Optimization of Composite Structures by Estimation of Distribution Algorithms

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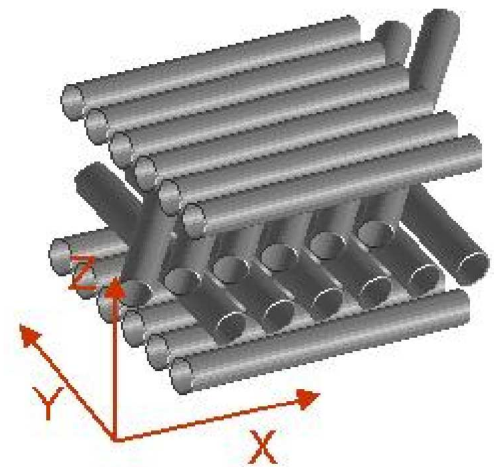

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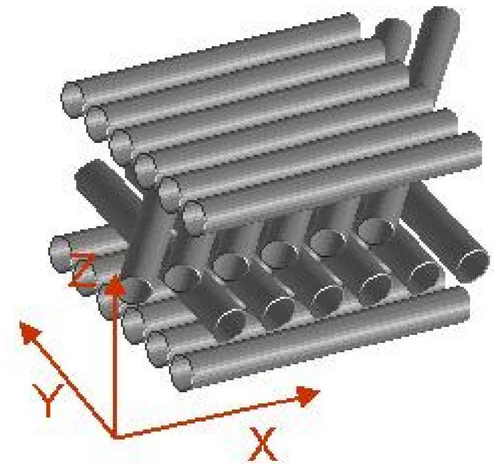

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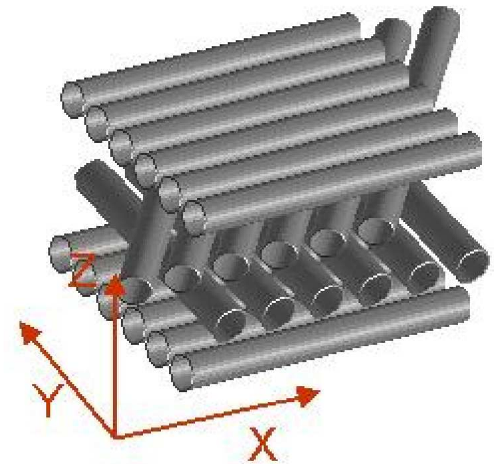

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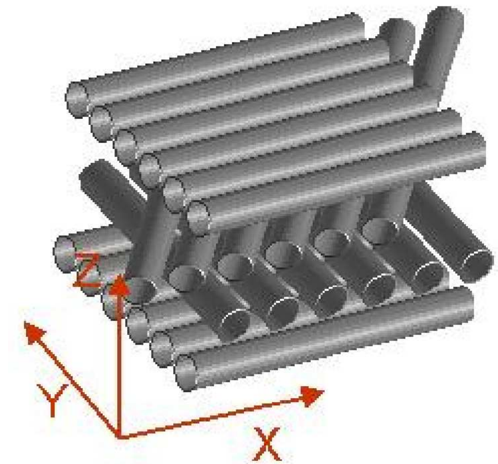

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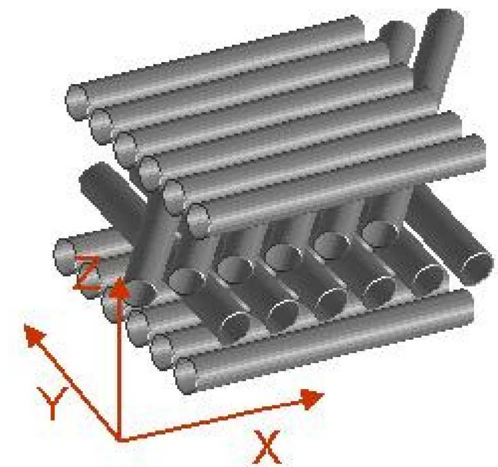

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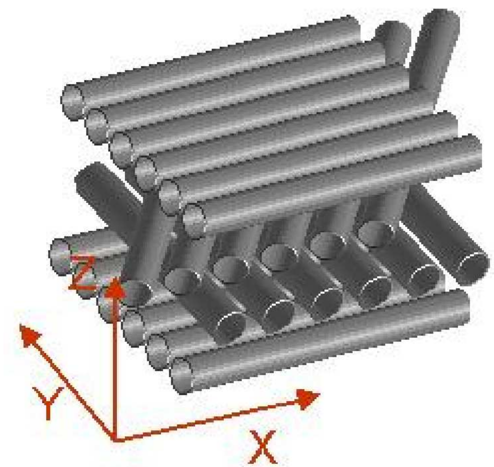

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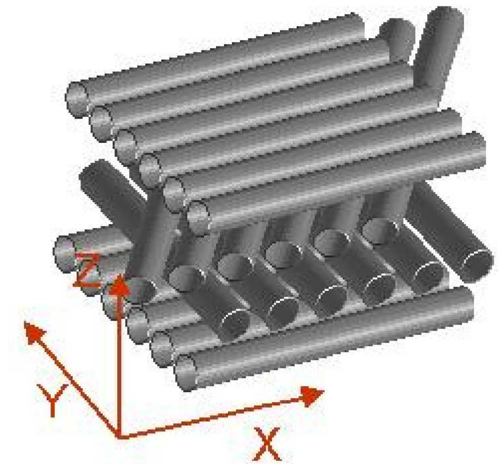

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  - déterminer la séquence d'empilement de la coque d'une voiture de course de manière à minimiser les vibrations

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- Méthodes traditionnelles basées sur les gradients mais problèmes continus uniquement et recherche locale

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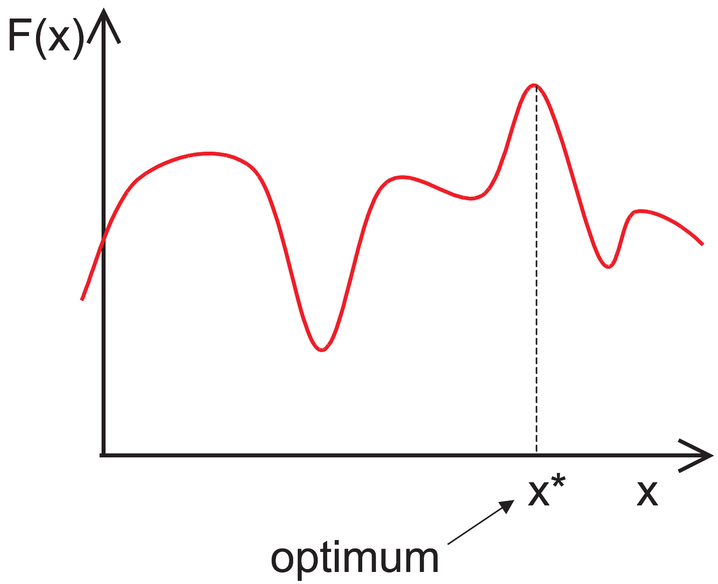
Objectifs de la thèse :

- Maturation des méthodes évolutionnaires depuis 20 ans (ES, EDA)
- Transférer ces nouvelles méthodes à l'optimisation de composites :
  - utilisation plus simple
  - méthodes plus efficaces

Algorithmes à estimation de distribution



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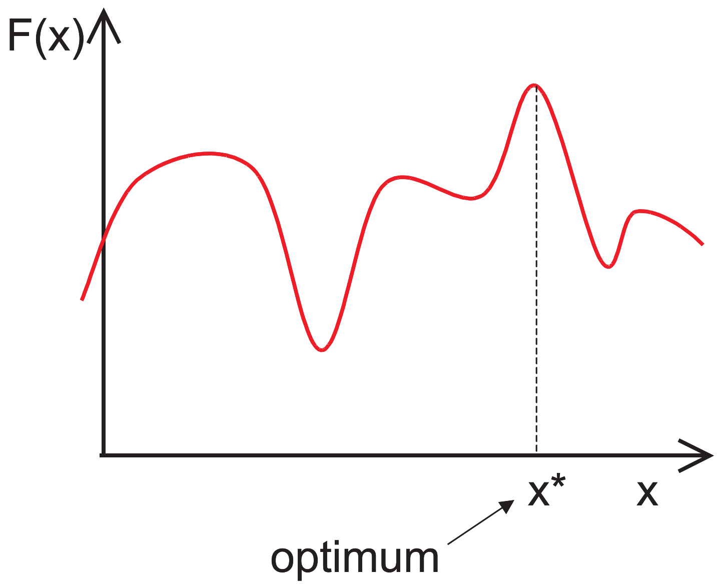

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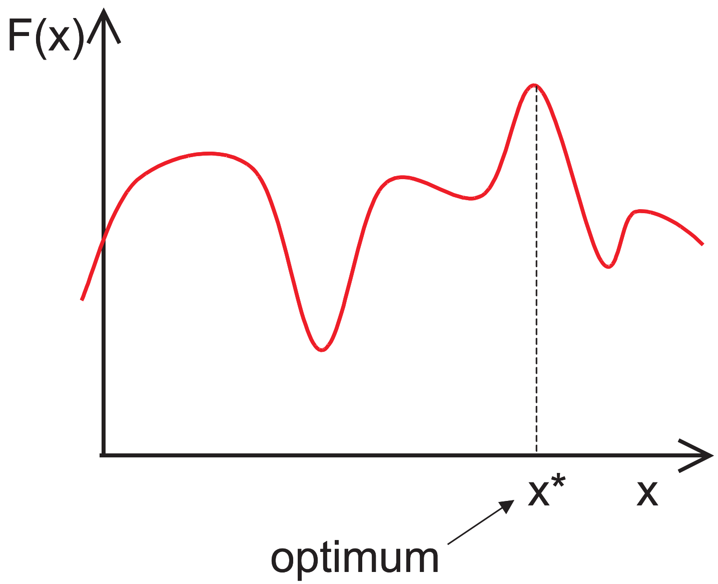

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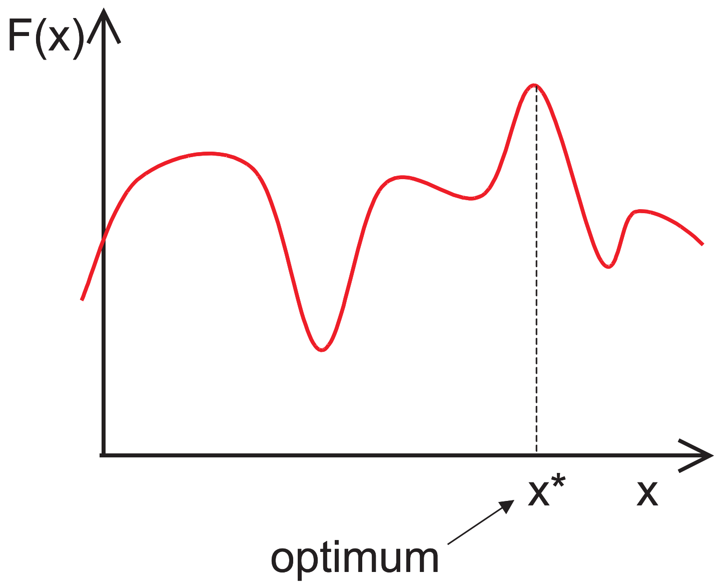

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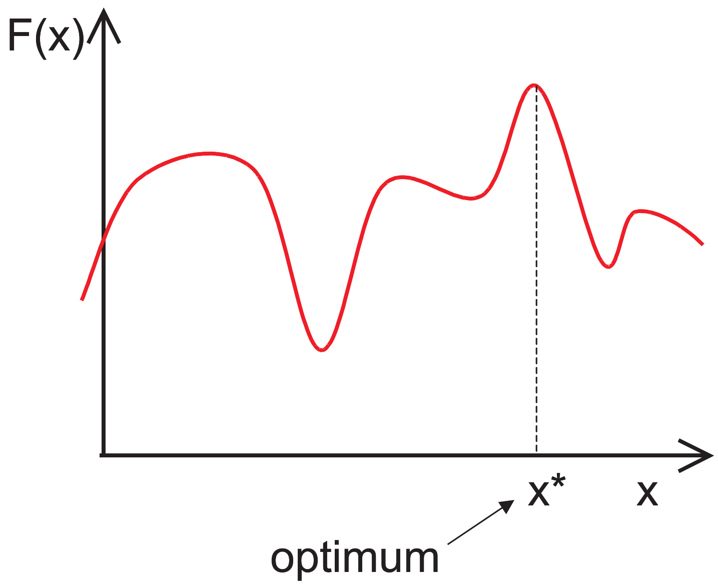

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  - les nouveaux points sont utilisés pour mettre à jour $p(\mathbf{x})$

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- La stratégie proposée conduit à une amélioration de l'efficacité sur les problèmes testés

Introduction to Composite Laminate Optimization

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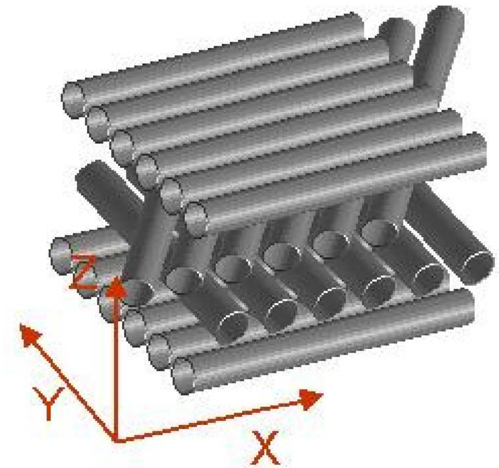
- Composite laminate: structure made of layers (plies) of fibrous material embedded in a matrix



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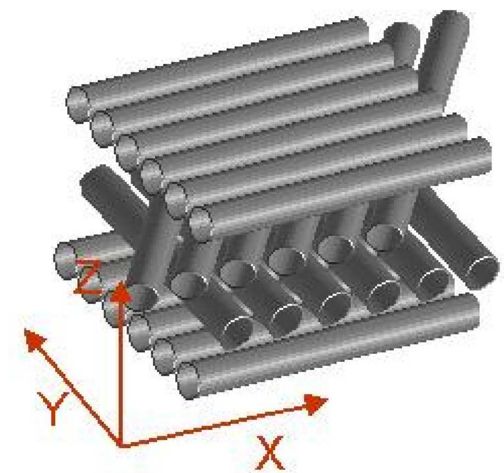

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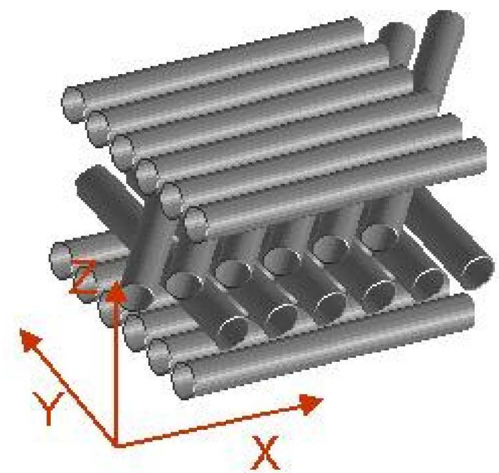
  - sporting goods (skis, sailing, tennis)
  - wind turbines
  - motorsports and performance cars



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⇨ Require specific optimization methods

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2. propose improvements to EDAs in the context of laminate optimization:
   use physics-based knowledge to improve efficiency
3. study the general behavior of EDAs and propose improvements to EDAs that can be used for other fields.

