorithm, a branch-and-cut algorithm, is proposed by Archetti et al. [5] for a related IRP in which deterministic demands, order-up-to level policy and no backlogging at the customers, and a single vehicle are considered. Solyalı and Süral [12] improve the results of [5] by proposing a strong MIP formulation within a branch-and-cut algorithm. These authors also develop an effective MIP based heuristic for the problem. Both [5] and [12] use a computationally attractive two-index vehicle flow formulation for the routing decisions. They differ in the formulation used for the lot sizing decisions of the customers: the former uses the standard inventory balance equations whereas the latter uses a strong shortest path formulation.

Let alone the fact that there does not yet exist any good formulation, even for the nominal (i.e. without uncertainty) case of the RIRP, the incorporation of robustness into the MIP formulations yields weaker formulations, as observed in [6]. Thus, it is crucial to develop a strong formulation for the exact solution of the RIRP. To this end, considering the RIRP as a combination of the lot sizing problem of the customers and the vehicle routing problem, we propose a new MIP formulation using effective mathematical programming representations for both. All existing studies in the IRP literature, except [12], consider standard inventory balance equations to model the corresponding lot sizing problem (see e.g. [1, 2]) which provide a weak link between the replenishment and the routing decisions, and thus a weak lower bound. The lot sizing problem of each customer can be seen as the uncertain version of the deterministic demand uncapacitated lot sizing problem with backlogging (ULSB), for which tight reformulations are known [11]. We use the facility location reformulation which defines the convex hull of feasible solutions of the lot sizing problem of each customer in the case of deterministic demand. For the vehicle routing problem, we use a two-index vehicle flow formulation which is one of the most computationally attractive formulations for the vehicle routing problem [10] and has been successfully applied to a deterministic IRP with order-up-to level policy [5, 12]. We first adapt the resulting formulation to the nominal case of the RIRP. Then, using the "budget of uncertainty" robustness approach of [6, 7], we formulate the RIRP as a tractable MIP formulation with slightly more constraints and variables than the formulation for the nominal case. Modifying the MIP formulation for the nominal case, we obtain a variation of that formulation which we use in developing another robust MIP formulation for the RIRP. The new robust formulation is indeed a nominal formulation with modified demands. We implement all these formulations through a branch-and-cut algorithm.

Computational results on instances adapted from the literature have revealed that our robust formulations can solve to optimality instances with up to 30 customers and seven periods within reasonable times. The robust solutions obtained provide immunization against uncertainty with a slight increase in total cost compared to the nominal case, especially when the average daily demand over vehicle capacity ratio is low, whereas the price of robustness is larger when the average daily demand over vehicle capacity ratio is high. The "budget of uncertainty" based robust formulation generally yields slightly better objective values than the robust formulation with modified demands, whereas the latter is faster than the former. Moreover, computational results on instances generated by [2] for the nominal case (i.e. a deterministic IRP with backlogging) show the superiority of our formulation and of our branch-and-cut algorithm over the MIP formulation of [1] and [2]. Specifically, our strong formulation within the branch-and-cut algorithm for the nominal case is able to optimally solve instances six times larger than the only previously available MIP formulation [1, 2].

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A Column Generation Heuristic for Dynamic Capacitated Lot Sizing with Random Demand under a Fillrate Constraint

Horst Tempelmeier

Abstract. This paper deals with the dynamic multi-item capacitated lot-sizing problem under random period demands (SCLSP). Unfilled demands are backordered and a fillrate constraint is in effect. It is assumed that, according to the static-uncertainty strategy of [1], all decisions concerning the time and the production quantities are made in advance for the entire planning horizon regardless of the realization of the demands. The problem is approximated with the set partitioning model and a heuristic solution procedure that combines column generation and the recently developed ABC$_\beta$ heuristic is proposed.

1 Introduction

We consider the stochastic version of the dynamic multi-item capacitated lot sizing problem (CLSP). The problem is to determine production quantities to satisfy demands for multiple products over a finite discrete time horizon such that the sum of setup and holding costs is minimized, whereby a capacity constraint of a resource must be taken into consideration. In contrast to the deterministic CLSP, we assume that for every product k and period t the demand is a random variable D_{kt} $(k = 1, 2, \ldots, K; t = 1, 2, \ldots, T)$. The period demands are non-stationary, which usually is the case in a material requirements planning (MRP) based environment. Demand that cannot be filled immediately from stock on hand is backordered. As the precise quantification of shortage penalty costs which involve intangible factors such as loss of customer goodwill is very difficult, if not impossible, we assume that management has specified a target service level. In particular, we assume that the fillrate criterion (β service level) is in effect, as this criterion is very popular in industrial practice.

We assume that, according to the static-uncertainty strategy of [1], all decisions concerning the time and the production quantities are made in advance for the entire planning horizon, whereby the limited capacity of the resource is respected. This strategy has the virtue that once a feasible production plan has been fixed, the production quantities remain stable regardless of the realization of the demands. Hence, the capacity will be met with certainty.

2 Problem Formulation

Consider K products that are produced to stock on a single resource. The planning situation is completely identical with that assumed in the classical dynamic capacitated lot sizing problem (CLSP) with one exception: For each product k and each time period t, the period demands D_{kt} are random variables with a known probability distribution and given period-specific expected values $E\{D_{kt}\}$ and variances $V\{D_{kt}\}$. These data, which in a dynamic planning environment vary over time, are the outcome of a forecasting procedure. Unfilled demands are backordered and the amount of backorders is controlled by imposing a fillrate per cycle constraint. We define the fillrate per cycle as the ratio of the expected demand observed during the coverage time of a production order that is routinely filled from available stock on hand and the actual lot size. More precisely, let τ be a production period of product k, let t be the last period before the next production of product k takes place and let $q_{k\tau}$ be the lot size produced in period τ covering the demand up to period t. Finally, let $F_{ki}(q_{k\tau})$ be the backorders of product k that occur during period i. Then for a target service level $\beta_k^\star$ it is required, that

$$1 - \frac{E\left\{ \sum_{i=\tau}^{t} F_{ki}(q_{k\tau}) \right\}}{E\left\{ \sum_{i=\tau}^{t} D_{ki} \right\}} \geq \beta_k^\star \qquad K = 1, 2, \ldots, K \quad (1)$$

This constraint is equivalent to the fillrate definition under stationary conditions which relates the average backorders per cycle to the average replenishment quantity. However, it is a sharper requirement, as not only in the long run, but also in each production cycle the fillrate target must be met. At the beginning of the planning horizon there is a known initial inventory I_{k0} $(k = 1, 2, \ldots, K)$.

A formulation of the stochastic dynamic capacitated lot sizing problem with product-specific fillrate constraints as a non-linear optimization problem is presented in [3]. However, as originally proposed for a variant of the deterministic CLSP by [2], the problem can be approximated by a set partitioning model as follows. Define for each product P_k $(k = 1, 2, \ldots, K)$ alternative production plans over the planning horizon T. Each production plan n is composed of lot sizes that cover an integer number of period demands under consideration of the fillrate constraint.

