Supply planning for two-level assembly systems with stochastic component delivery times: trade-off between holding cost and service level

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Abstract: A supply planning for two-level assembly systems under lead time uncertainties is considered. It is supposed that the demand for the finished product and its due date are known. It is assumed also that the component lead time at each level is a random discrete variable. The objective is to find the release dates for the components at level 2 in order to minimize the expected component holding costs and to maximize the customer service level for the finished product. A multiobjective approach is considered and numerical results are reported.

1 Introduction

In the literature of production planning and inventory control, most papers examine inventory systems where lead times are supposed to be equal to zero or constant. In reality, lead times are rarely constant; unpredictable events can cause random delays. Most often, lead time fluctuations strongly degrade system performance. For different reasons (machine breakdowns, transport delays, or quality problems, etc.), the component lead times, i.e. time of component delivery from an external supplier or processing time for the semi-finished product at the previous level, have often an uncertain duration. To minimize the influence of these stochastic factors, firms implement safety stock (or safety lead time), but stock is expensive. In contrast, if there is not enough stock, stockout occurs with unsatisfied customer service level. So, the problem is to find a trade-off between the holding costs and the customer service level. The optimal planned lead times, for a one-level assembly system, are derived by [1]. [7] investigated the problem of planned lead time calculation for multilevel serial systems under lead time uncertainties. [1] considered only the case of two and three stage (level) serial systems. Both [7] and [1] proposed continuous inventory control models.

This paper deals with an extension of these approaches for both multi-level serial (one type of component in each level) and assembly (several components are assembled at each level) systems. Taking into account the fact that MRP approach uses planning buckets (discrete time), a discrete inventory control model where decision variables are integers is developed.

In literature, few works model lead times as discrete random variables. In [2], a one-level inventory control problem with random lead times and fixed demand is considered, for a dynamic multi-period case. The authors give a Markov model for this problem. In [5], under the additional restrictive assumptions that the lead times of the different types of components follow the same probability distribution, and the unit holding costs per period are the same for all types of components, the optimal solution is obtained as a generalized Newsboy model. In [6] a multi-period planning for one-level assembly systems where lead time density functions for components are known in advance is considered. The aim is to minimize the total expected cost composed of component holding costs and finished product backlogging costs, and a lower bound of the cost function is proposed. Recently, we proposed in [4] a genetic algorithm dealing with large size instances of this problem.

In this paper, we consider the cases in which the impact of stockout cannot be reduced to backlogging costs and they are instead replaced by a customer service level.

2 Problem description

The supply planning for two level assembly systems is considered: the finished product is produced from components themselves obtained from other components (see Figure 1).

The finished product demand D is supposed to be known and the due date T requested by the customer is the end of the period (so also known). This is a single period problem. The unit inventory cost for each type of component is known. The lead times for various component orders are independent. The assembly of the components at each level is carried out as soon as the necessary components are available (just in time).

Let us introduce the following notations:

- T : due date for the finished product ; without loss a generality, let T=0
- D : demand for finished product for the date T ; without loss a generality, let D=1
- $c_{i,j}$: component i of level j (j=1 or 2) of bill of material (BOM)
- N_j : number of types of components of level j (j=1or 2)
- $P_{i,j}$: set of the “sons” of $c_{i,j}$ in a BOM tree
- $L_{i,j}$: random lead time for component $c_{i,j}$

- $h_{i,j}$: unit holding cost for component $c_{i,j}$ per unit of time
- $x_{i,j}$: planned lead time for component $c_{i,j}$
- $-X_{i,2}$: release date for component $c_{i,2}$ (this type of variable is defined only for level 2)
- $F_{i,j}(.)$: cumulative distribution function of $L_{i,j}$
- $u_{i,j}$: maximum value of $L_{i,j}$; each $L_{i,j}$ varies in $[1, u_{i,j}]$
- $U_{k,2} = u_{k,2} + u_{i,1}$:maximum value of $L_{k,2}+ L_{i,1}$, $c_{k,2} \in P_{i,1}$; $(L_{k,2}+ L_{i,1})$varies in $[2, U_{k,2}]$,
- U: $\max(U_{k,2})$, $k=1,2,\ldots,N_2$
- $H = \sum_{i=1}^{N_1} h_{i,1}$: sum of holding cost of components at level 1.
- $M_{i,1} = \max_{c_{k,2}\in P_{i,1}} \left(L_{k,2} - X_{k,2}\right)$: date to assemble $c_{i,1}$, $i=1,2,\ldots,N_1$.