Introduction to Estimation of Distribution Algorithms

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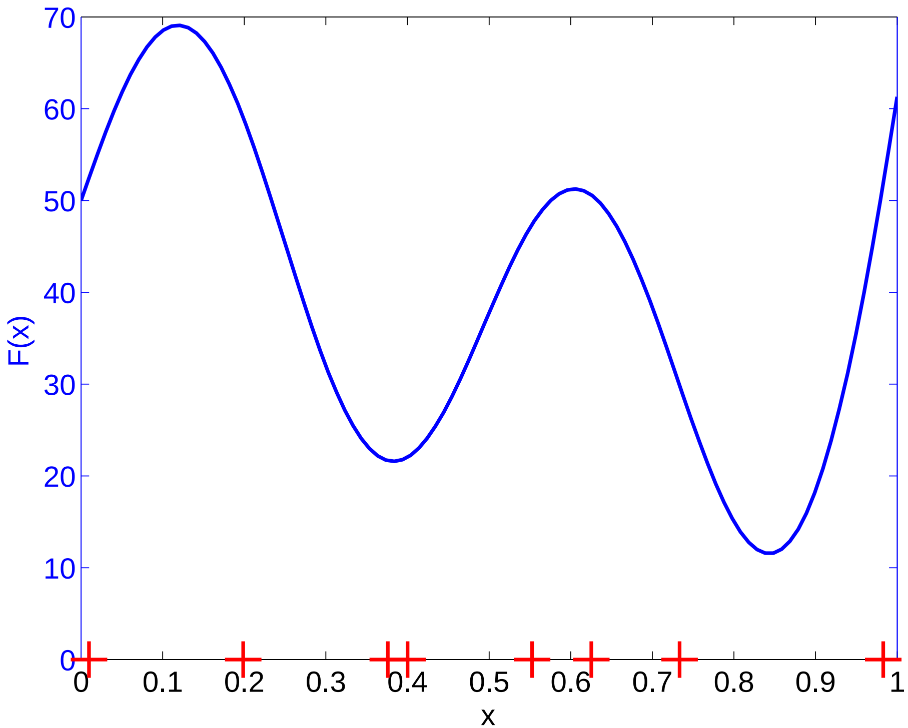
  - Environment $\rightarrow$ Function $F(\mathbf{x})$ to maximize ("fitness")
  - Population of individuals $\rightarrow$ population of points $\mathbf{x}_i$, $i = 1, \ldots, \lambda$
  - Natural selection $\rightarrow$ Selection based on F
  - Reproduction $\rightarrow$ recombination (crossover) and perturbation (mutation) operators

GA Mechanism



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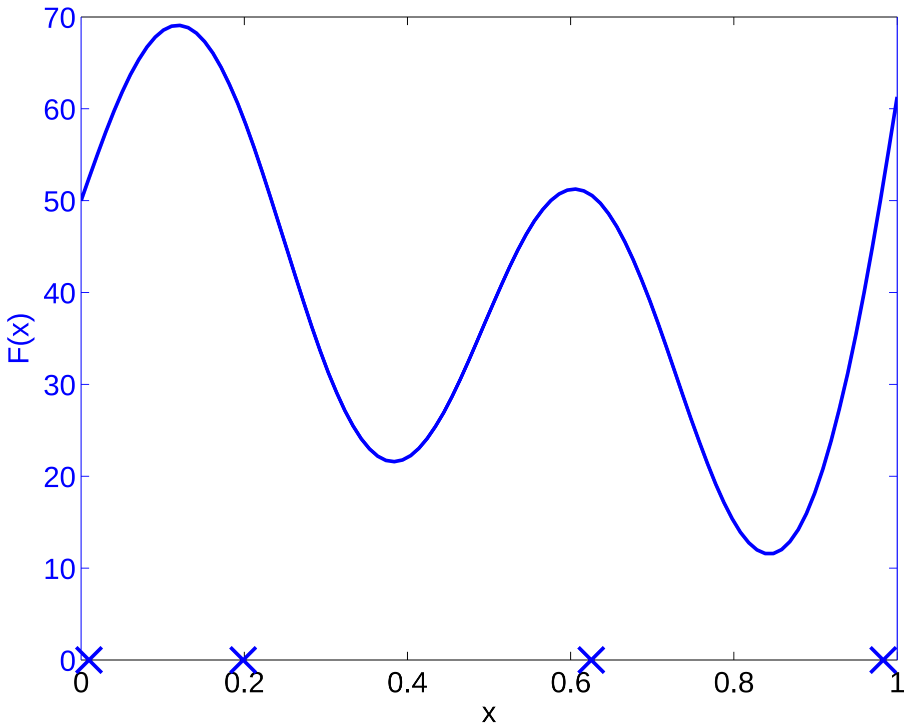

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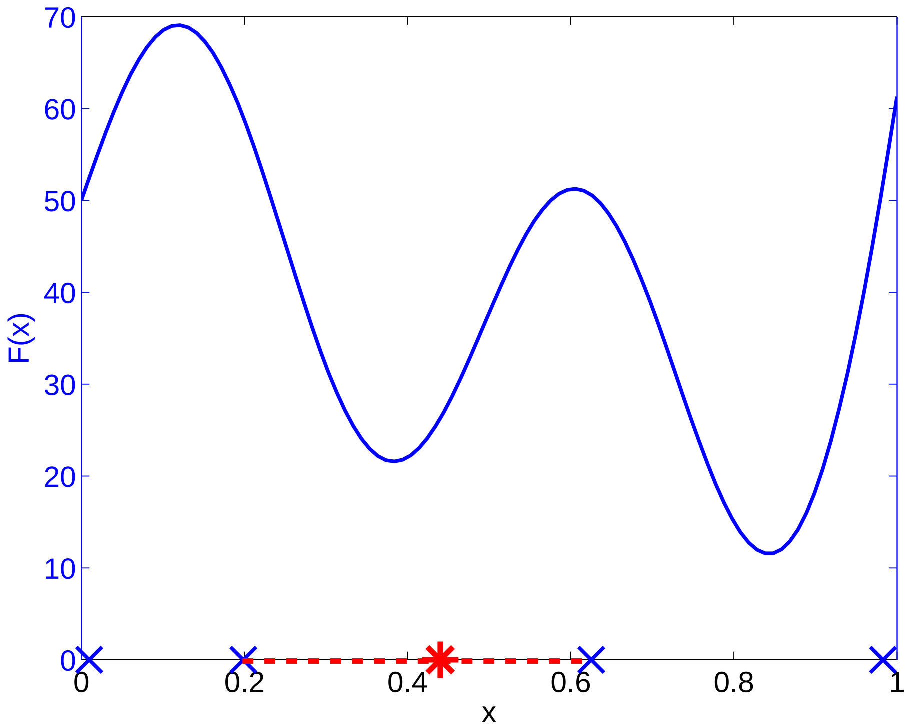

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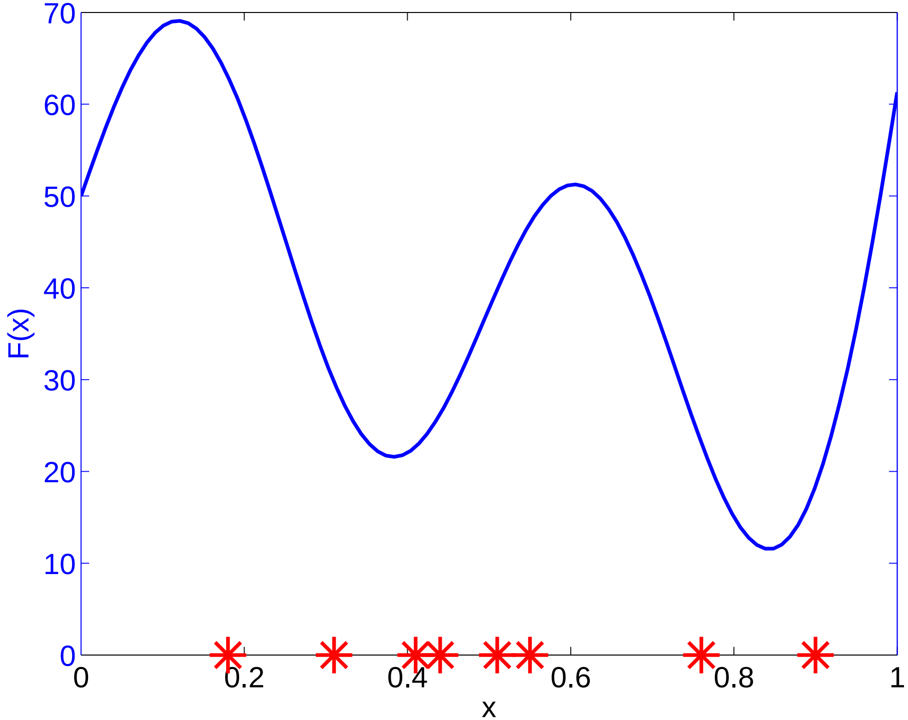

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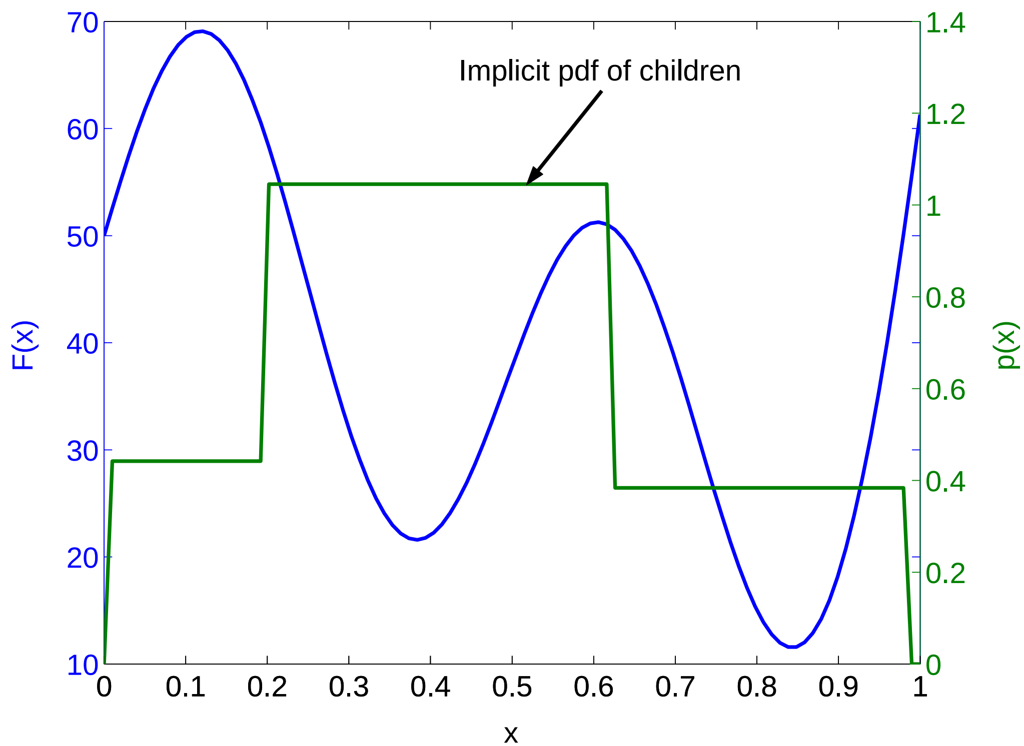

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	fitness function F		implicit
+	selection procedure	}	⇨ probability distribution $p(\mathbf{x})$
+	variation operators		over the design space

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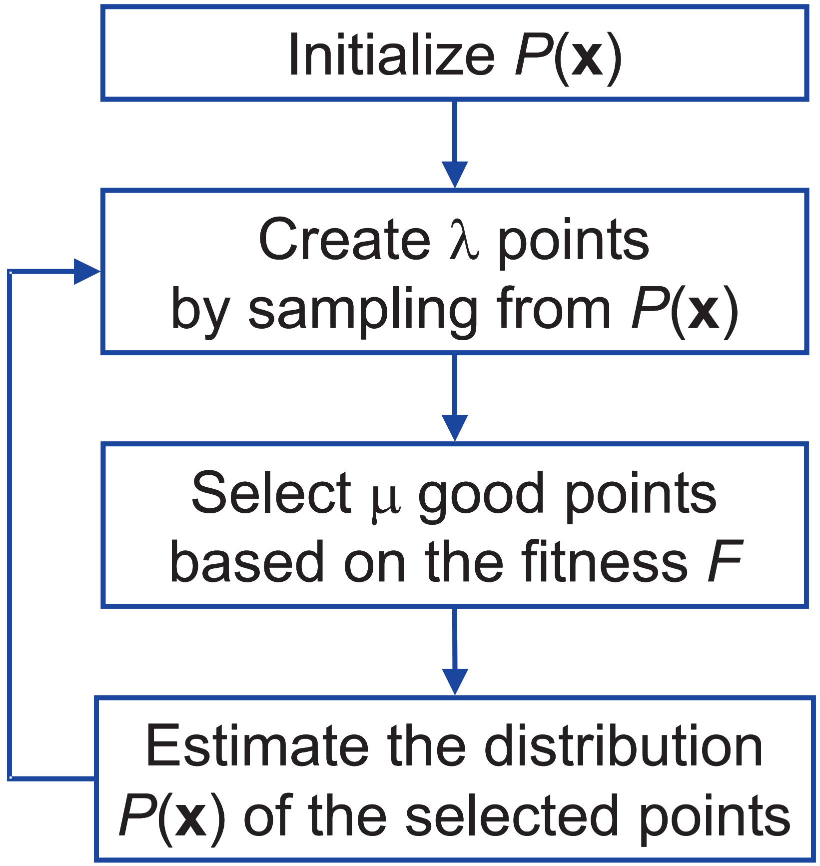

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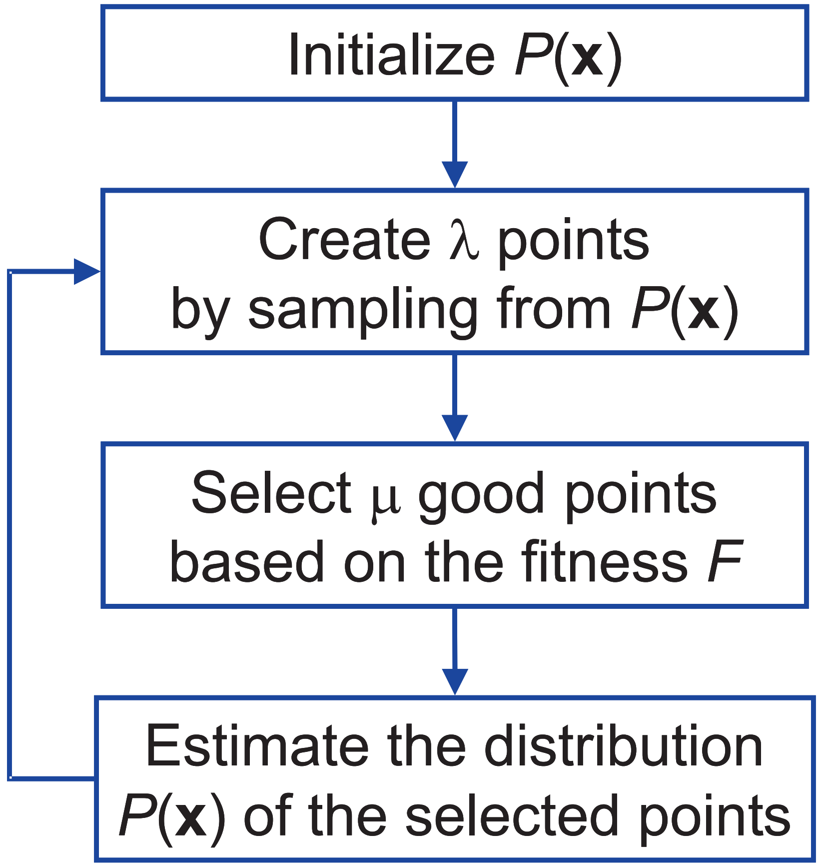

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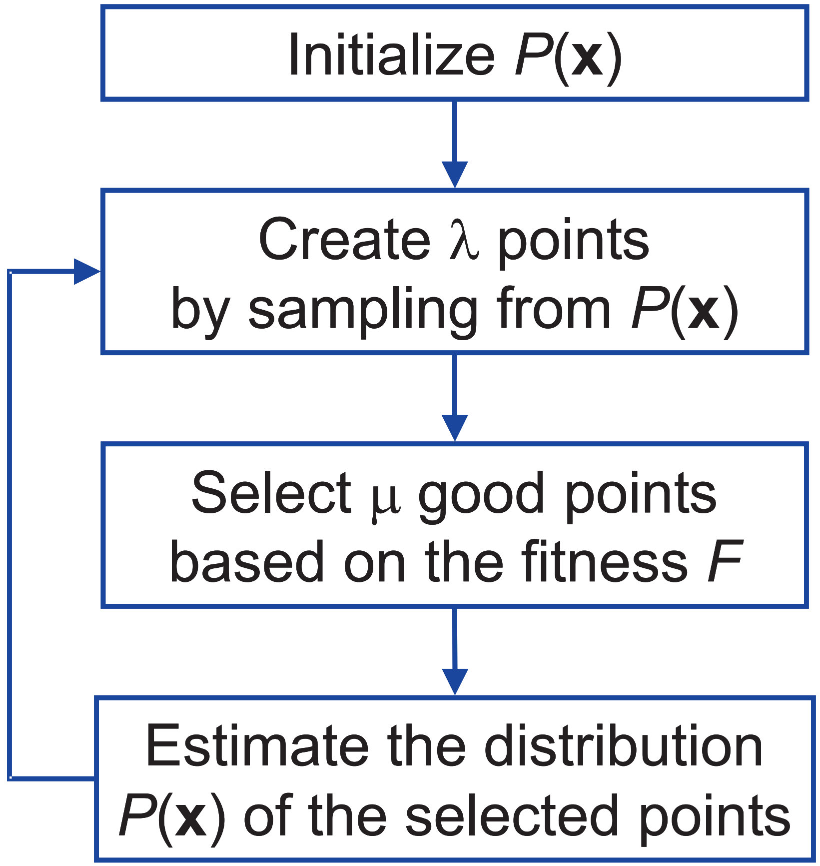