With a given production plan n of product k, the expected total setup and holding costs c_{kn} and the exact capacity requirements κ_{knt} in period t $(k = 1, 2, \ldots, k; n = 1, 2, \ldots, P_k; t = 1, 2, \ldots, T)$ can be determined. The problem is then to select for each product exactly one production plan alternative such that in all periods the capacity constraint is respected. In the following we assume that a single resource with period capacities b_t $(t = 1, 2, \ldots, T)$ is used. The resulting set partitioning model formulation is then:

Model SCLSP$_{\mathbf{SPP}}$

$$\text{Minimize } Z = \sum_{k=1}^{K} \sum_{n=1}^{P_k} c_{kn} \cdot \gamma_{kn} \tag{2}$$

s. t.

$$\sum_{k=1}^{K} \sum_{n=1}^{P_k} \kappa_{knt} \cdot \gamma_{kn} \leq b_t \qquad t = 1, 2, \ldots, T \quad (\pi_t) \tag{3}$$

$$\sum_{n=1}^{P_k} \gamma_{kn} = 1 \qquad k = 1, 2, \ldots, K \quad (\sigma_k) \tag{4}$$

$$\gamma_{kn} \geq 0 \qquad k = 1, 2, \ldots, K; n = 1, 2, \ldots, P_k \tag{5}$$

The objective function minimizes the sum of the expected costs of all selected production plans. γ_{kn} is a binary variable that selects production plan n of product k. Constraint (3) ensures that the period capacity of the resource in period t is respected, whereby κ_{knt} is the capacity requirement resulting from production plan n of product k in period t. Equation (4) states that for each product exactly one production plan must be selected. The coefficient c_{kn} which represents the expected costs of production plan n of product k is the result of an embedded optimisation problem, which for each setup period determines the minimum lot size required to achieve the target service level at the end of the associated production cycle.

Assume that for production plan n of product k a setup pattern is given with J_n setups in periods τ_j $(j = 1, 2, \ldots, J_n)$. Then for each setup j the problem is to find the minimum lot size q_j, that respects the service level constraint for the current production cycle. This problem can be stated as follows:

Model MINQ$_j$

$$\text{Minimize } q_j \tag{6}$$

s. t.

$$1 - \frac{E\left\{ \displaystyle\sum_{i=\tau_j}^{\tau_{j+1}-1} F_{ni}(q_j) \right\}}{E\left\{ \displaystyle\sum_{i=\tau_j}^{\tau_{j+1}-1} D_{ki} \right\}} \geq \beta_k^\star \tag{7}$$

$(k = 1, 2, \ldots, K; n = 1, 2, \ldots, P_k; j = 1, 2, \ldots, J_n)$ with $\tau_{J_n+1} = T + 1$. For a given setup pattern with J_n setups and associated lot sizes the expected costs c_{kn} of production plan n of product k can be easily computed.

3 Solution Approach

3.1 General Structure

With respect to the number of production plans considered we follow the same approach as the set partitioning based solution procedures proposed for the deterministic CLSP, i. e. we consider only production plans with integer numbers of demand periods covered by any production lot. This is a simplifying assumption, as with capacity constraints the optimum solution may comprise a production lot that covers less than the full demand of a period. It is well-known that even with this simplifying assumption the number of production plans becomes prohibitively large even for small problem instances. Therefore, generating all production plans in advance is only feasible for very small problem instances. However, as Model $\text{SCLSP}_{\text{SPP}}$ has the same formal structure as its deterministic counterpart, we propose to use a column generation approach that generates the candidate production plans as required. In the current problem the set partitioning problem serves as the master problem. The corresponding subproblem comprises K product-specific uncapacitated dynamic lot sizing problems with random demand and a fillrate per cycle constraint. These are solved with the exact solution procedure of [4].

In each iteration, first the LP relaxation of the master problem is solved which provides actual dual variables σ_k and π_t $(k = 1, 2, \ldots, K; t = 1, 2, \ldots, T)$. These are used to modify the objective function of the subproblems, which are subsequently solved to optimality. The new production plans generated in this way are introduced into the master problem, only if their reduced costs are negative. The procedure is repeated until no more schedules that improve the objective function of the master problem can be generated. Finally, all production plans with integer selection variables γ_{kn} in the optimal solution of the last specification of the LP relaxation of model $\text{SCLSP}_{\text{SPP}}$ are fixed and their capacity requirements are subtracted from the available period capacities. For the remaining products and the residual capacities, the resulting stochastic CSLP is solved with the help of the ABC_β heuristic proposed by [3].

3.2 Solution of the Subproblems

A subproblem for product k can be cast as a shortest-path problem with $T+1$ nodes labeled $(1, 2, \ldots, T+1)$. An edge originating at node τ and ending at node j specifies that the inventory on hand after production in period τ covers the demands from period τ to $j-1$ to the extent dictated by the target fillrate. The next setup is then scheduled for period j.

According to the general structure of a column generation procedure that uses an LP relaxation of model $\text{SCLSP}_{\text{SPP}}$, the costs associated to an edge starting at node τ and ending at node j are given as

$$c_{\tau j} = E\{C_{\tau j}(P_\tau^{\text{opt}})\} - \pi_\tau \cdot tb_k \cdot q_{\tau j}, \tag{8}$$

where tb_k denotes the capacity requirements for one unit of product k. The term $E\{C_{\tau j}(\cdot)\}$ represents the corresponding expected setup and holding costs that occur when the production (setup) in period τ covers the demand up to period $(j-1)$. As the production quantity required in period τ to guarantee the target service level depends on the net inventory at the beginning of period τ, which in turn is influenced by the optimum production plan up to that period (denoted as P_τ^{opt}), the costs cannot be computed in advance (as in the deterministic counterpart of the problem) but must be calculated during the solution procedure. The details of the solution procedure for this problem are given in [4].

Let $\bar{c}_k^{\text{opt}}$ denote the objective value of the optimum solution of the shortest path problem for product k. Then the reduced cost of this optimum production plan for product k are $\bar{c}_k = \bar{c}_k^{\text{opt}} - \sigma_k$. If $\bar{c}_k < 0$, the current production plan for product k is added to the set partitioning model. Once for each product a subproblem has been solved, the next instance of model $\text{SCLSP}_{\text{SPP}}$ is generated and its LP-relaxation is solved. The optimum solution provides new values of the dual variables π_t and σ_k which are then used to generate new product-specific subproblems. The procedure ends when no new production plans are generated.

At this point, all production plans with integer γ_{kn}-variables are fixed and their capacity requirements are subtracted from the available period capacities. For the remaining products with fractional γ_{kn}-variables and the residual period capacities the heuristic ABC_β procedure proposed by [3] is applied.

4 Numerical Results

In order to test the quality of the proposed heuristic, we generated problem instances assuming regular demands with up to 20 periods and up to 70 products. The period demands were assumed to be normally distributed. The parameters of the data set are shown in Table 1.