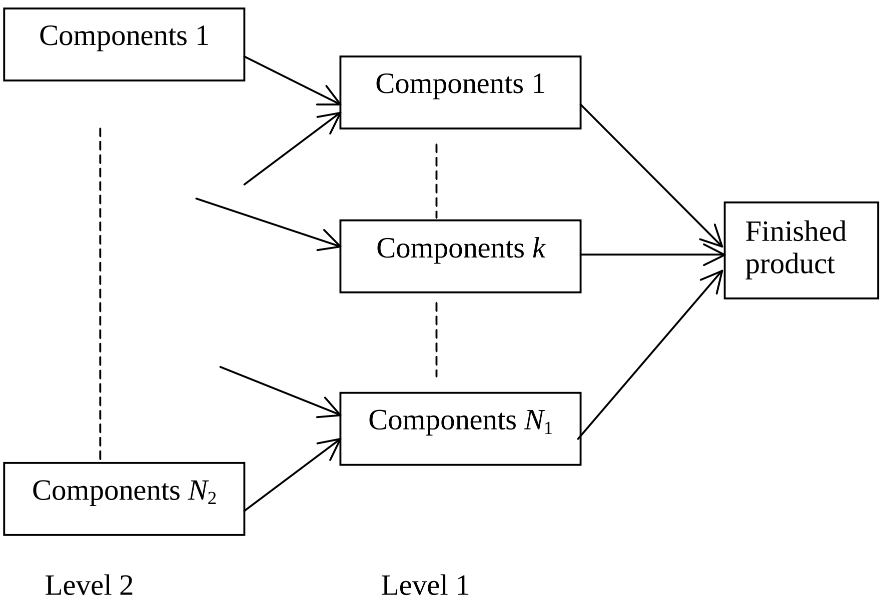

Figure1: A two-level assembly system

For such a system, level 2 delivers the components to level 1 with a random discrete lead time. When the semi-finished product arrives at the level 1 it undergoes the necessary operations and afterwards the finished product is delivered to the customer in order to satisfy the demand D. It is assumed that each component of level 2 is used to assembly only one type of components of level 1. We assume also that the probability distributions of lead times for the components of all levels are also known in advance. Let the component lead times be random discrete variables with known distributions: $\Pr(L_{i,j}=k)$, $k= 1,2,\ldots,u_{i,j}$, for $j= 1, 2$ and $i=1,2,\ldots,N_j$. The variable $L_{i,j}$

takes into account all processing times at the level j plus transportation time between levels j and j-1. The release date $-X_{i,2}$, i=1,2,…,N_2, for the component $c_{i,2}$ can be calculated as follows:

$$-X_{k,2} = -x_{i,1} - x_{k,2}, \text{ where } c_{k,2} \in P_{i,1},$$

3 Mathematical model

Taking into account the fact that the different components on the same level do not arrive at the same time, there are stocks at level 1 and 2. The expected holding cost is given by (1). There is a stockout if the finished product is assembled after the due date (T=0). The probability of stockout is given by (2).

The problem is to minimize the cost given by (1) while maximizing the service level, i.e. minimizing the stockout probability given by (2):

Minimize $EC(X)$

Minimize $SP(X)$

Subject to:

$$\begin{aligned} EC(X) = H \times \sum_{s \in \mathrm{N}} \left(1 - \prod_{i=1}^{N_1} \left(\sum_{o_1, o_2 \in Z | o_1 + o_2 = s} \left(\Pr(L_{i,1} = o_1) \times \prod_{c_{k,2} \in P_{i,1}} F_{i,2}(X_{i,2} + o_2) \right) \right) \right) \\ + \sum_{i=1}^{N_1} H_{i,1} \times \left(\sum_{s \in \mathrm{N}} \left(1 - \prod_{c_{k,2} \in S_{i,1}} F_{k,2}(X_{k,2} + s) \right) \right) - \sum_{i=1}^{N_1} H_{i,1} \times \left(\sum_{s \in \mathrm{N}} \left(1 - \prod_{c_{k,2} \in S_{i,1}} (1 - F_{k,2}(X_{k,2} - s - 1)) \right) \right) \\ - \sum_{i=1}^{N_1} \left(\sum_{c_{k,2} \in P_{i,1}} h_{k,2} \{ E(L_{k,2}) - X_{k,2} \} \right) - \sum_{i=1}^{N_1} h_{i,1} E(L_{i,1}) \end{aligned} \quad (1)$$

$$SP(X) = 1 - \prod_{i=1}^{N_1} \left(\sum_{s \geq 0} \left(\Pr(L_{i,1} = s) \times \prod_{c_{k,2} \in P_{i,1}} F_{k,2}(X_{k,2} - s) \right) \right) \quad (2)$$

where,

$$2 \leq X_{i,2} \leq U_{i,2}, \quad j = 1,2 \quad \text{and } i = 1,2,\dots, N_j, \quad (3)$$

$$X = (X_{1,2}, \dots, X_{k,2}, \dots, X_{N_2,2}),\ H = \sum_{i=1}^{N_1} h_{i,1},\ H_{i,1} = -h_{i,1} + \sum_{c_{k,2} \in P_{i,1}} h_{k,2} \ \text{ and } \ F_{i,j}(x) = \Pr(L_{i,j} \leq x)$$

Obviously the two objective functions usually do not have the same optimum (actually they are somehow conflicting). Minimizing in this context means identifying a set of trade-off solutions.

4 Optimization algorithm

The problem considered in this paper has two non linear objective functions with integer variables. Due to the combinatorial explosion, the explicit enumeration of the entire search space becomes impossible when the size of the problem is significant. In [4], a metaheuristic based on genetic algorithm improved by local search was suggested. This procedure can solve large scale problems. Computational experiments show the efficiency of the genetic algorithm as well as the ability of the local search to speed up its convergence.