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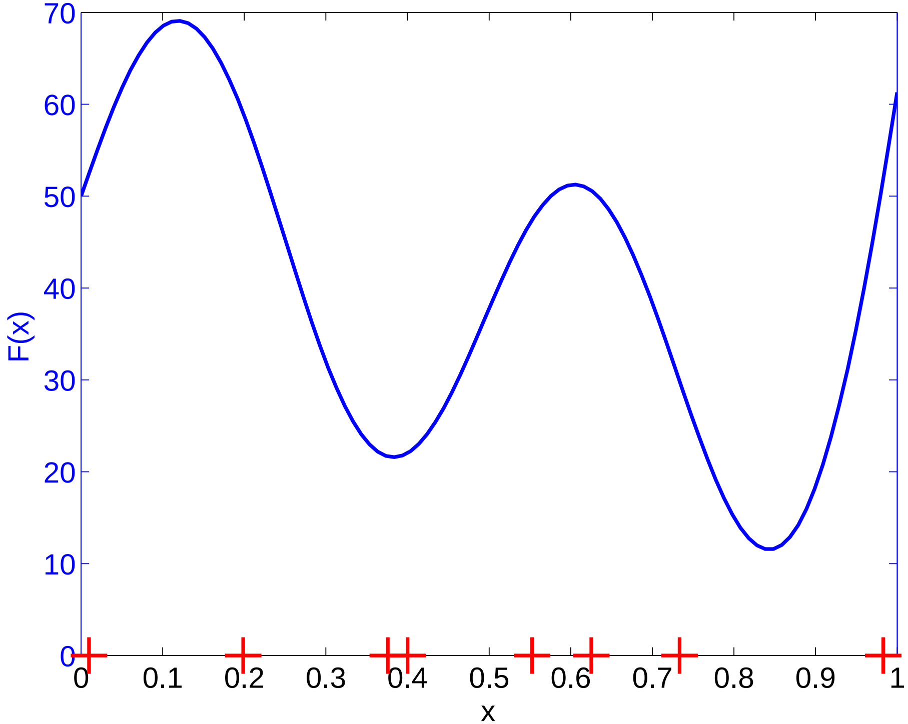
Estimation of Distribution Algorithm: Illustration



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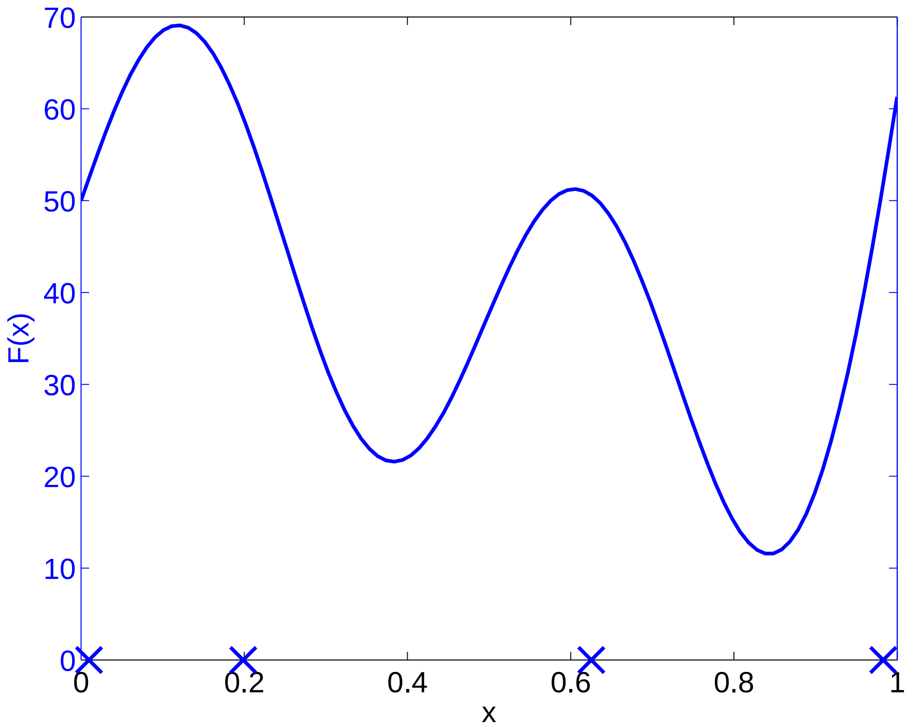

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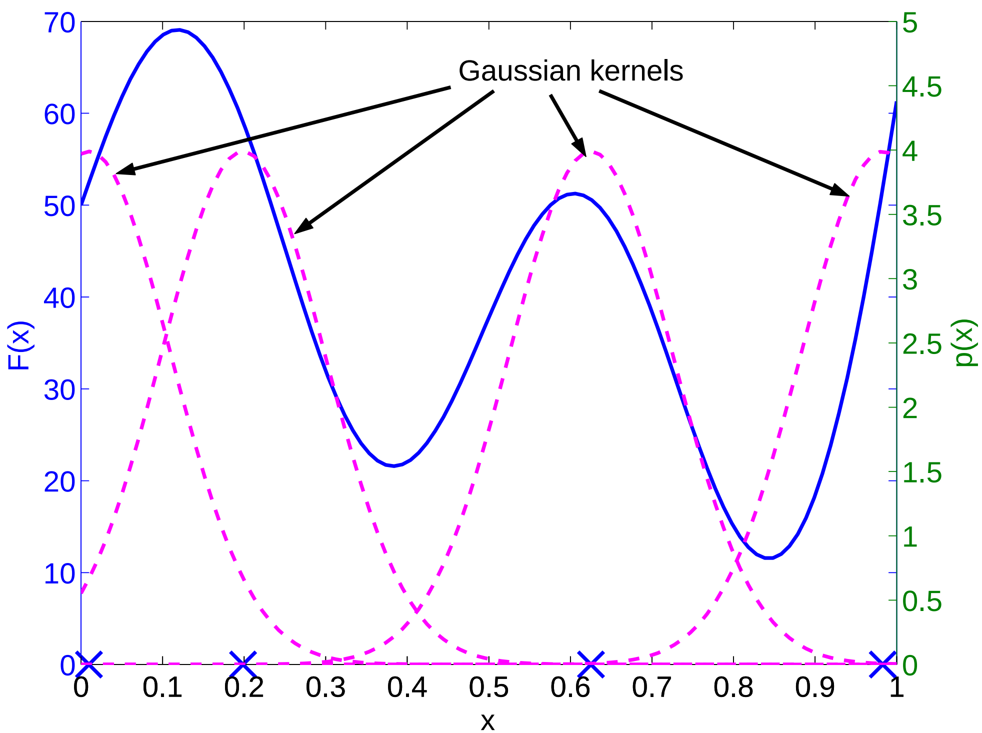

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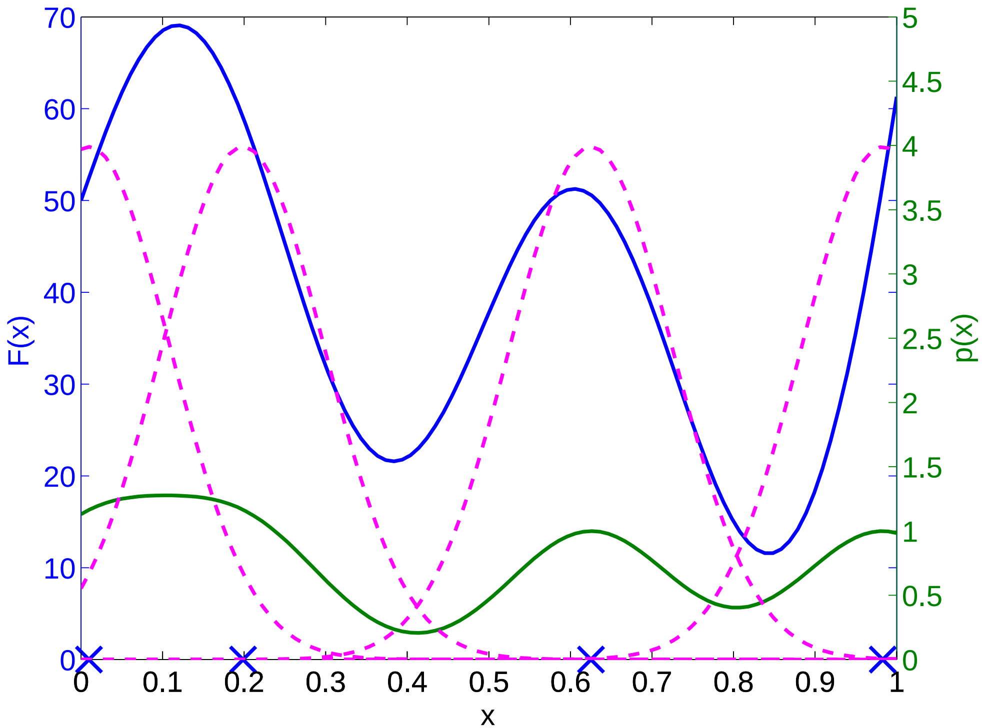

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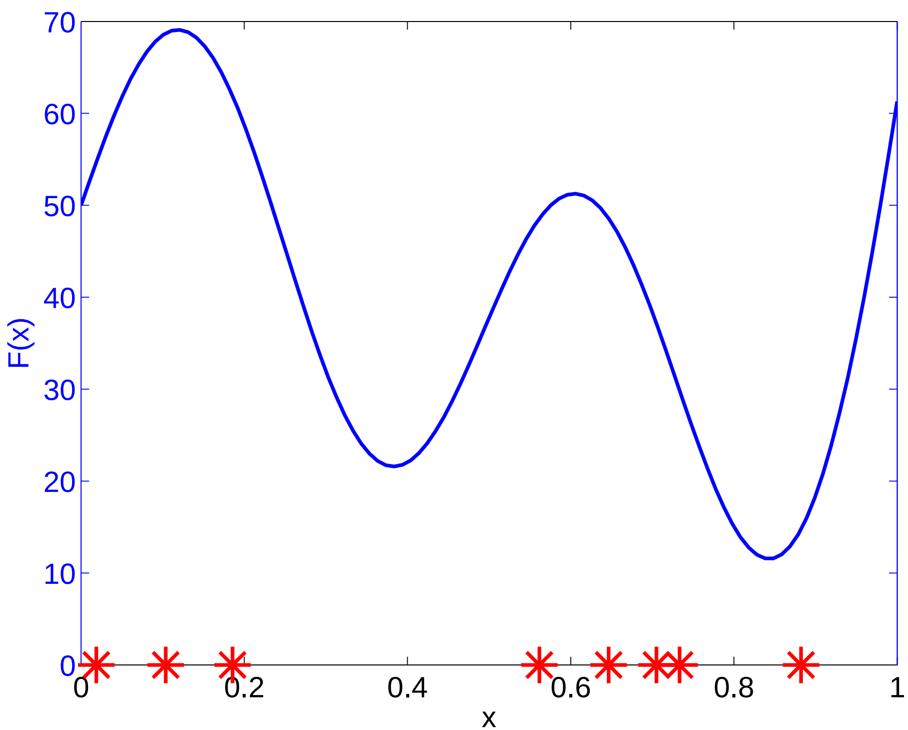

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Application of a Simple EDA to Laminate Optimization

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- Changes: non-binary alphabet, mutation to compensate for estimation error

Application to Laminates: Frequency problem

- Constrained maximization of the first natural frequency of a simply-supported rectangular laminated plate:

maximize $f_1(\theta_1, \ldots, \theta_{15})$
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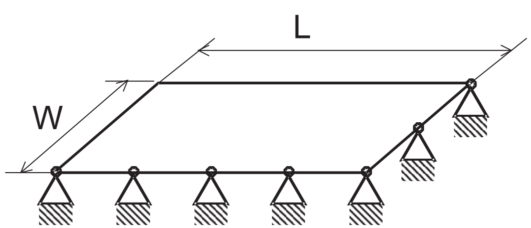
Poisson's ratio = deformation observed in the transverse direction when a unit deformation is applied in the longitudinal direction



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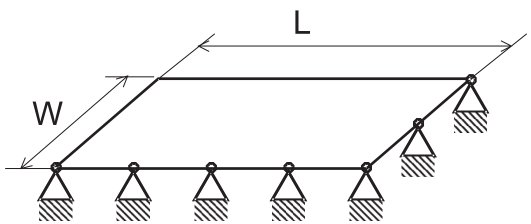

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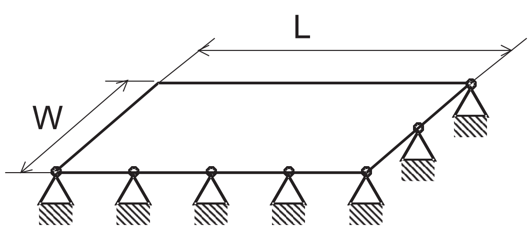

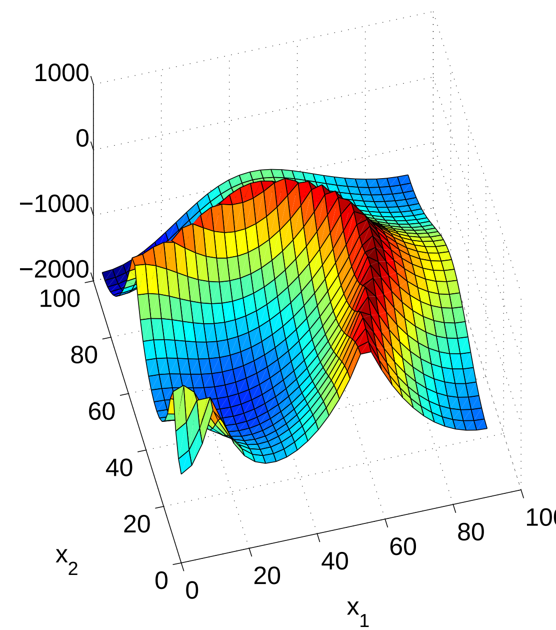

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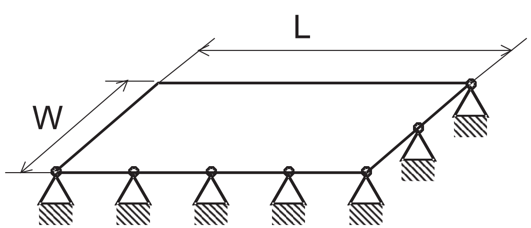

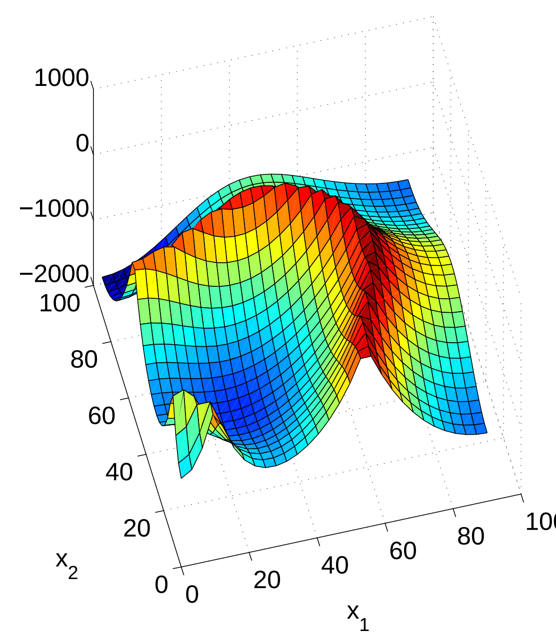

- $\theta_k \in \{0°, 15°, 30°, 45°, 60°, 75°, 90°\}$
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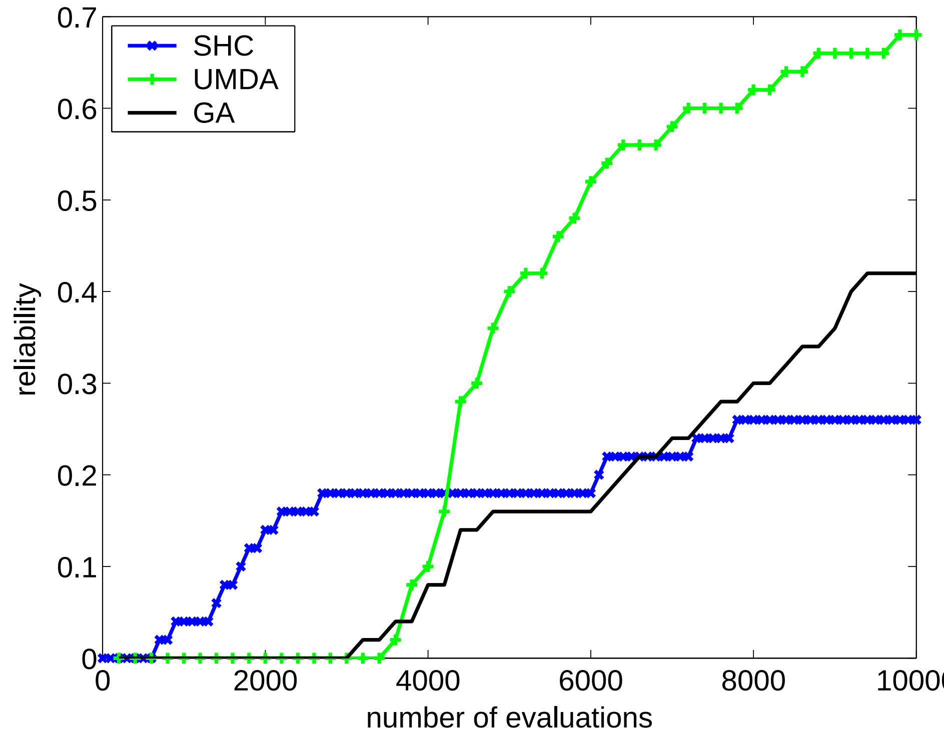