For each combination of the parameters T, K, $\beta_k^\star$ and 'Capacity' ten replications were generated, whereby the expected demands per product and period were drawn from a continuous uniform distribution. For each product, the coefficient of variation (used for the complete demand series of this product) was selected from a discrete uniform distribution with the possible outcomes $\{0.15, 0.2, 0.25, 0.3, 0.35\}$. The TBO-values of the products were randomly drawn from the discrete uniform distribution with the possible outcomes $\{1, 5, 10\}$. The holding costs of product k were calculated as $h_k = \frac{2 \cdot s}{\overline{d_k} \cdot \mathrm{TBO}_k^2}$ where $\overline{d_k}$ is the average demand per period and the setup costs were $s = 500$ for all products. The period capacity of the resource was calculated as follows. First, the average workload $w = \sum_{k=1}^{K} \overline{d}_k$ was computed. According to the considered workload scenario {low, medium, high}, the capacity b_t was then set as $b_t^{\mathrm{low}} = 1.10 \cdot w$, $b_t^{\mathrm{medium}} = 1.50 \cdot w$, and $b_t^{\mathrm{high}} = 2 \cdot w$ $(t = 1, 2, \ldots, T)$. Depending on the target fill rate, the resulting utilizations of the solved problem instances spanned between 47% and 97%. In addition, some combinations of fill rate and capacity resulted in problem instances with capacity over-utilization for which consequently no solution was found. In total, 3240 individual problem instances were generated.

Each problem instance was solved with the proposed column generation heuristic (combined with the ABC_β heuristic for the remaining problem, as described above) and with the ABC_β heuristic of [4] alone. For the latter, we used the parameter combination SH/SM/E which means that the products were sorted according to the setup costs over the holding costs ratio, the cost criterion was the Silver-Meal criterion and the east direction was used. [4] found that this parameter combination generated good solutions compared to the other available parameter combinations.

For 2804 problem instances a feasible solution could be found with both heuristics. The ABC_β heuristic solved 148 problem instances which could not be solved with the column generation heuristic.

For 97.95% of all solved problem instances the proposed column generation heuristic found the best solution. For these problems, the solution quality of the column generation heuristic was on the average 25.14% better than the solution found with the ABC_β heuristic. Table 2 shows the relative cost increase of the ABC_β heuristic compared to the solution found with the column generation (CG) heuristic, i. e. the ratio $\left(\frac{ABC_\beta}{CG} - 1\right)$.

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Number of periods, T	$\{10, 20\}$
Number of products, K	$\{10, 40, 70\}$
Fill rate, $\beta_k^\star$	$\{0.5, 0.6, \ldots, 0.9, 0.98\}$
Capacity	{low, medium, high}
Mean period demand	continuous uniform $U(0, 100)$
Coefficient of variation	discrete uniform $U(0.15, 0.2, 0.25, 0.3, 0.35)$
Time-between-orders, TBO	discrete uniform $U(1, 5, 10)$

Table 1 Parameters used

		Average cost increase of ABC_β		Total
	Capacity	$T = 10$	$T = 20$	
$K = 10$	1.10$\cdot w$	15.91%	14.96%	15.44%
	1.50$\cdot w$	12.53%	9.30%	10.91%
	2.00$\cdot w$	10.77%	9.05%	9.91%
$K = 40$	1.10$\cdot w$	45.74%	39.71%	42.73%
	1.50$\cdot w$	29.00%	23.56%	26.28%
	2.00$\cdot w$	19.58%	16.28%	17.93%
$K = 70$	1.10$\cdot w$	51.78%	51.72%	51.75%
	1.50$\cdot w$	33.20%	29.34%	31.27%
	2.00$\cdot w$	21.47%	18.65%	20.06%
				25.14%

Table 2 Results

Capacitated lot-sizing: Synchronization issues for multi-stage systems

Christian Almeder and Bernardo Almada-Lobo

1 Extending the multi-level capacitated lot-sizing problem (MLCLSP)

The majority of the models and algorithms proposed for the MLCLSP rely on one of the following two assumptions: either lead times are neglected, thus allowing predecessors and successors to be produced in the same period; or lead times account for at least one period for each component, forcing the throughput time (in number of periods) of the finished products to be at least equal to the number of levels of the bill-of-materials (BOM). Buschkühl at al. [1] report in their review that out of 16 papers dealing with metaheuristic solutions to MLCLSP, only one study considers lead time, while all others neglect it. The present work is motivated by the observation that the majority of the solutions published for well-known MLCLSP instances suffer from this assumption. Indeed, the zero lead time assumption leads to plans that are not implementable, as the lower-level scheduling problem is likely to be infeasible. This is more pronounced as the capacity tightens. On the other hand, positive lead time usually results in huge amounts of work-in-process, which tends to increase with a larger number of levels in the BOM.

The standard MLCLSP can be extended such that synchronization between stages is possible within a single period, i.e. a production lead time of less than a period is possible. In each time period, sizing and sequencing of production lots are simultaneously addressed in such a way that precedence relations of products are respected. We investigate two different cases: **batch production** (new products are only available after the whole production lot is finished) and **lot-streaming** (products can be retrieved while a lot is processed).

In order to be able to consider precedence relations within a period, it is necessary to keep track of the actual production sequence. We deploy a formulation for sequence-dependent setup times and costs described in [2]. In addition we use a set of continuous variables to denote the exact start and finish time of each production lot. We use a similar set of constraints as in [3] to determine those timing variables. In such a way, it is possible express the inventory levels within a period. By constraining those levels to be non-negative, we avoid that the production of an item is scheduled before all the necessary input materials from the previous stages are available.

2 Comparison with classical MLCLSP

2.1 MLCLSP with no lead time and no setup carryover

Considering that the proposed model incorporates more details than MLCLSP, we first evaluate if solutions derived from the traditional MLCLSP model would also be optimal or feasible for our approach. For this purpose we considered the two smallest classes of instances proposed by [4] with 10 items produced on 3 machines and a planning horizon of 4 periods.

The classical MLCLSP does not consider setup carryover. To avoid setup carryover in our formulation we introduce additional items (one for each machine) with no demand and force that setup carryovers are only possible for those additional items. The setup times and costs into these items are set to zero. The production quantities obtained by solving the MLCLSP are fixed in our formulation such that we only determine the production sequences and start and finish times. We let CPLEX run for at most ten minutes and classify the outcomes in two different categories: infeasible (no capacity feasible scheduling possible) and feasible (a feasible scheduling is possible)

Table 1 shows the results of these tests. We clearly observe that the vast majority of the solutions are infeasible. Since the batching constraint is more restrictive, fewer solutions are feasible under this assumption. But even without considering batching, which implies that predecessor and successor can be produced simultaneously, more than 90% of the instances are infeasible. It seems that the zero lead-time assumption leads to solutions that cannot be implemented in practice. On the other hand, having only a few periods, a 1-period lead time is also not meaningful.

Table 1 Checking the feasibility of solutions of the classical MLCLSP model.

	batching		lot-streaming	
class	infeasible	feasible	infeasible	feasible
A	99.67%	0.33%	94.80%	5.20%
B	99.67%	0.33%	93.67%	6.33%

2.2 MLCLSP with one period lead time and setup carryover

One way to overcome the feasibility problem of the MLCLSP is to consider a minimum lead time of one period. As aforementioned this lead time may cause a substantial increase of inventory. [5] used an extension of the class B test instances for MLCLSP with lead time and setup carryover. In order to guarantee feasibility two additional periods are added at the beginning of the planning horizon. We will denote the new class of instances as B6.