The idea in this paper is to use this genetic algorithm embedded within a parameterized procedure to perform successively and independently the search on different scalarization functions, leading to different trade-offs between the two objective functions. Thus we can use the genetic algorithm to solve the following problem (4-5):

$$\text{Min} \quad \alpha \times EC(X) + (1-\alpha) \times SP(X), \text{ with } \alpha \in [0,1] \tag{4}$$

$$\text{Where,} \quad 2 \leq X_{i,2} \leq U_{i,2}, \; j = 1, 2 \text{ and } i = 1, 2, \ldots, N_j \tag{5}$$

The whole set of solutions which are in the final population for one of the α considered are kept. Then we can compute the subset of solutions which are efficient trade-off in the sense of Pareto (i.e., there is no other solution in the set which is better on both objective functions).

The algorithm is tested for 120 randomly generated instances. For theses instances, the number of components at level 2 varies from 10 to 120. For each number of components, 10 different instances were generated. The Table 1 reports the average number (for the instances with same size) of efficient trade-off solutions obtained with this algorithm considering 20 different values for α. Obviously, since the multiobjective approach considered is basic, its performance quickly decreases when the size of the problem increases. However it permits to highlight that some very different trade-off can be obtained for a large number of instances. This behavior is illustrated in Figure 2 where the solutions obtained for one instance are presented. Outside of the particular case of the solution with a stockout probability equal to 1 which has a very low practical utility, a set of 30 different trade-off have been obtained within a large range of stockout probability (from 0% to 32.8%).

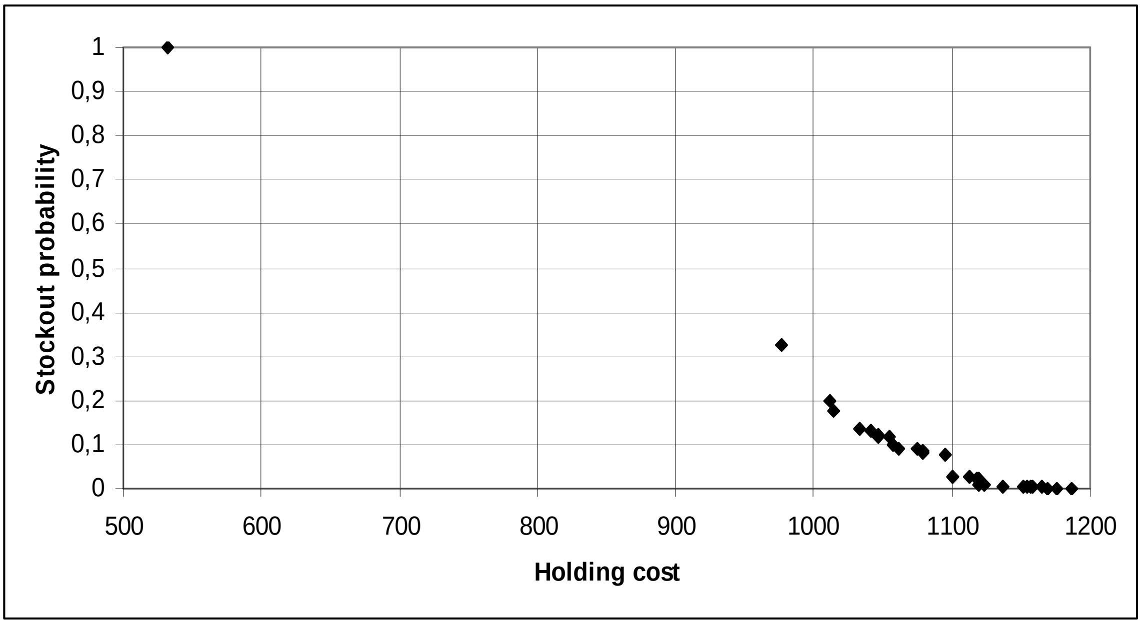

Fig 2: Example of trade-off between stockout probability and holding cost

for one run on instance of size 20

Table 1: Average number of trade-off solutions obtained with the genetic algorithm

N_2	10	20	30	40	50	60	70	80	90	100	110	120
Average number of trade-off	28.5	8.8	5.3	5.8	6.2	4.7	5.3	5.9	5.3	5.4	5.2	5.4

5 Conclusion and perspectives

A problem, dealing with supply planning for two level assembly systems under random actual lead times was studied. A new model was proposed with two criteria: the inventory cost and the customer service level. In contrast with the known approaches which are usually based on continuous time models, the suggested method uses discrete optimization techniques with integer decision variables. This is more appropriate for MRP environment where the planning horizon is divided into discrete periods (bucket times). For this problem, a basic multiobjective metaheuristic was proposed to find trade-off between holding cost and stockout probability.

Further research should be focused on the development of more effective multiobjective metaheuristic (see e.g. [3]). Another path for future research would deal with multilevel assembly systems, i.e. with multi-level bill of material. Logically, difficulty will increase because of dependence among levels in addition to the dependence among inventories.

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