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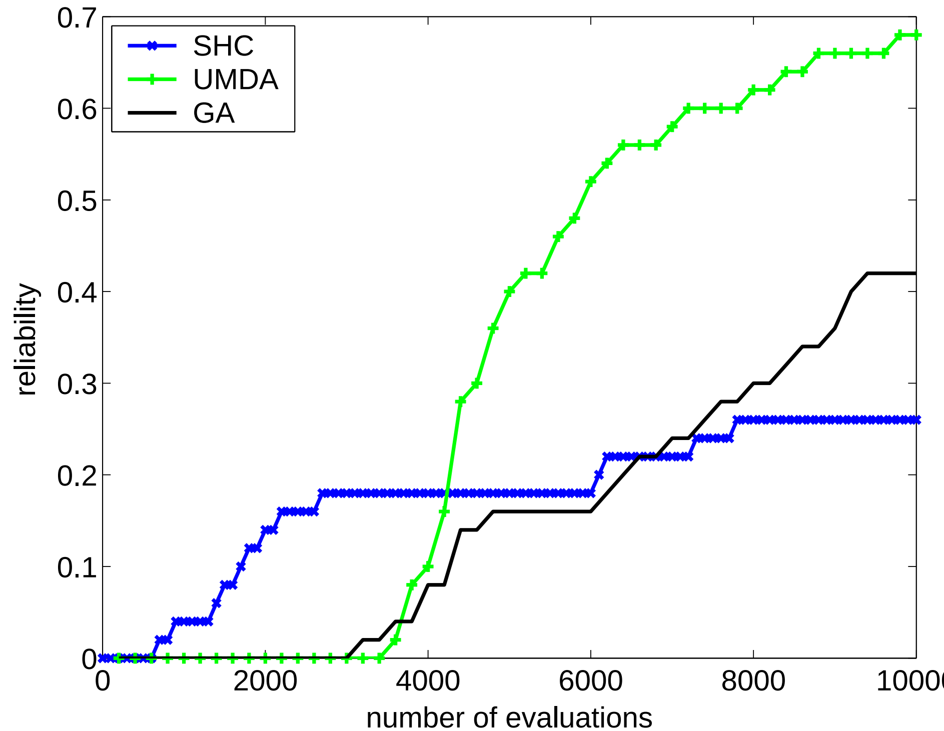

- ➪ UMDA outperforms SHC because its distribution approach (global) allows it to handle narrow search spaces
- ➪ the performance of UMDA is substantially higher than that of GA for this problem

Improvement of the Statistical Model of Selected Points through Auxiliary Variables: the Double-Distribution Optimization Algorithm

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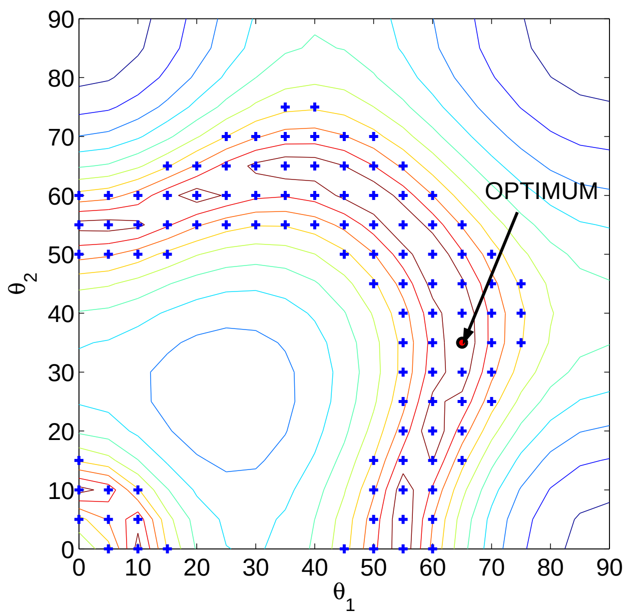

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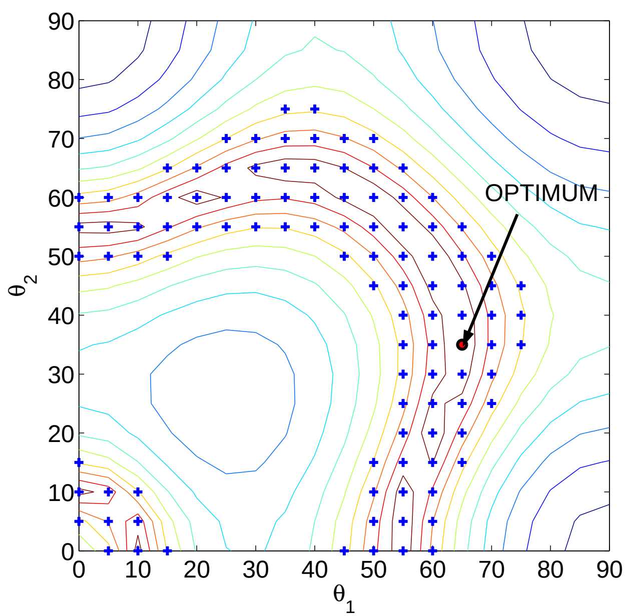

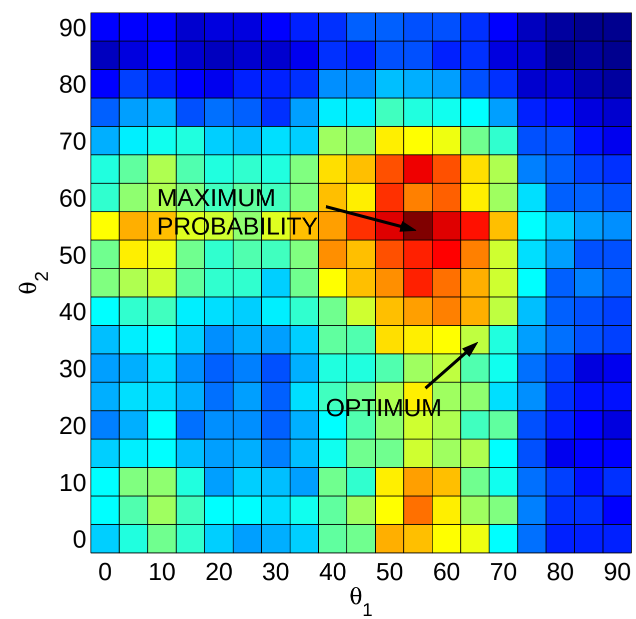

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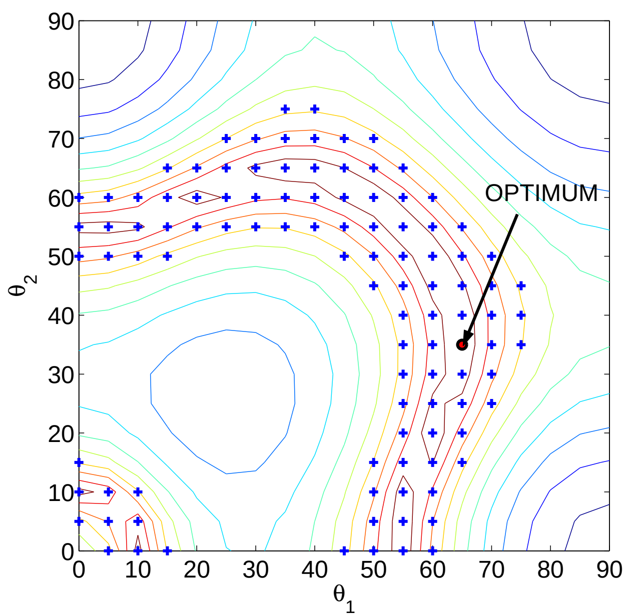

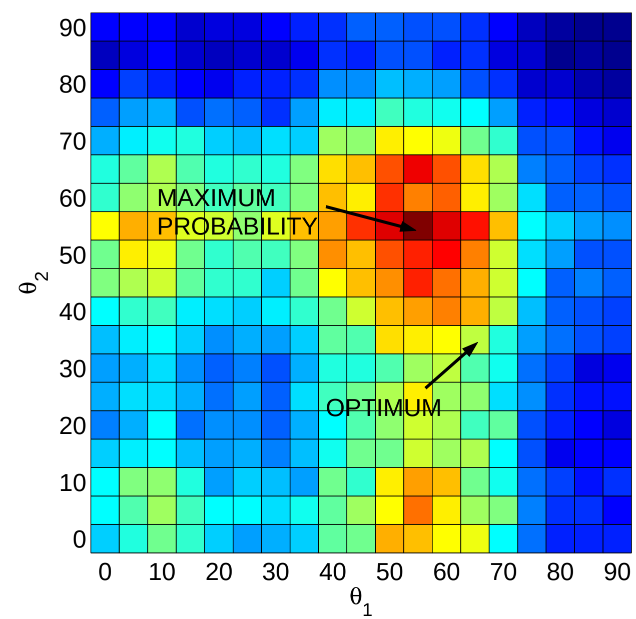

- High-probability areas do not coincide with high-fitness regions

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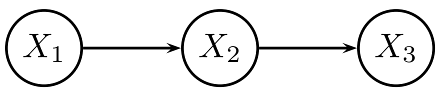

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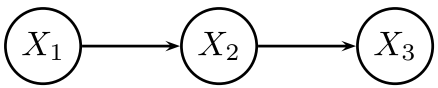

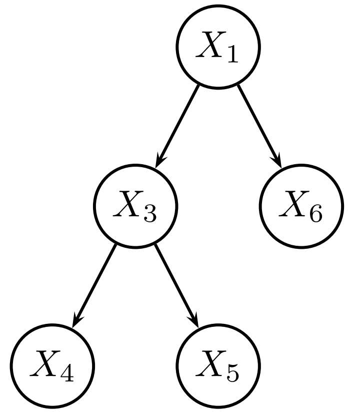

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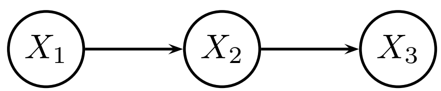

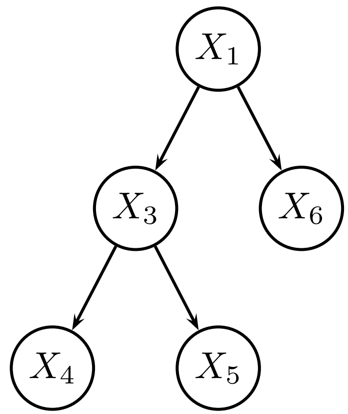

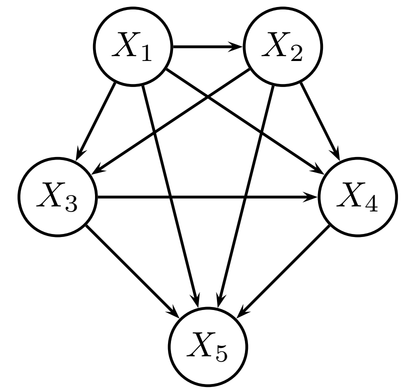

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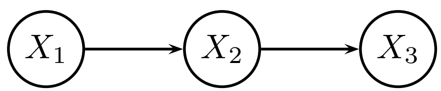

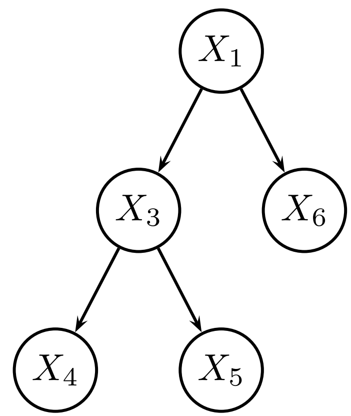

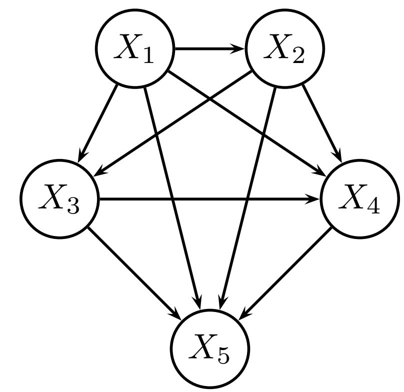

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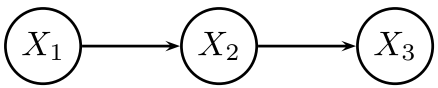
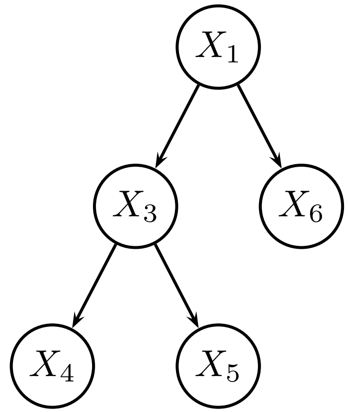
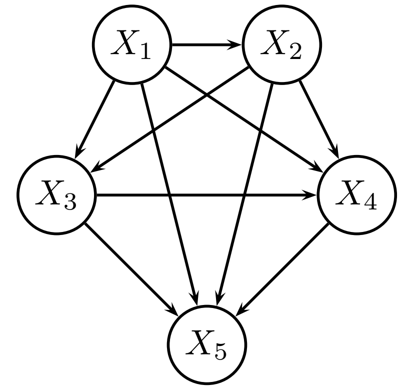


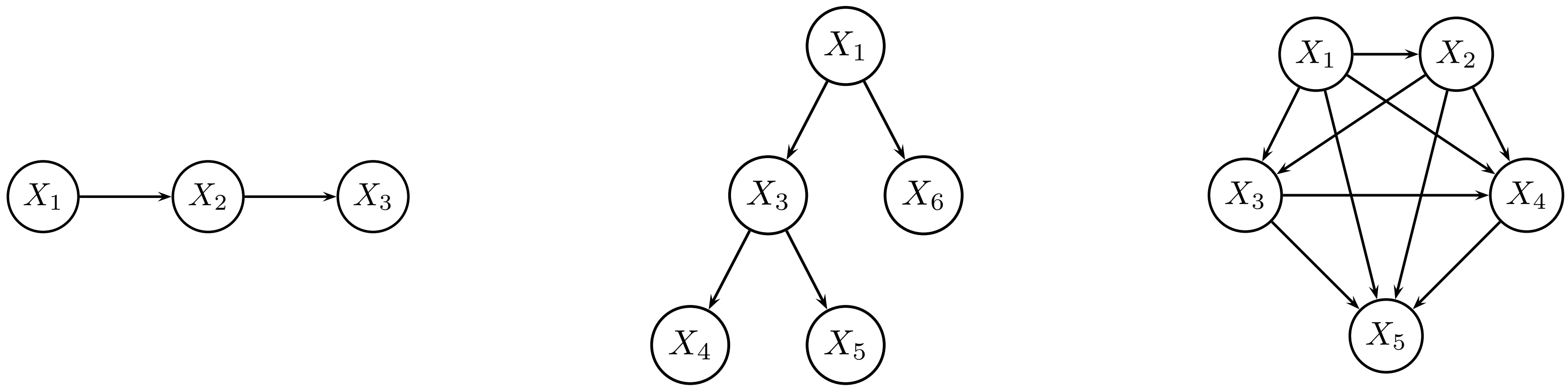




- Disadvantages:
  - The number of parameters m_j to estimate increases rapidly with the model complexity