We solve the test instances using the classical MLCLSP without synchronization, but with one period lead time (as for example described by [5]), as well as using our new model formulation with synchronization. Since the model with synchronization is much harder to solve, we apply a 10 minute run time limit to CPLEX. Table 2 compares the new solutions obtained by using the synchronization feature. The columns show the number of solutions solved to optimality, the number of feasible solutions, the number of instances where no solution has been obtained, the number of improved solutions and the average cost improvement of the synchronization model compared with the MLCLSP with lead time.

Table 2 Comparing the MLCLSP with one period lead time with the new model with synchronization for the class B6 test instances.

	batching					lot streaming				
class	opt.	feas.	unknown	improved	UB impr.	opt.	feas.	unkn.	improved	UB impr.
B6	114	483	3	549 (92%)	27.5%	148	452	0	599 (100%)	37.6%

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Tight Formulations for the Two and Three Level Serial Lot-Sizing Problems

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Abstract. This study considers the two and three level uncapacitated serial lot-sizing problems, where the aim is to determine when and how much to produce at all levels over a T-period planning horizon so that the external known demand occurring at the last level is satisfied without backlogging and the total cost composed of fixed setup, production and inventory holding costs at all levels is minimized. Nonzero initial inventories are explicitly considered. Using new shortest path representations, strong formulations, which are tight in the case of zero initial inventories, are proposed for these problems besides showing that the two-level and the three-level problems can be solved in $O(T^3)$ and $O(T^4)$ time, respectively, in the absence of initial inventories.

1 Introduction

We consider two (Two-LSL) and three (Three-LSL) level uncapacitated serial lot-sizing problems, where the aim is to determine when and how much to produce at all levels over a T-period planning horizon so that the external known demand occurring at the last level is satisfied without backlogging and the total cost composed of fixed setup, production and inventory holding costs at all levels is minimized. Let L be the number of levels with 1 denoting the first level and L denoting the last level, and d_t be the demand of the last level in period t. Let I_{i0} denote the initial inventory available at level i ($1 \leq i \leq L$). Note that I_{L0}, without loss of generality, is taken equal to zero since nonzero I_{L0} can be deducted from external known demands starting from period 1 until it depletes and thus it can be treated as zero by adding its associated cost to the objective value. However, this is not valid for other levels because demands occuring at other levels are actually quantities produced in their next levels which are not known a priori. Thus, we explicitly consider nonzero initial inventories at all levels except the last level. Let p_{it}, h_{it} and f_{it} be the unit production cost, unit inventory holding cost, and fixed setup cost at level i in period t, respectively. Define D_{tk} as the total demand of the last level from period t through $k-1$, i.e. $D_{tk} = \sum_{r=t}^{k-1} d_r$.

The Two-LSL and Three-LSL problems are the special cases of multi-level uncapacitated lot-sizing problem in a serial supply chain, for which Zangwill [6] proposed a polynomial time dynamic programming (DP) algorithm that runs in $O(T^3 + (L-2)T^4)$ time (see [5]) assuming zero initial inventories at all levels. Later, van Hoesel et al. [5] extended Zangwill's work by proposing polynomial DP algorithms to a more general problem in which a stationary capacity on production is considered in the first level of the supply chain. van Hoesel et al. [5] showed that the DP recursion in [6] does not apply in the presence of initial inventories at any level except the last level. Recently, Melo and Wolsey [1] proposed a DP algorithm running in $O(T^2 \log T)$ time and a corresponding tight formulation (derived using the DP recursion) with $O(T^3)$ variables and $O(T^2)$ constraints for the Two-LSL problem assuming zero initial inventories at both levels. Similar to [6], the DP recursion of [1] does not apply and the developed tight formulation is not valid in the presence of initial inventory at the first level. Solyalı and Süral [2] considered a variant of the Two-LSL problem assuming that when the demand at the second level is satisfied by the first level, its inventory level has to be brought up to a particular level, which is called the order-up-to level policy. They proposed a DP algorithm running in $O(T^3)$ time for solving the problem. Solyalı et al. [3] suggested a tight formulation for this problem, which describes the convex-hull of the feasible set of the problem. Solyalı and Süral [4] presented different MIP formulations for the Two-LSL problem with multiple locations of demands at the second level, including a new shortest path based strong reformulation, and analyzed the strength of their LP relaxations with respect to each other. Both studies considered the nonzero initial inventories in the first level explicitly.

In this paper, using new shortest path representations, we propose strong formulations, which are tight in the case of zero initial inventories, for the Two-LSL and Three-LSL problems besides showing that the Two-LSL and Three-LSL problems can be solved in $O(T^3)$ and $O(T^4)$ time, respectively, in the absence of initial inventories.

2 Two-LSL Problem

In the following, we transform the Two-LSL problem into an equivalent shortest path network problem, which represents the convex hull of feasible solutions of the problem when the initial inventory at the first level is zero.

Let $G^{2L} = (V^{2L}, A^{2L})$ be an acyclic directed graph for the Two-LSL problem where V^{2L} is the set of nodes and A^{2L} is the set of arcs. A node of V^{2L} is denoted by $(a1, a2)$ where $a1$ and $a2$ denote the time period for the first and second level, respectively. There are three types of arcs in A^{2L} which are vertical, horizontal and zero demand arcs. Vertical arcs represent production quantity decisions while horizontal arcs determine whether a fixed setup cost at the first level is incurred or not. Let W_{qtk} represent the vertical arc from node (q,t) to node (q,k) with cost C_{qtk} meaning that the total demand of the second level from period t through $k-1$ is satisfied by producing in period q at the first level (by initial inventory at the first level if $q = 0$) and in period t at the second level, where $0 \le q \le t < k \le T+1, t \ge 1$. Let U_{qkt} represent the horizontal arc from node (q,t) to node (k,t) meaning that a fixed setup cost f_{1k} is incurred at level 1 in period k, where $0 \le q < k \le t \le T$. To account for the zero demand case, we define an arc from node $(0,0)$ to node (t,t) represented by Z_{tt} with cost f_{1t} for each $1 \le t \le T$. Let $C_{0tk} = f_{2t} + H_{2tk} - D_{tk}\sum_{l=t}^{T} h_{0l}$ and $C_{qtk} = f_{2t} + D_{tk}H_{1qt} + H_{2tk}$, where $H_{1qt} = p_{1q} + \sum_{l=q}^{t-1} h_{1l}$ and $H_{2tk} = p_{2t}D_{tk} + \sum_{l=t}^{k-2} h_{2l}D_{l+1,k}$ for $1 \le q \le t < k \le T+1$. Defining $(0,0)$ as the source node of the network, we can construct a shortest path network G^{2L} where

$$
\begin{aligned}
V^{2L} &= \{(t,k) | 0 \le t \le k \le T+1, t \le T\} \text{ and} \\
A^{2L} &= \{(0,0) \to (0,1)\} \cup \{(0,0) \to (t,t) | (t,t) \in V^{2L}, 1 \le t \le T\} \\
&\quad \cup \{(q,t) \to (q,k) | (q,t) \in V^{2L}, (q,k) \in V^{2L}, 0 \le q \le t < k \le T+1, t \ge 1\} \\
&\quad \cup \{(q,t) \to (k,t) | (q,t) \in V^{2L}, (k,t) \in V^{2L}, 0 \le q < k \le t \le T\}.
\end{aligned}
$$

Theorem 1 *When $I_{10} = 0$, the Two-LSL problem can be solved in $O(T^3)$ time.*

Proof When $I_{10} = 0$, nodes $\{(0,t) | 1 \le t \le T+1\}$ and all incoming and outgoing arcs associated with these nodes are eliminated from G^{2L}. Since the resulting graph is an acyclic directed graph with $O(T^3)$ arcs, it can be solved in $O(T^3)$ time. □

Below we give the formulation of the Two-LSL problem based on the network structure defined above, referred to as NF-2L. Then, we show that the formulation is tight in that its linear programming (LP) relaxation always yields an integral solution when $I_{10} = 0$. NF-2L is as follows.

$$
\text{NF-2L: } \min \sum_{i=1}^{2}\sum_{t=1}^{T} f_{it}y_{it} + \sum_{q=0}^{t}\sum_{t=1}^{T}\sum_{k=t+1}^{T+1} C'_{qtk}W_{qtk} + \sum_{t=1}^{T} h_{0t}I_{10} \tag{1}
$$

$$
\text{s.t. } W_{001} + \sum_{t=1}^{T} a_{tt}Z_{tt} = 1 \tag{2}
$$

$$
\sum_{t=1}^{T} W_{ttk} - a_{tt}Z_{tt} = 0 \qquad \forall (t,t) \in V, 1 \le t \le T \tag{3}
$$

$$
\sum_{k=t+1}^{T+1} W_{qtk} + \sum_{k=q+1}^{t} U_{qkt} - \sum_{k=q}^{t-1} W_{qkt} - \sum_{k=1}^{q-1} U_{kqt} = 0 \quad \forall (q,t) \in V^{2L} \setminus \{(t,t)\}, 0 \le q \le t \le T \tag{4}
$$

$$
\sum_{q=0}^{t-1}\sum_{k=t}^{T} U_{qtk} + a_{tt}Z_{tt} \le y_{1t} \qquad 1 \le t \le T \tag{5}
$$

$$
\sum_{q=0}^{t}\sum_{k=t+1}^{T+1} W_{qtk} \le y_{2t} \qquad 1 \le t \le T \tag{6}
$$

$$
\sum_{t=1}^{T}\sum_{k=t+1}^{T+1} D_{tk}W_{0tk} \le I_{10} \tag{7}
$$

$$
W_{qtk} \ge 0, Z_{tt} \ge 0, U_{qrl} \ge 0 \quad 0 \le q \le t < k \le T+1, t \ge 1, 0 \le q < r \le l \le T \tag{8}
$$

$$
y_{it} \in \{0,1\} \qquad 1 \le i \le 2, 1 \le t \le T, \tag{9}
$$

where $C'_{qtk} = C_{qtk} - f_{2t}$, $a_{11} = 1$, $a_{tt} = 1$ if $D_{1t} = 0$ and 0 otherwise for $t \ge 2$.

The objective function (1) is the sum of fixed setup, production and inventory holding costs at levels one and two. Constraints (2)–(4) are the shortest path flow balance constraints representing the integrated lot-sizing problems of both levels. Constraints (5) guarantee that a fixed setup cost is incurred at the first level if the first level produces in a period. Constraints (6) stipulate that a fixed setup cost is incurred at the second level if the second level produces in a period. Constraint (7) ensure that the total amount of demand satisfied by the initial inventory at the first level does not exceed the available amount. Constraints (8) are for the nonnegativity of variables while constraints (9) are for the integrality of variables.

Theorem 2 *When $I_{10} = 0$, NF-2L defines the convex hull of feasible solutions of the Two-LSL problem.*

Proof When $I_{10} = 0$, the W_{0tk} $(1 \le t < k \le T+1)$ and U_{0rl} $(0 < r \le l \le T)$ variables, and constraint (7) are eliminated from NF-2L. Note that constraints (5) and (6) will be satisfied as equality since $f_{it} \ge 0$ $(1 \le i \le 2)$. Thus, the y_{1t} and y_{2t} variables in the objective function, respectively, can be replaced with $\sum_{q=1}^{t-1} \sum_{k=t}^{T} U_{qtk} + a_{tt}Z_{tt}$ and $\sum_{q=1}^{t} \sum_{k=t+1}^{T+1} W_{qtk}$ as defined by (5) and (6), and the y_{it} variables as well as constraints (5) and (6) can be eliminated from the formulation. It is immediate that coefficient matrix of the remaining constraints is totally unimodular as all coefficients are elements of $\{0, -1, 1\}$, each variable appears in those constraints twice with coefficients of -1 and 1, and there exists a partition $(R_1 = R, R_2 = \emptyset)$ of the set R of rows such that the difference between the summation of coefficients in R_1 and the summation of coefficients in R_2 is zero for each variable. □

3 Three-LSL Problem

Similar to the Two-LSL problem, we transform the Three-LSL problem into an equivalent shortest path network problem, which represents the convex hull of feasible solutions of the problem when the initial inventory at the first and second levels are zero. We basically incorporate an additional attribute to each node and arc of the Two-LSL problem's network representation (resp. an additional index to the U, W, and Z variables of NF-2L) to account for the decisions of the additional level in the Three-LSL problem. This leads to an acyclic network representation with $O(T^4)$ arcs. Thus, like Two-LSL problem, we show that the Three-LSL problem can be solved in $O(T^4)$ time and the corresponding formulation is tight when the initial inventory at the first and second levels are zero.

As can be seen in transition from the Two-LSL problem to the Three-LSL problem, our approach increases the number of arcs and thus the complexity of the algorithm by $O(T)$. Thus, applying our approach to the general L-level uncapacitated serial lot-sizing problem yields an acyclic network representation with $O(T^{L+1})$ arcs which can be solved in $O(T^{L+1})$ time in the case of zero initial inventories.

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Performance Assessment of Production Planning Approaches in Semiconductor Manufacturing

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Abstract. In this talk, we discuss a production planning problem that is motivated by real-world problems found in semiconductor manufacturing. We describe the planning problem that consists in determining appropriate wafer quantities for several products, facilities, and time periods by taking demand fulfillments, i.e. confirmed orders and forecast, and capacity constraints into account while maximizing the difference between revenue and total costs. In addition, fixed costs are used to minimize the production partitioning. A mixed integer programming formulation is presented. Two heuristic procedures are proposed, namely a simple product-based decomposition scheme and a genetic algorithm. We introduce a simulation-based architecture for performance assessment. The performance of the heuristics is assessed in a rolling horizon setting. Some computational results are presented.

1 Introduction and Motivation

Complex manufacturing systems, such as semiconductor wafer fabrication facilities (wafer fabs), are characterized by a diverse, over time changing product mix, re-entrant process flows due to expensive machinery, different process types, and different kinds of internal and external disruptions [7, 8]. A single wafer fab has typically several hundred machines that are used to process up to 800 different lots. The current generation of semiconductor products often requires up to 700 unit processing steps that can take up to three months to complete. Recently, manufacturing networks become very popular and crucial in the semiconductor industry because of the fierce competition among manufacturers. This development is caused by the fact that front-end operations, i.e. the production of chips on silicon wafers, are often performed in highly industrialized nations, while assembly and test operations are typically carried out in countries where labor rates are cheaper. There are many papers that deal with production control issues, especially dispatching and scheduling of lots on the shop-floor (cf. [9] for a recent survey on dispatching approaches in semiconductor manufacturing). Production planning problems for semiconductor manufacturing are less often considered in the literature. The corresponding papers are related to long-term, strategic capacity planning [1, 6] or consider only a single wafer fab [4, 3]. The models in Denton et al. [2] take too much details into account.