Representation of variable dependencies

- Complex statistical models have been tried: chain models, tree models, full Bayesian networks (*Pelikan, 1999*)



- Disadvantages:
  - The number of parameters m_j to estimate increases rapidly with the model complexity
  - ➪ the population size needed to estimate these parameters ensure (with a constant confidence) increases with the model complexity because flexible models do not generalize well the information contained in the sample to other regions

Representation of joint actions of the variables via auxiliary variables

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- These meaningful quantities $\mathbf{V}$ reflect joint actions of the variables $\mathbf{x}$ and can be used as auxiliary variables to capture such interactions

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- The distribution $p(\mathbf{x})$ is represented by a simple model (UMDA)
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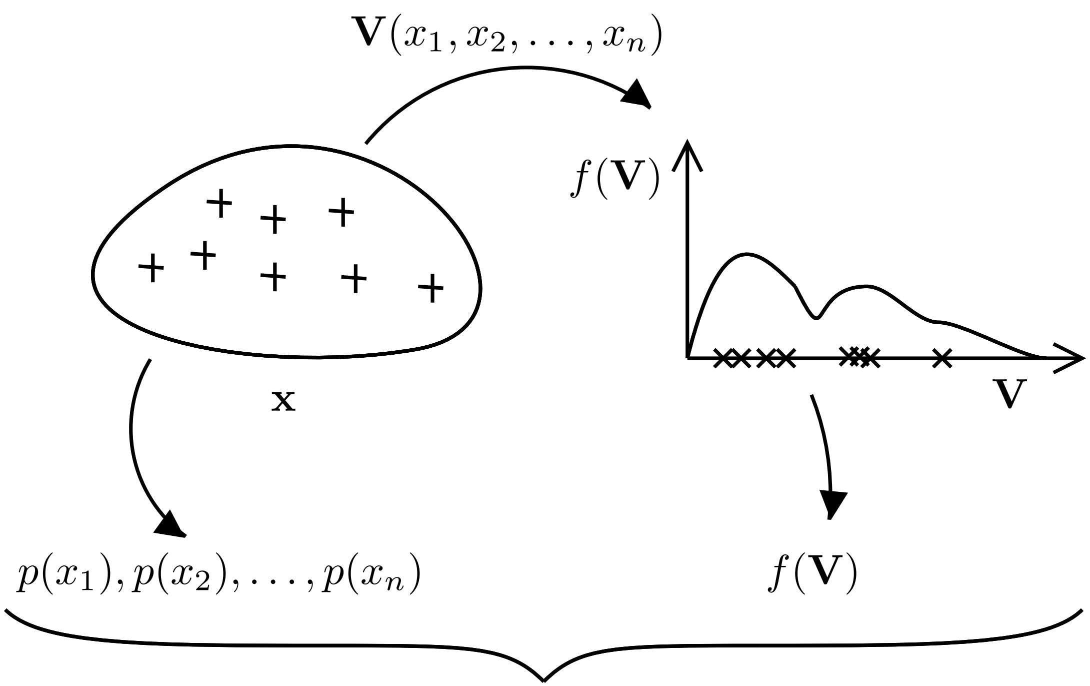

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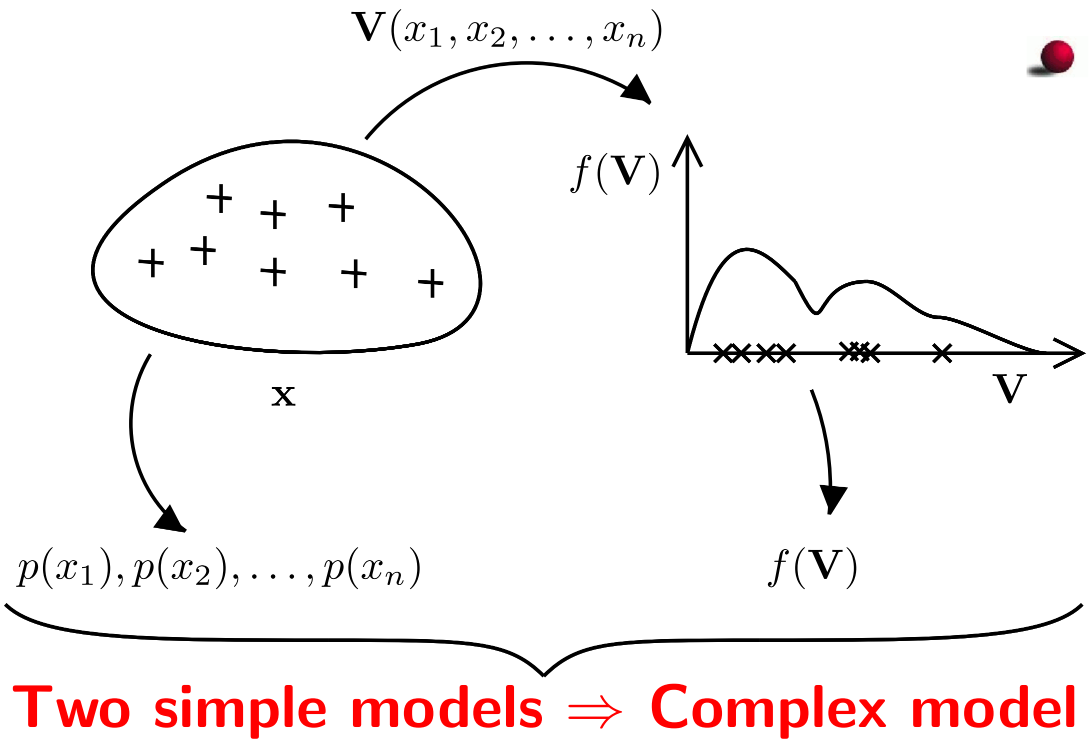

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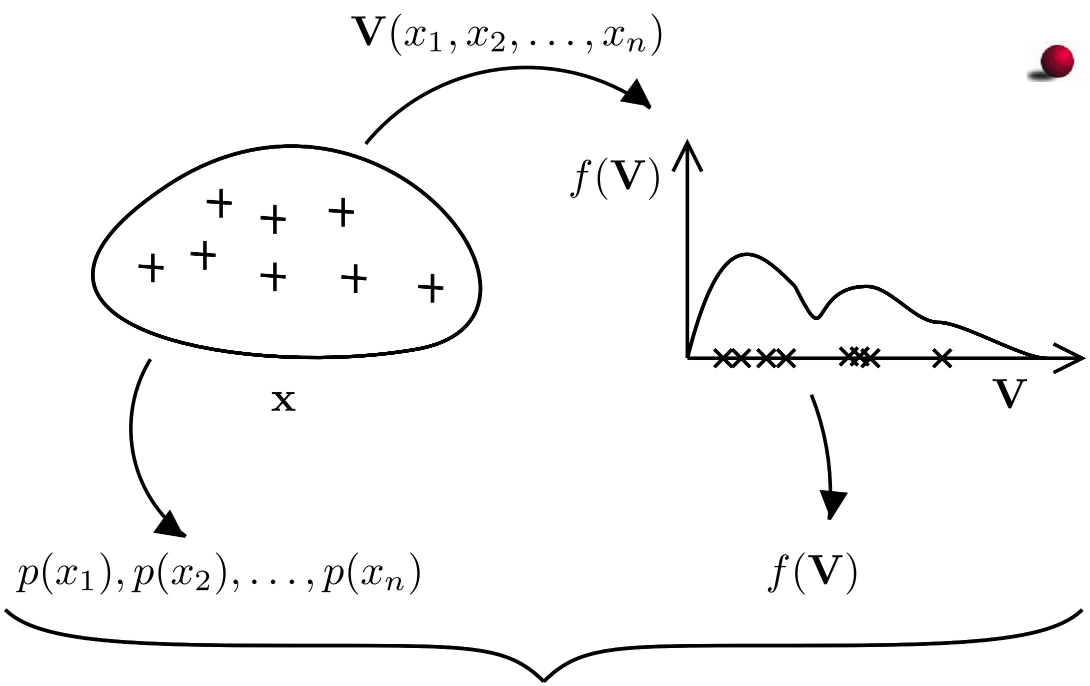

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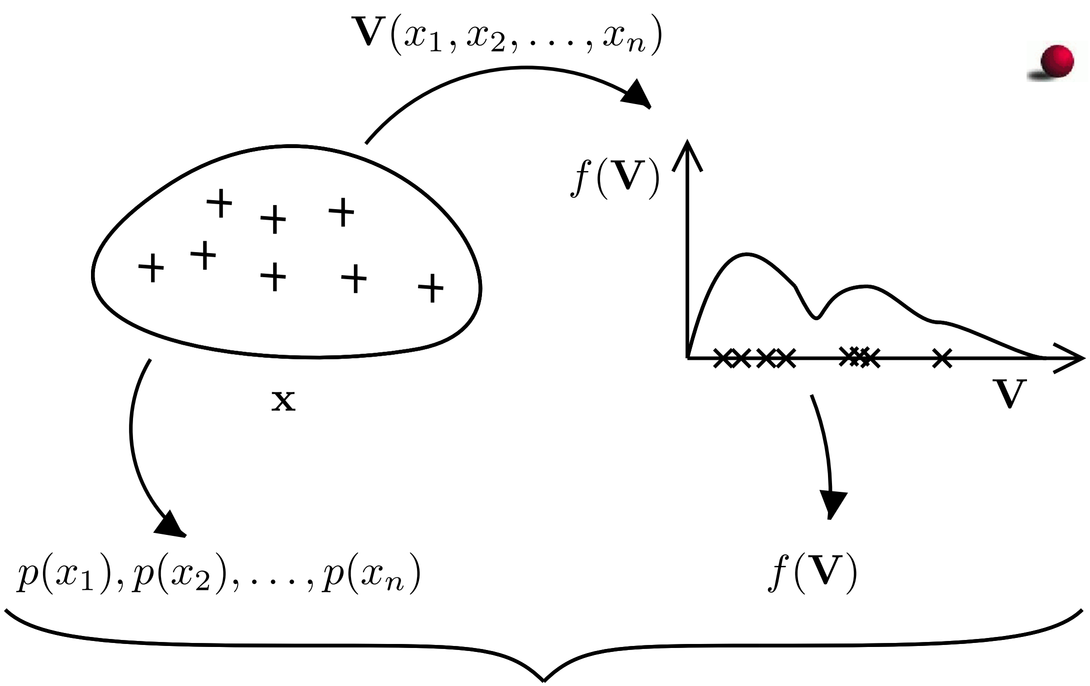

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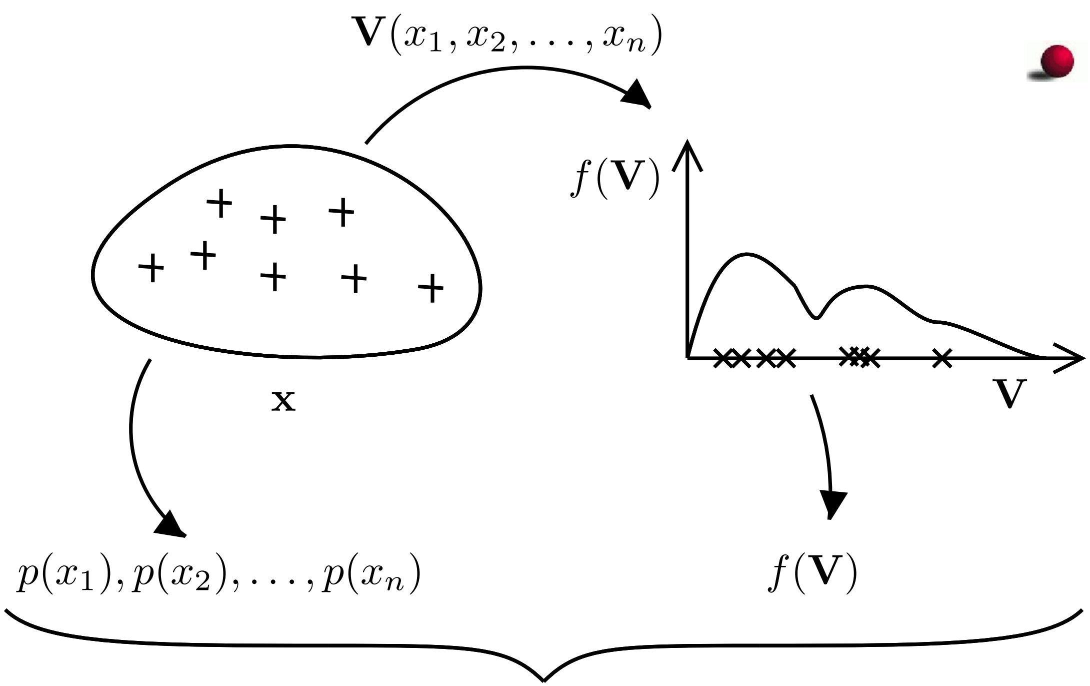

Two simple models $\Rightarrow$ Complex model

- Required attributes of the V's:
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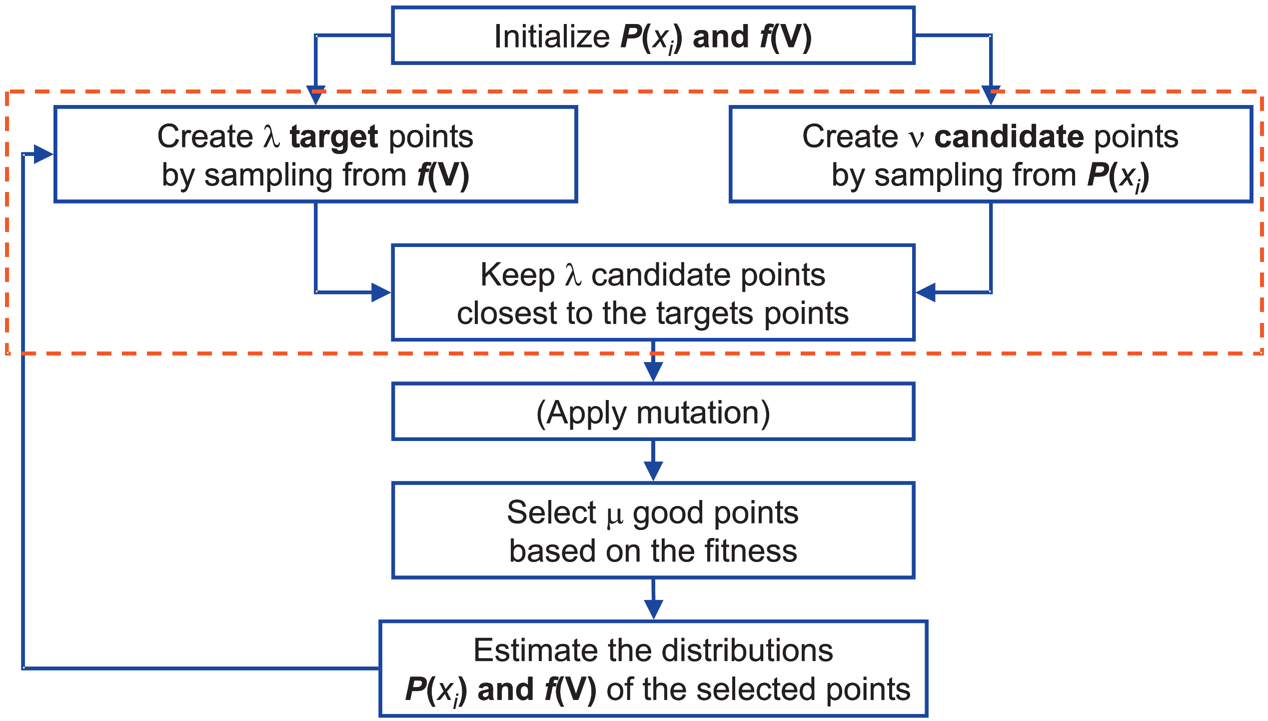
  3. inexpensive to compute

The Double-Distribution Optimization Algorithm



Goal: create points whose distribution that reflects both $p(\mathbf{x})$ and $f(\mathbf{V})$:

- $p(x_i)$ provides a pool of points that have correct marginal distributions.
- $f(\mathbf{V})$ is used as a filter that favors promising regions.
- The relative influence of the 2 distributions is adjusted through ν/λ

Application to Laminate Optimization Problems

Case of composite laminates: Auxiliary variables = “Lamination Parameters”

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h: total laminate thickness, U_i's: material invariants

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- Symmetric balanced laminates $[\pm\theta_1, \pm\theta_2, \ldots, \pm\theta_n]_s$:

$$V^*_{\{1,3\}} = \frac{2}{h}\int_0^{h/2} \{\cos 2\theta, \cos 4\theta\} dz = \frac{1}{n}\sum_{k=1}^{n} \{\cos 2\theta_k, \cos 4\theta_k\}$$