2 Problem and Modeling Issues

We are interested in determining appropriate wafer quantities for several products, several production sites, and several periods of time. The production plan has a horizon of six months divided in weekly time buckets. Since market demand is not entirely known while planning a couple of weeks or months ahead, we have to distinguish between customer orders and additional forecast. On the one hand, explicit customer requirements are confirmed, postponed, or reduced by the order management process based on available supply. One the other hand, the demand planning process performed monthly by sales and marketing departments aims to foresee the rest of the market needs. In our model, orders are fulfilled at first, while forecast is satisfied only if capacity is available due to difference in the demand certainty. We assume that unmet customer orders in period t are postponed as backlogs to period $t + 1$. On the contrary, unfulfilled forecasts are ignored for the rest of the horizon. It is also assumed that inventory is used to store finished goods for later fulfillments of demand.

The capacity modeling is a crucial point for production planning as it represents the main quantity restriction of the problem. The capacity limits are related to bottleneck work centers. We assume that products have a fixed average cycle time, but we are interested in relaxing this assumption in future research by using iterative simulation. Given the completion period of a wafer and its fabrication process, we are able to compute when it arrives at a certain bottleneck work center. Then, we accumulate the time that the product spends on processing on the machines of the considered bottleneck. This method allows us to take re-entrant process flows of the wafers into account. This capacity modeling is similar to other approaches used for capacity planning in semiconductor manufacturing (cf. Barahona et al. [1]).

A MIP formulation is proposed that takes the different constraints into account. For a small number of products, periods, and facilities we are able to solve problem instances optimally. But because of the size of real-world problem instances, we have to look for efficient heuristics.

3 Heuristics

Two heuristics are proposed. The first one performs a product-based decomposition. Products are first sorted according to their criticality, e.g. products with high demand and high backlog cost have high priority. Then, a bundle of products is selected from the ranked list to form a subproblem. The subproblems are solved by taking the demand for those products and the actual remaining capacity into account. Afterward, the values of the decision variables are frozen, and the maximum capacity limit is decreased according to the loading that has already been planned. Finally, we increment the global objective function value. The algorithm continues until all product subsets have been considered. The second heuristic is a genetic algorithm (GA). It is hybridized with some local search. The GA outperforms the decomposition-based heuristic.

4 Simulation-based Performance Assessment

Discrete-event simulation is used to assess the performance of the planning heuristics in a rolling horizon setting. Because we have to consider a network consisting of several wafer fabs, we use reduced simulation models of wafer fabs as proposed by Hung et al. [5] for a single wafer fab. Our method to decrease the level of detail of the simulation models without altering the quality of the results is briefly sketched. The center point of the used architecture is a data-layer between the simulation model and the production planning algorithms. The simulations experiments show that the more sophisticated heuristics lead to larger objective function values, while the simple heuristics offer more stable plans.

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Transportation lot sizing: Research topics and practical aspects

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Practical application of lot sizing in transportation settings implies distinguishing features in terms of specific cost functions and capacity restrictions. Based on experiences from a Scandinavian case study, we present and discuss some of these characteristics, suggest model formulations, and report results from computational testing using a standard Mixed Integer Programming software.

The study is based on a logistics company that is responsible for distribution of beverages in Scandinavian countries, facing a supply chain planning problem that integrates lot sizing and transportation mode selection in a combined Full TruckLoad (FTL) and Less Than Truckload (LTL) setting with discount schedules. In practice, shippers may choose among different transportation alternatives and switch from one to another as needed, but it is often assumed that only a single transportation mode is available. Hence there is an improvement potential of considering availability of multiple modes and by allowing combined use of several modes for shipping the order quantity simultaneously.

Based on this, we propose and test a dynamic lot-sizing model that allows combination of multiple modes with various cost functions, and perform a study to investigate the economical benefits of transportation mode combinations and the effect of various model parameters. Consideration of multiple transportation modes in the same model adds a new layer of decision variables, and therefore increases the computational complexity. The proposed model supports decisions on the timing of order placement and the order quantity allocated to each transportation mode. The study of economical effects suggests that it may be important for managers to take transportation mode decisions together with lot-sizing decisions and to understand the impact of simplified decisions, such as for example use of a single mode instead of combining modes, on the total cost.

Lot-sizing with carbon emission constraints

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1 Introduction

Legislations are evolving in order to enforce control on carbon emissions. This will probably be done by constraining companies to emit less than a given amount of carbon dioxide by product unit that is produced and transported. This amount of carbon emission will probably figure on the item packaging in the near future. Companies will face new constraints that will force them to reduce carbon emissions while still minimizing production and transportation costs. There are few papers addressing production planning and transportation problems that take into account environmental constraints. Generally, environmental constraints are integrated as cost components in the objective function, and resulting problems are solved using multi-criteria approaches (see [3] and [1]). The limit of these models is that classical cost components (production and transportation costs) have the same behavior than environmental cost components (e.g. reducing the number of vehicles, the total distance, etc.). We only found one work addressing the integration of carbon emission constraints in lot-sizing problems [2]. The authors add a new capacity constraint that limits carbon emissions. This constraint links all carbon emissions related to production and storage over the planning horizon. The limit of this constraint is that producers can create large carbon emissions at the beginning of the horizon by producing large quantities, and balance the total carbon emission by producing nothing at the end of the horizon. In this paper, we address multi-sourcing lot-sizing problems with carbon emission constraints. We investigate the integration of these news constraints depending on expected legislations. We introduce new lot-sizing models integrating carbon emission constraints. These new constraints are induced from a maximum allowed carbon dioxide emission coming from legislations, green taxes or initiatives of companies.

2 Mathematical programming models

Consider a multi-sourcing lot-sizing problem faced by a company that has to determine over a given planning horizon of T periods, when, where and how much to produce N items to satisfy a deterministic demand. Different production locations and transportation modes are possible to satisfy a given demand. In this work, we consider M modes where a mode corresponds to the combination of a production location and a transportation mode. We study four types of carbon emission constraints: (1) Periodic carbon emission constraint, (2) Cumulative carbon emission constraint, (3) Rolling carbon emission constraint and (4) Global carbon emission constraint. We shall see that the first and fourth constraint types are actually special cases of the third one. To model these new constraints, we define the following parameters and variables.

Parameters

d_{it}: Demand of item i at period t.

h_{it}: Unitary holding cost of item i at period t.

p_{it}^m: Unitary production and transportation cost of item i at period t using mode m.

f_{it}^m: Production and transportation setup cost of item i at period t using mode m.

e_i^m: Environmental impact (carbon emission) related to producing and transporting one unit of item i using mode m.

$MaxE_{it}$: Maximum unitary environmental impact allowed for item i at period t.

Variables

X_{it}^m: Quantity of item i produced and transported at period t using mode m.

Y_{it}^m: Binary variable equal to 1 if item i is produced and transported at period t using mode m, and 0 otherwise.

I_{it}: Inventory of item i carried from period t to period $t+1$.