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Distribution in the $\mathbf{V}$-domain

- Continuous variables
- Use kernel density estimate:

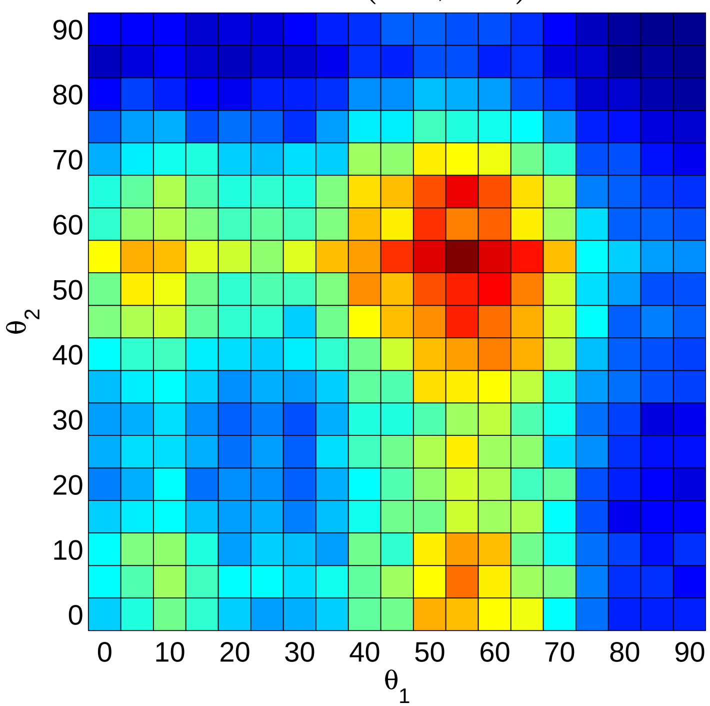
$$f(\mathbf{V}) = \frac{1}{\mu} \sum_{i=1}^{\mu} K\left(\mathbf{V} - \mathbf{V}_i\right)$$

In this work, we used Gaussian kernels:

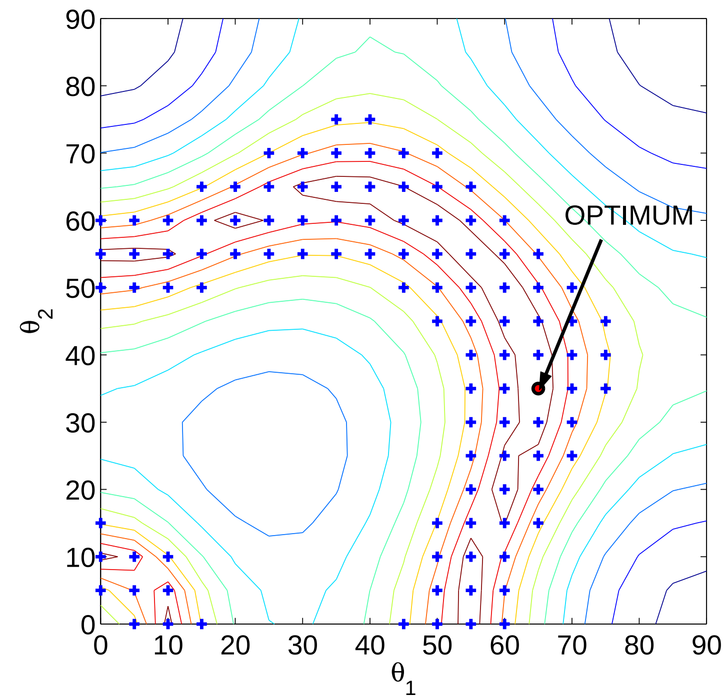
$$K(\mathbf{u}) = \frac{1}{(2\pi)^{d/2}\sigma^d} \exp\left(-\frac{\mathbf{u}^T\mathbf{u}}{\sigma^2}\right)$$

Estimated Distributions: UMDA and DDOA

UMDA: probability concentrated around $(55, 55)$



Fitness function and selected points



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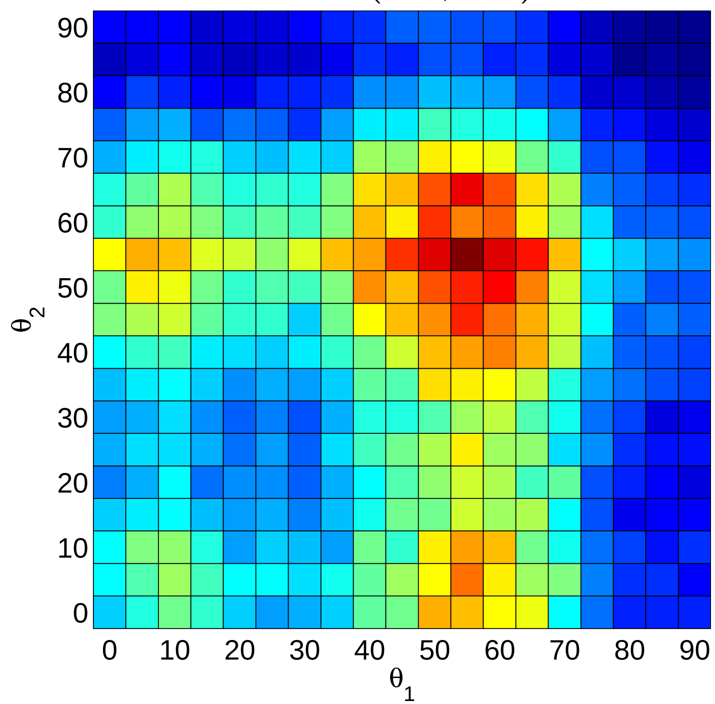
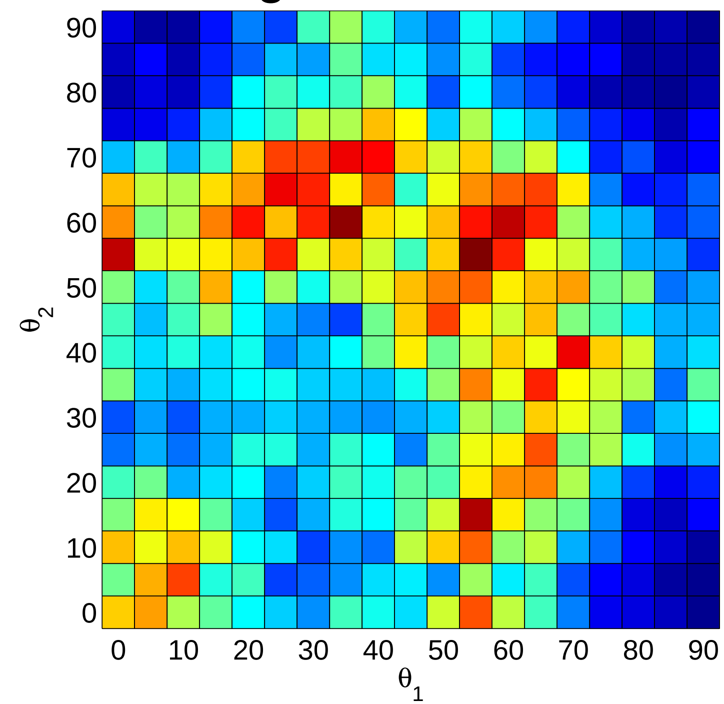


DDOA: the mass is distributed along the “tunnel”



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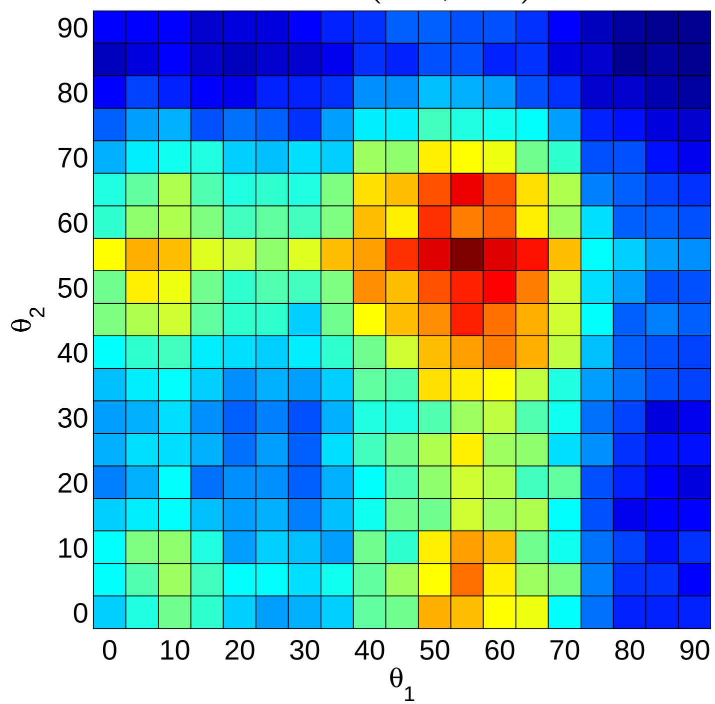

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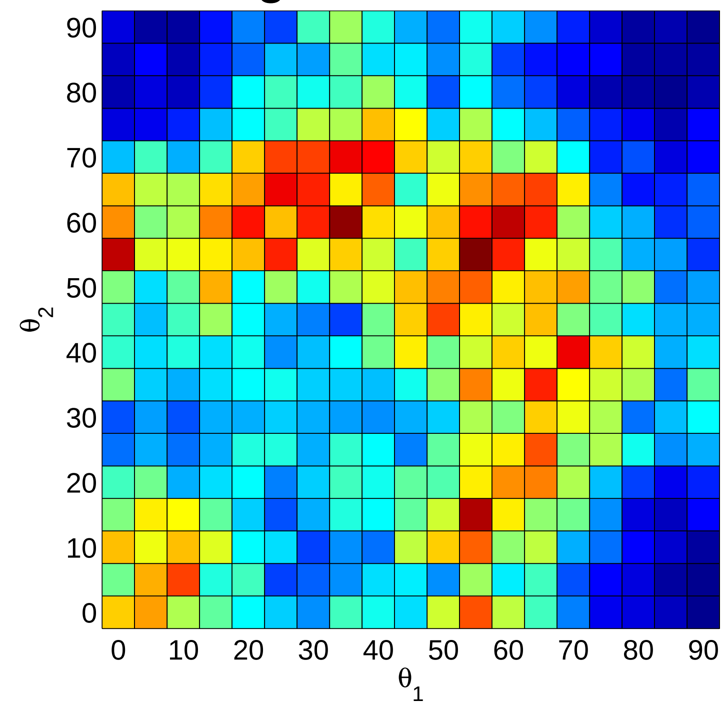

- Sampling from $p(\theta_1), \ldots, p(\theta_n)$ leads to an inaccurate distribution
- $\mathbf{V}$-based selection improves the distribution by introducing variable dependencies

Application I: Design of “zero-CTE” laminates

- Objective: minimize the longitudinal coefficient of thermal expansion (CTE) subject to a constraint on the first natural vibration frequency, for a 50 in $\times$ 15 in plate

minimize $|\overline{\alpha}_x|$
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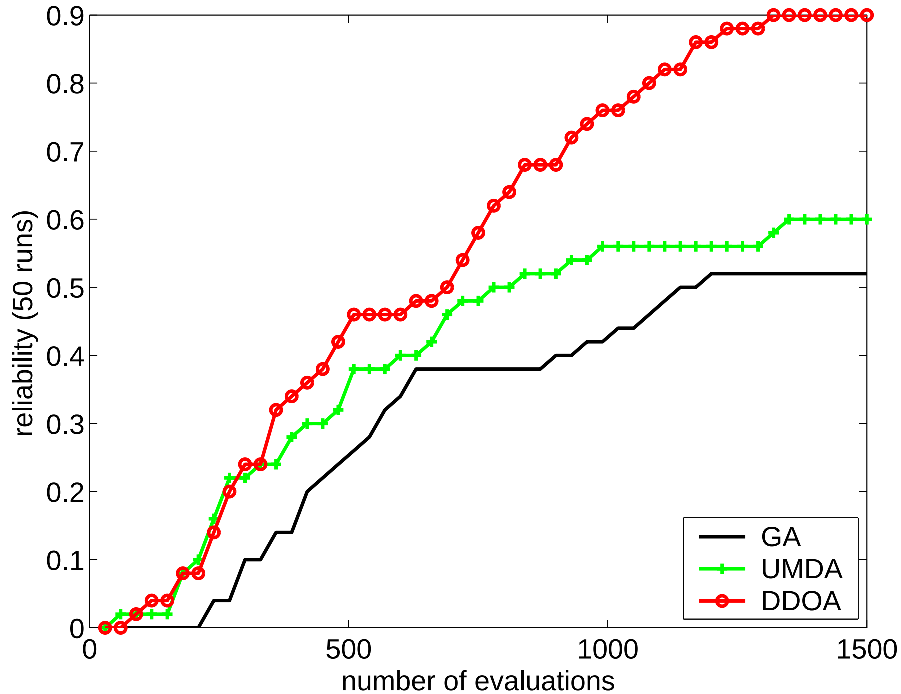
- Particularity: response is a function of the lamination parameters only
  ➾ V’s provide reliable information about the optimum

Comparison with UMDA and GA



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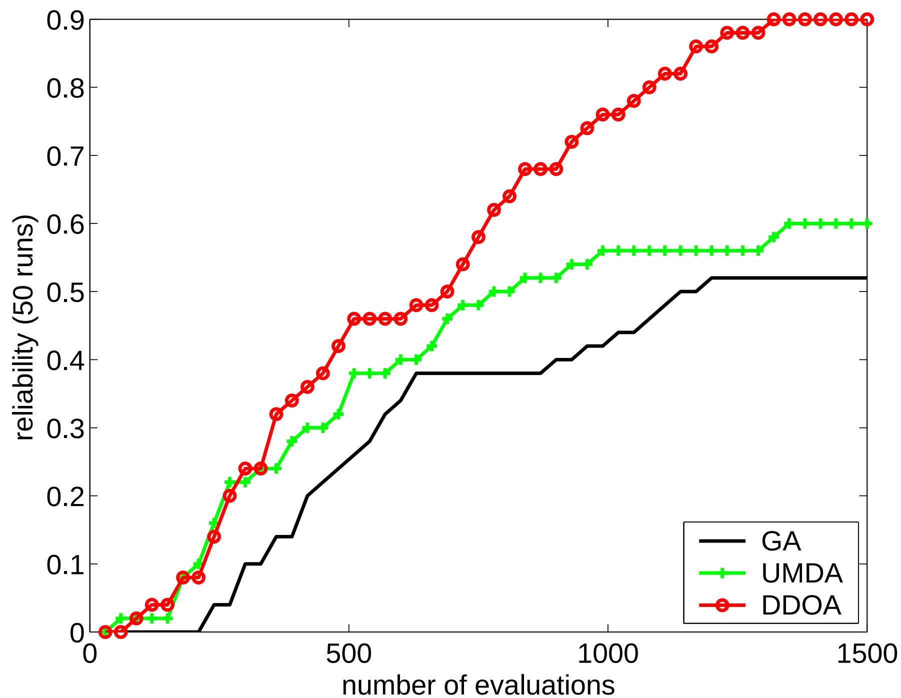

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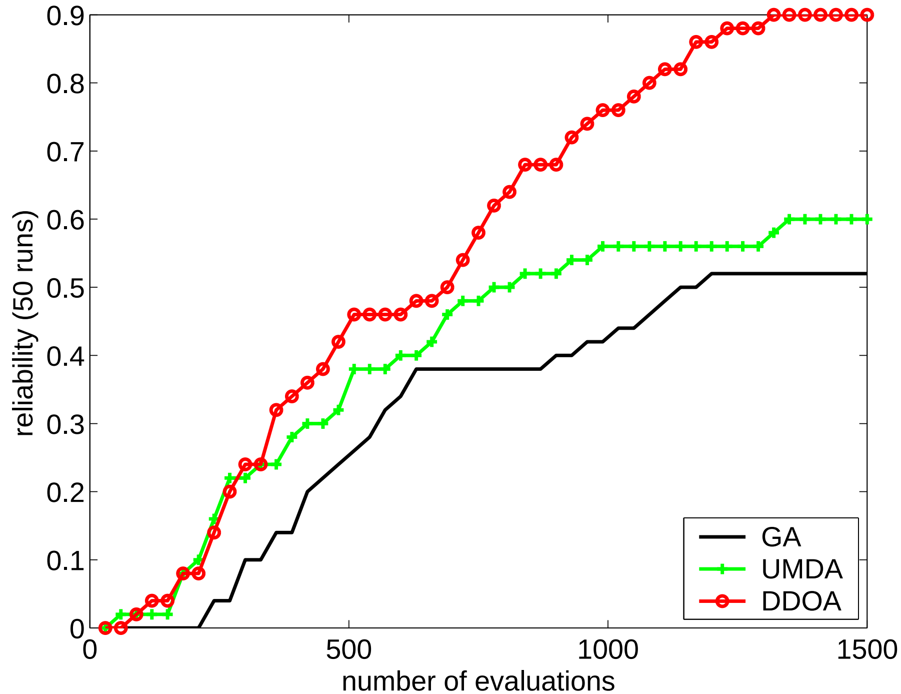