Note that $MaxE_{it}$ depends on the period t, since carbon emissions will probably be forced to be decreased by stages and not to a specific level right away. The variations of $MaxE_{it}$ will depend on the level (strategic, tactical or operational) at which the lot sizing models are used.

To model our problem, we use: the classical flow conservation constraints (1), production and transportation constraints (2) and finally carbon emission constraints ((4), (6), (8) or (9)). The objective function, given by (3), minimizes production and transportation costs (fixed and variable) and holding costs.

$$\sum_{m=1}^{M} X_{it}^{m} - I_{it} + I_{i,t-1} = d_{it},\ i = 1, \ldots, N, t = 1, \ldots, T. \tag{1}$$

$$X_{it}^{m} \leq \left(\sum_{t'=t}^{T} d_{it'} \right) Y_{it}^{m},\ i = 1, \ldots, N, t = 1, \ldots, T. \tag{2}$$

$$\min \sum_{m=1}^{M} \sum_{i=1}^{N} \sum_{t=1}^{T} p_{it} X_{it}^{m} + f_{it} Y_{it}^{m} + \sum_{i=1}^{N} \sum_{t=1}^{T} h_{it} I_{it} \tag{3}$$

2.1 Periodic carbon emission constraint

This constraint is very tight, and assumes that the amount of carbon emission that is not used in a given period is lost. This constraint can be formulated as follows:

$$\frac{\sum_{m=1}^{M} e_i^m X_{it}^m}{\sum_{m=1}^{M} X_{it}^m} \leq MaxE_{it},\ i = 1, \ldots, N, t = 1, \ldots, T. \tag{4}$$

This constraint forces the average amount of carbon emission for a given item i at a given period t to be lower than the maximum unitary environmental impact allowed. This constraint can be rewritten as follows:

$$\sum_{m=1}^{M} (e_i^m - MaxE_{it}) X_{it}^m \leq 0,\ i = 1, \ldots, N, t = 1, \ldots, T. \tag{5}$$

2.2 Cumulative carbon emission constraint

Constraint (6) below is less strong than Constraint (4). The amount of unused carbon emission of a given period can be used in future periods without overpassing cumulative capacities.

$$\sum_{t'=1}^{t} \sum_{m=1}^{M} (e_i^m - MaxE_{it'}) X_{it'}^m \leq 0,\ i = 1, \ldots, N, t = 1, \ldots, T. \tag{6}$$

This constraint could be modeled using inventory variables representing the amount of unused carbon emission. We define J_{it} the amount of unused amount of carbon emission that could be used in future periods (with $J_{it} \geq 0$ and $J_{i0} = 0$).

$$J_{it} = J_{i,t-1} + \sum_{m=1}^{M} (e_i^m - MaxE_{it}) X_{it}^m,\ i = 1, \ldots, N, t = 1, \ldots, T. \tag{7}$$

2.3 Rolling carbon emission constraint

Constraint (6) assumes, at each period t, that the horizon from 1 to t can be used to compensate carbon emissions between periods. In Constraint (8) below, we suppose that this is only possible on a rolling horizon of R periods. This seems more realistic, and makes the problem less dependent on the planning horizon T. Note that the periodic carbon emission constraint (4) is equivalent to Constraint (8) when $R = 1$.

$$\sum_{t'=t-R+1}^{t} \sum_{m=1}^{M} (e_i^m - MaxE_{it'}) X_{it'}^m \leq 0,\ i = 1, \ldots, N, t = R, \ldots, T. \tag{8}$$

Inventory variables J_{it} can still be used, but constraints on perishable inventories must be introduced, since unused carbon emission in period t cannot compensate carbon emission in periods after $t + R$. This is not detailed in this abstract.

2.4 Global carbon emission constraint

This constraint extends Constraint (6) on the whole horizon, and is thus weaker, and is equivalent to Constraint (8) when $R = T$. It means that unitary carbon emission of an item i over the whole horizon cannot be larger than the maximum unitary environmental impact allowed for item i. In this case, the maximum unitary environmental impact $MaxE_{it}$ does no longer depend on the horizon and is fixed, i.e. $MaxE_{it} = MaxE_i \ \forall t = 1, \dots, T$.

$$\sum_{t=1}^{T} \sum_{m=1}^{M} (e_i^m - MaxE_i) X_{it}^m \leq 0, \ i = 1, \dots, N. \tag{9}$$

One of the main limitations of Constraint (9) is that the solution strongly depends on the horizon length T. Generally speaking, this constraint has similar drawbacks than the ones of the constraint used in [2], discussed in Section 1.

3 Conclusion and further research directions

We believe the integration of carbon emission constraints in lot-sizing problems lead to relevant and original problems. This paper is a first step to model such problems from which several new academic lot-sizing problems could arise. Some properties of resulting single-item lot-sizing problems will be presented.

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Collaborative Planning in Supply Chains

Alf Kimms

The starting point of our work is the capacitated lot sizing problem (CLSP) which is an $\mathcal{NP}$–hard optimization problem where one single decision maker i has to determine a production plan for T periods of time. Several items are to be produced which share a common resource with availability R_{it}. Let K be the index set of the items under consideration and d_{ikt} be the dynamic demand of item k in period t that is faced by the decision maker i. Let b_{ikt} be the decision maker's production coefficient for item k in period t, i.e. the number of resource units required to produce one unit of product k by i in t. Given these parameters, the decision variables q_{ikt} can be used to represent the production plan. The variable q_{ikt} is the quantity of item k that is produced by i in period t. Let the parameter $c^{\mathcal{P}}_{ikt}$ be the unit production cost coefficient. The objective is to find a production plan with minimum costs where a tradeoff exists between setup costs and holding costs. A fixed setup cost $c^{\mathcal{S}}_{ikt}$ is incurred whenever i produces a positive quantity of product k in period t. A binary decision variable x_{ikt} can be used to indicate whether or not a setup takes place. Items can be produced and kept in stock until needed. For each unit of k in stock at the end of period t a holding cost $c^{\mathcal{H}}_{ikt}$ is charged and a decision variable I_{ikt} can be used to represent the number of items in stock. I_{ik0} is a parameter and stands for the initial inventory of decision maker i regarding item k. For decision maker i the $CLSP_i$ can be formulated as a mixed–integer program (with decision variables q_{ikt}, I_{ikt}, and x_{ikt}) as follows:

$$\min \sum_{k\in K}\sum_{t=1}^{T}(c^{\mathcal{S}}_{ikt}x_{ikt}+c^{\mathcal{P}}_{ikt}q_{ikt}+c^{\mathcal{H}}_{ikt}I_{ikt}) \tag{1}$$

s.t.

$$I_{ikt}=I_{ik,t-1}+q_{ikt}-d_{ikt} \qquad k\in K; t=1,\ldots,T \tag{2}$$

$$\sum_{k\in K} b_{ikt}q_{ikt}\le R_{it} \qquad t=1,\ldots,T \tag{3}$$

$$q_{ikt}\le M_{ikt}x_{ikt} \qquad k\in K; t=1,\ldots,T \tag{4}$$

$$q_{ikt}, I_{ikt}\ge 0 \qquad k\in K; t=1,\ldots,T \tag{5}$$

$$x_{ikt}\in\{0,1\} \qquad k\in K; t=1,\ldots,T \tag{6}$$

The objective (1) is to minimize the total costs. Inventory balance constraints are stated by (2). (3) are the capacity constraints. Due to (4) production cannot be done without a setup. The parameter M_{ikt} is a large number and can be chosen as $M_{ikt}=\sum_{\tau=t}^{T} d_{ik\tau}$. The domains of the decision variables are given by (5) and (6).

The standard CLSP–model formulation (1)–(6) can easily be extended for additional aspects such as setup times, backlogging, overtime, limited stocking capacity, and multi–level product structures. For the sake of simplicity we do not integrate these extensions into our model, but it is worth emphasizing that the paper can be adapted straightforwardly if desired.

The focus of our attention is a cooperative CLSP setting. Papers on our work which cover the topics of our presentation are [1, 2].

The idea behind a cooperative CLSP is that several decision makers, who are called players from now on, work together in the sense that players cannot only produce to meet own demand, but also to fulfill other players' demand.

Let N be the index set of all players under consideration. Such a cooperative situation seems to be realistic if the capacity of some players is insufficient to fulfill their demand while other players can help out. It seems to be reasonable as well if some players have lower costs than others (and sufficient capacity). This setting however means that items must be transported among the players. Let $c^{\mathcal{T}}_{ijkt}$ be the per unit transportation cost for shipping product k from player i to player j in period t and a_{ijkt} the transported quantity. If a set $S\subseteq N$ of players forms a coalition the cooperative optimization problem with decision variables q_{ikt}, I_{ikt}, x_{ikt}, and a_{ijkt} is the following (let $c(S)$ denote the total cost of the coalition S, c is called the characteristic function of the cooperative game):

$$c(S) = \min \sum_{i \in S} \sum_{k \in K} \sum_{t=1}^{T} \left(c^{\mathcal{S}}_{ikt} x_{ikt} + c^{\mathcal{P}}_{ikt} q_{ikt} + c^{\mathcal{H}}_{ikt} I_{ikt} + \sum_{j \in S} c^{\mathcal{T}}_{ijkt} a_{ijkt} \right) \tag{7}$$

s.t.

$$I_{ikt} = I_{ik,t-1} + q_{ikt} + \sum_{j \in S} a_{jikt} - d_{ikt} - \sum_{j \in S} a_{ijkt} \qquad \begin{array}{l} i \in S; k \in K; \\ t = 1, \ldots, T \end{array} \tag{8}$$

$$\sum_{k \in K} b_{ikt} q_{ikt} \leq R_{it} \qquad i \in S; t = 1, \ldots, T \tag{9}$$

$$q_{ikt} \leq M_{kt} x_{ikt} \qquad i \in S; k \in K; t = 1, \ldots, T \tag{10}$$

$$q_{ikt}, I_{ikt} \geq 0 \qquad i \in S; k \in K; t = 1, \ldots, T \tag{11}$$

$$a_{ijkt} \geq 0 \qquad i, j \in S; k \in K; t = 1, \ldots, T \tag{12}$$

$$x_{ikt} \in \{0, 1\} \qquad i \in S; k \in K; t = 1, \ldots, T \tag{13}$$

The objective (7) remains that of minimizing total costs where transportation costs are included here. Inventory balance constraints (8) take into account that transshipments to or from a player — see the decision variables defined in (12) — can take place. The underlying assumption is that such transshipments can occur only within a cooperation. All other constraints and decision variables are taken from the standard CLSP model, and $M_{kt} = \sum_{i \in N} \sum_{\tau=t}^{T} d_{ik\tau}$ is a sufficiently large number. Note that $c(\emptyset) = 0$ by construction and $c(S) \geq 0$. To be well–defined let us assume $c(S) = \infty$ if the optimization problem has no feasible solution for the coalition S. It should be noted that extensions like limited transportation capacity or fixed transportation costs can easily be added to the model.

The focus of our talk will be on solving the cost sharing problem of the total cost $c(N)$, i.e. distributing cost shares among the players in a fair and stable way.

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Integrated Case Pack Design and Procurement Planning

Shuang Chen and Joseph Geunes

In retail supply chains, merchandise is often packed into cases containing multiple stock keeping units (SKUs). These *case packs* reduce the number of individual touches a product receives in distribution, which streamlines handling costs. Case packs often serve as the lowest level of packaging hierarchy in a distribution system, and are designed to flow through from vendor to retail store. Retail distribution systems have been increasing the use of case pack deliveries to stores, instead of receiving bulk packages for each SKU at distribution centers (DCs), and then breaking, sorting, and repackaging them for delivery to stores. This trend is a result of the fact that breaking bulk typically involves a great deal of work at the DC, including associated infrastructure and personnel, while DCs can quickly process case packs at minimal cost, often through the use of cross-docking to transfer them to individual stores.

While the inclusion of multiple items in one case pack reduces the number of touches individual items experience in the distribution chain (thus reducing overall handling costs), determining the best case pack composition is a major challenge, and the use of case packs can severely increase the complexity of procurement planning at the retail level. This complexity arises because different case packs (and the items within them) share certain (fixed) order costs, and the composition of the case pack requires ordering in defined combinations of multiple SKUs. Fully exploiting the advantages of using case packs in distribution requires understanding how case packs reduce handling costs as well as how case pack composition affects replenishment and inventory holding costs. The corresponding relevant research questions are:

1. How many case pack patterns should be used in a given system, and which SKUs should be contained in each case pack (and in what quantities)?
2. How should inventory replenishment be planned to ensure that a retailer meets demand with as little leftover inventory as possible, while minimizing total procurement-related costs?

Designing larger case pack configurations containing a greater number of SKUs implies handling cost advantages, but also reduces ordering flexibility and consequently increases the likelihood of overstocking and/or the need for breaking bulk at the DC. Thus, it is necessary to balance these tradeoffs in order to take full advantage of the reduced DC handling costs associated with case packs.

We consider case pack design and procurement planning for an assortment of items, when multiple items are assigned to case packs (defined by an assortment of items included within a larger case or container). Each item has a period-dependent, per-unit ordering and holding cost. Furthermore, since stores receive replenishments containing multiple SKUs from a DC on a common truck, a fixed order cost is incurred that is virtually independent of the number of different case packs contained in the order (any additional fixed cost for ordering each individual case pack is practically negligible). The goal is to jointly determine the case pack composition and the number of case packs of each type to order such that the demand for all items is satisfied within each scenario and the expected cost is minimized. We develop a model that addresses this complex and practical problem class, and provide algorithms for solving it in acceptable computing time.

In this talk, we will focus on the single case pack variant of the problem, i.e., we wish to determine the optimal configuration of a single case pack containing some fixed quantity of each of the relevant SKUs that a retailer must order from a given supplier. We will first discuss related literature, and then describe and formulate the joint case pack design and procurement problem, which is an integer nonlinear (nonconvex) optimization problem that contains an embedded multi-item lot sizing problem with shared order costs. We propose an exact linearization method, as well as an iterative heuristic that alternatively fixes case pack composition and procurement variables and solves the remaining subproblem optimally. To capture the benefits of integrated decision making, we further study an integrated approach combining a geometric programming method and a dynamic lot sizing approach. We present results of the implementation of our solution methods for the single case pack version, and then discuss concluding remarks and the scope of our ongoing research in this area.

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