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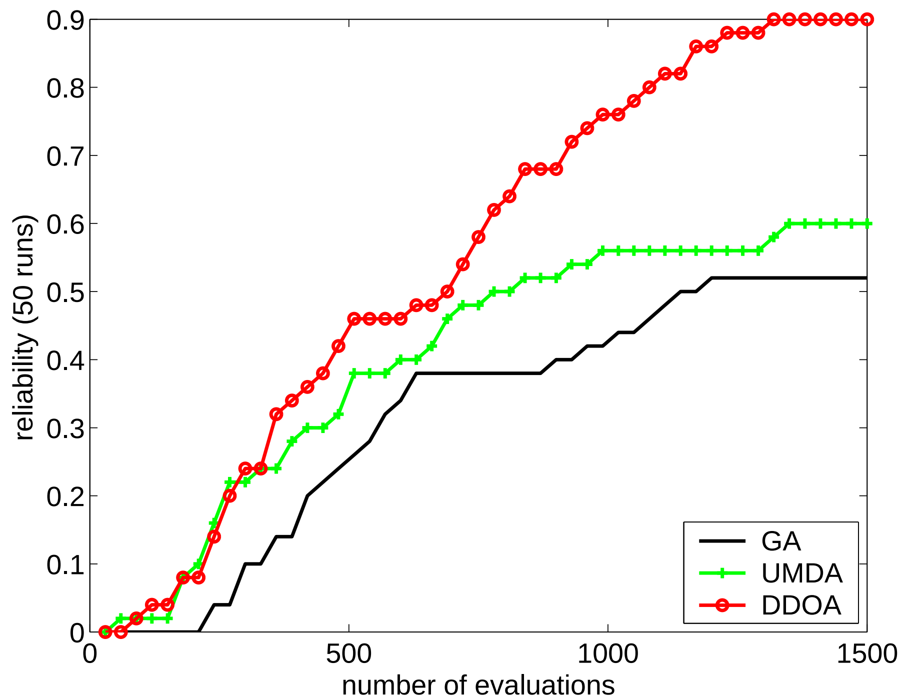

- GA and UMDA's progress falls off after 600 analyses
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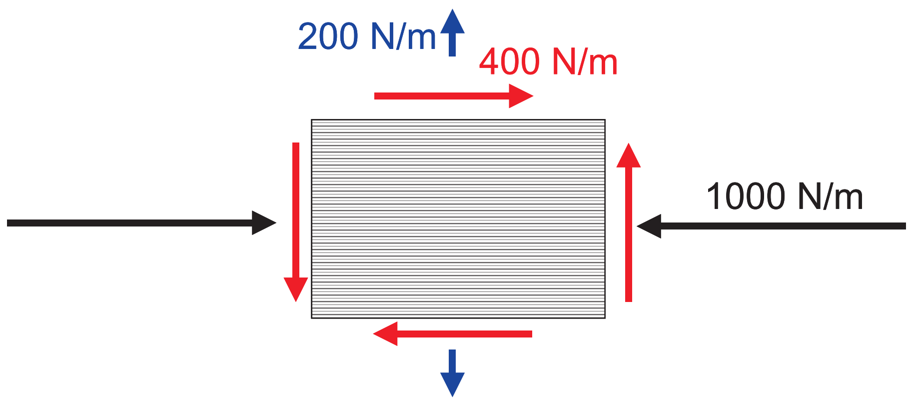
- For this problem, DDOA benefits from the use of auxiliary variables (incorporation of physics-based information improves accuracy of $p(\mathbf{x})$)

Application II: Strength Maximization

- Strength maximization problem:

$$\textbf{maximize } \lambda_s = \min_{k=1}^{n}\left(\min\left(\frac{\epsilon_1^{\text{ult}}}{\epsilon_1(k)}, \frac{\epsilon_2^{\text{ult}}}{\epsilon_2(k)}, \frac{\gamma_{12}^{\text{ult}}}{\gamma_{12}(k)}\right)\right)$$



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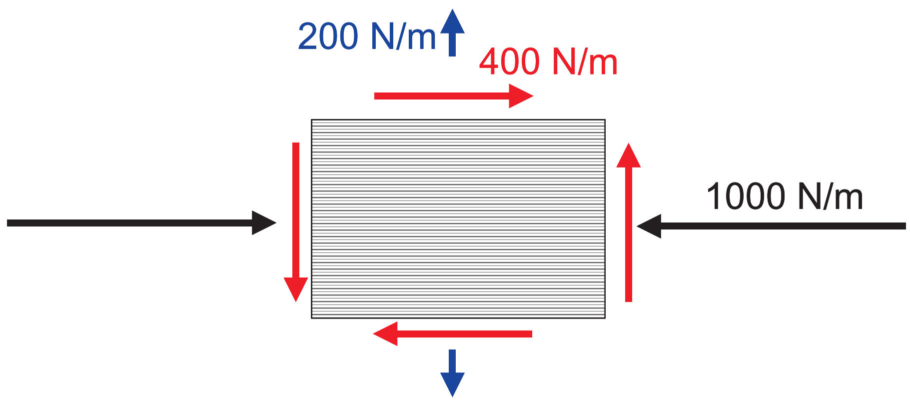

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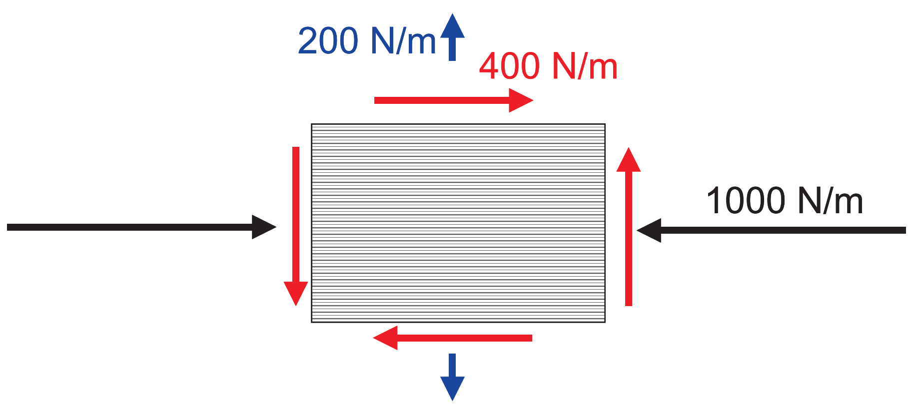

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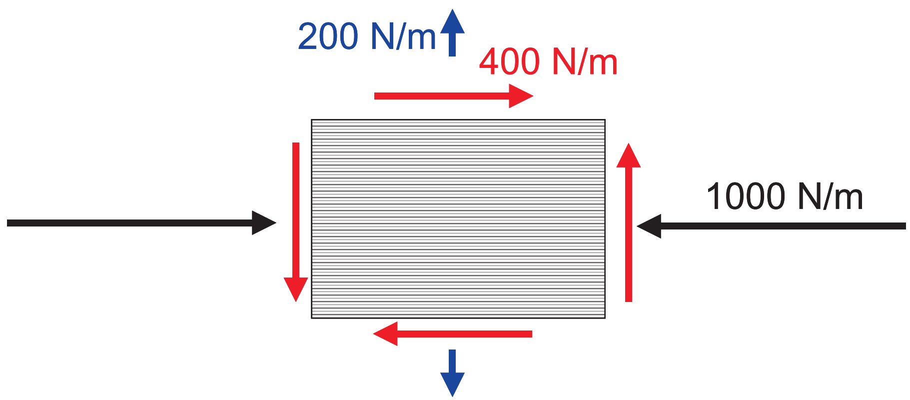

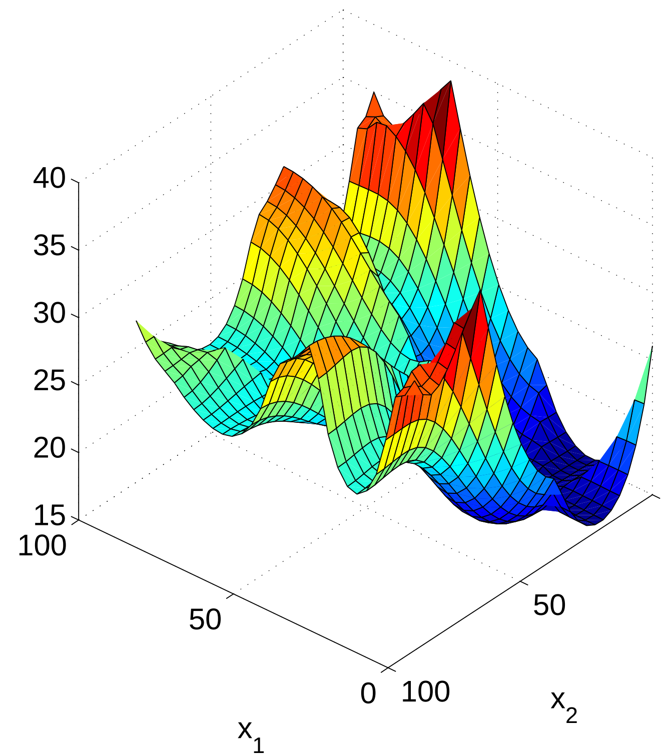

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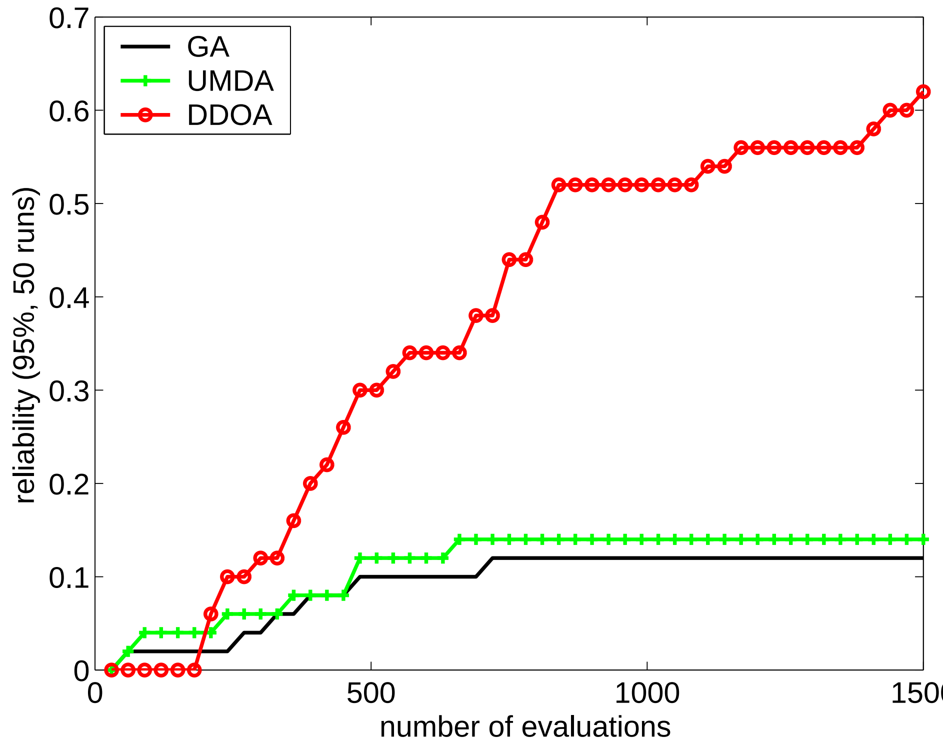

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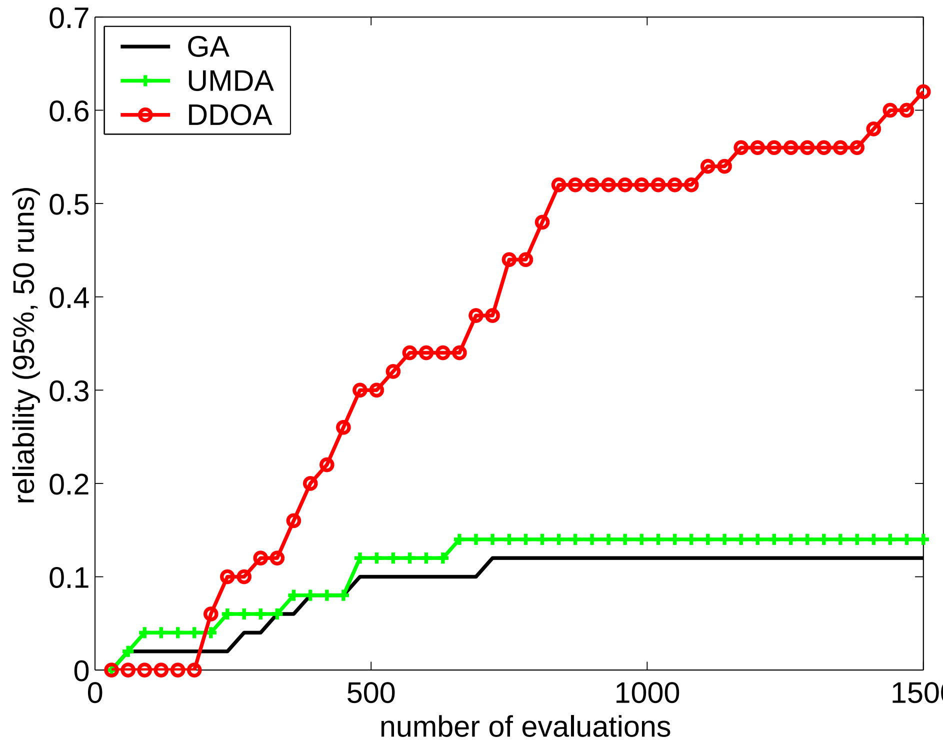

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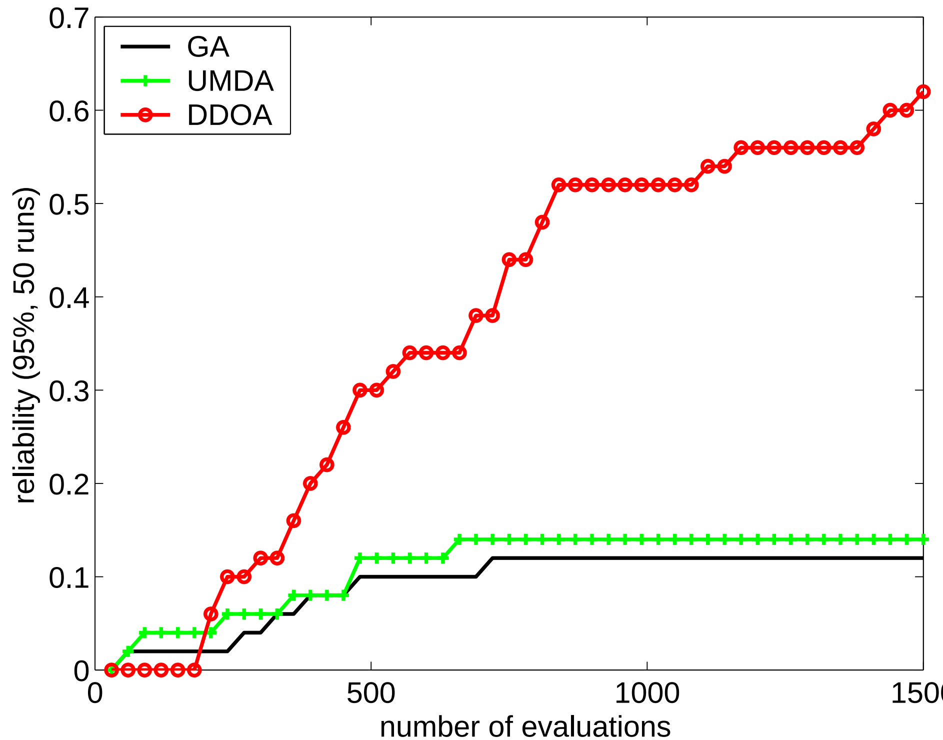
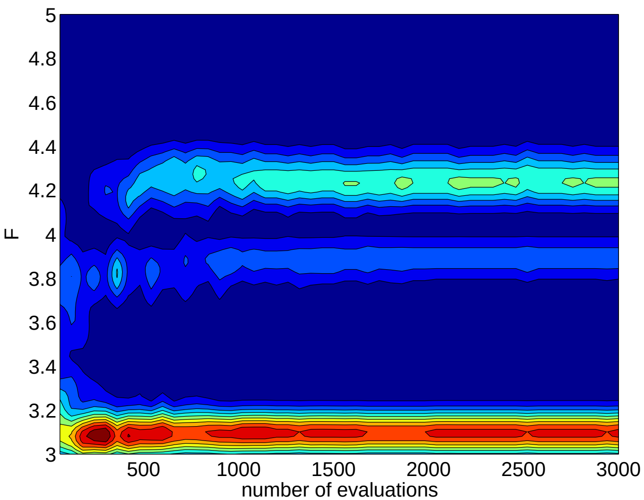


- UMDA and GA locally improve the initial candidate solutions, but fail to converge to high-fitness solutions
- DDOA reliably finds the optimum

Distribution of the solutions found



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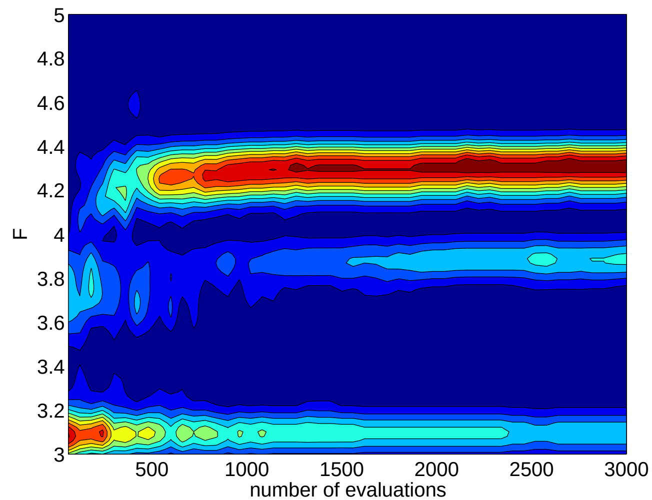

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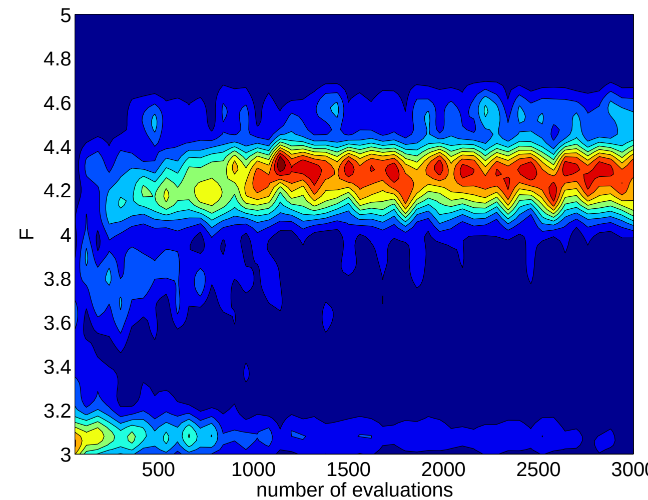

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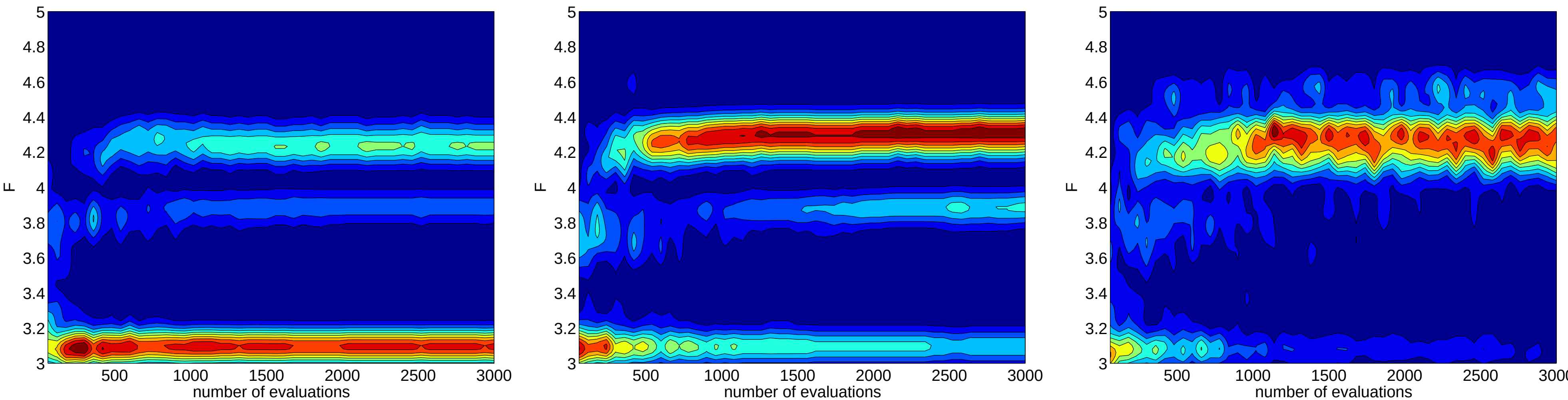

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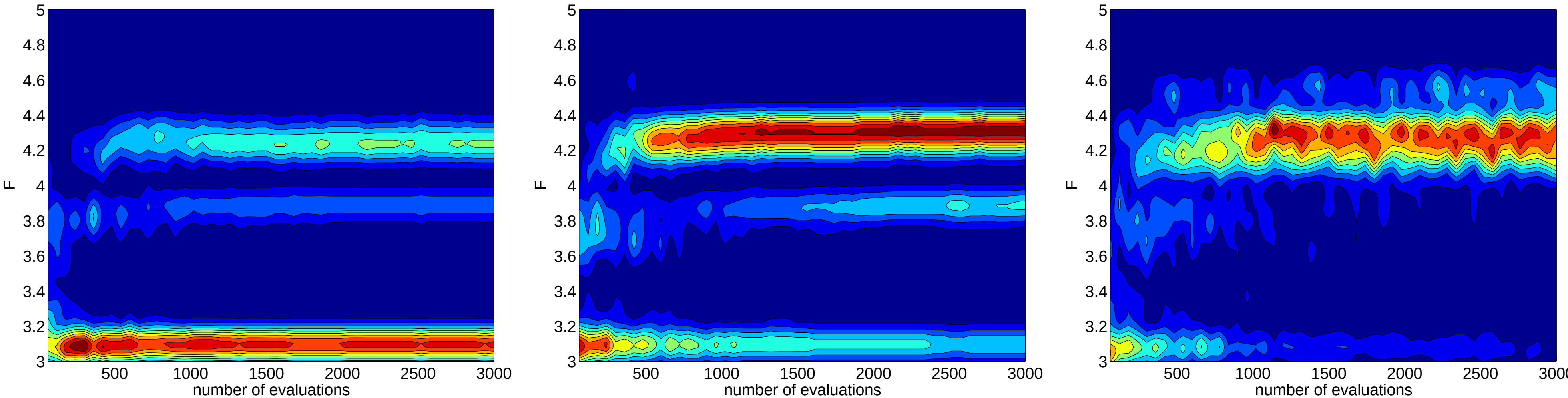

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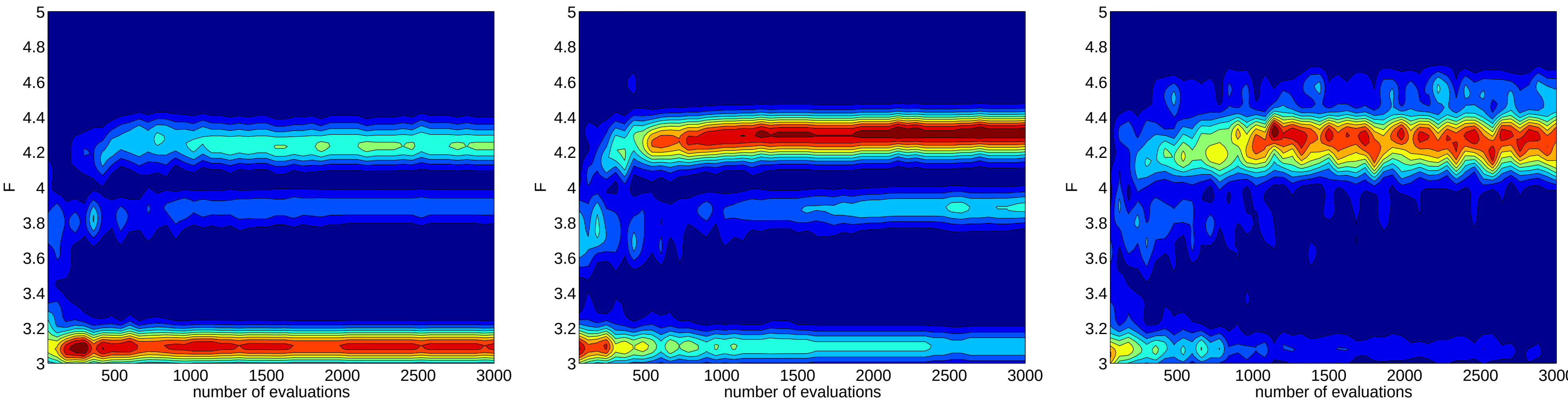

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- DDOA focuses the search on high-fitness regions. The two lower basins of attraction present in GA and UMDA bearly appear

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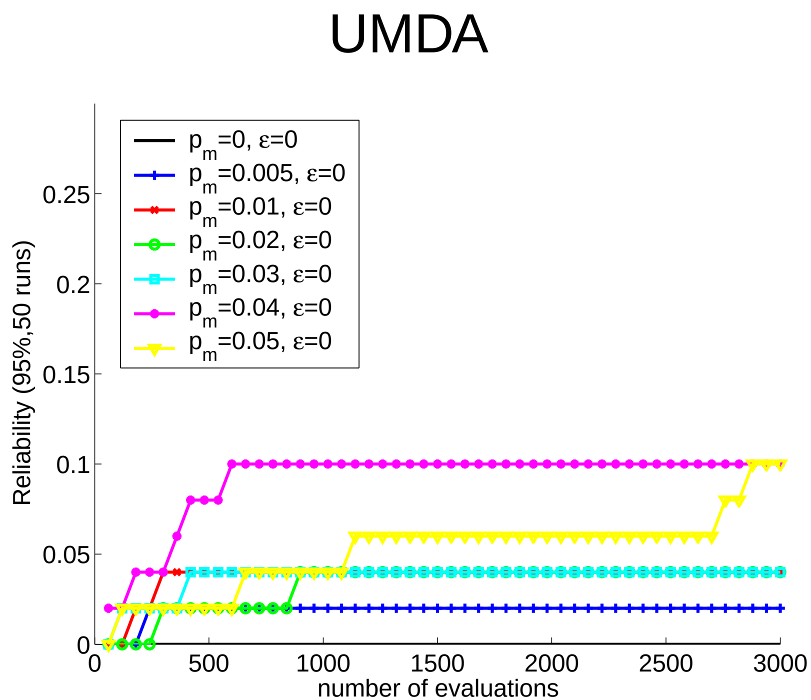
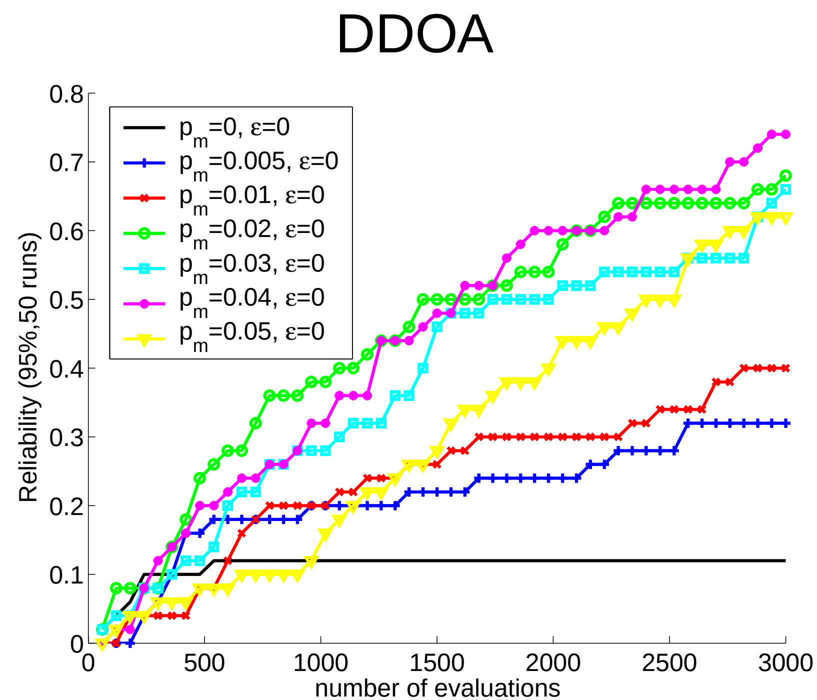
  - lower bound on marginal probabilities: the probability $p(\theta_k = c_l)$ is not allowed to fall below a threshold ϵ

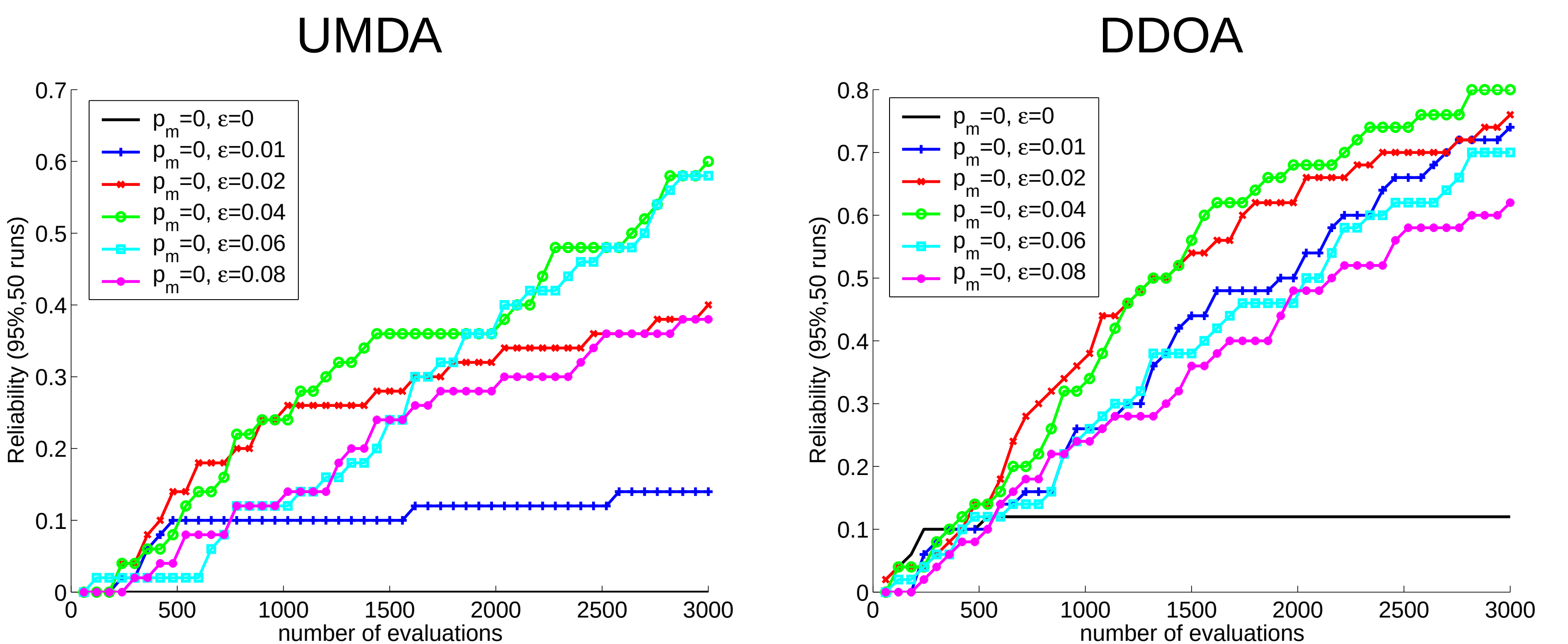
Effect of mutation





- even with a large mutation rate, UMDA does not reliably find the optimum (probability of obtaining a good point by chance very low with $n = 12$)
- mutation greatly improve DDOA's performance: the auxiliary variable scheme filters out poor candidates created by mutation

Effect of the lower bound on marginal distributions



- preventing probabilities to vanish improves UMDA's reliability but it remains inferior to DDOA.
- DDOA greatly benefits from bounding the probabilities. The performance is not sensitive to the value of ϵ
  same explanation as for mutation

Performance with optimized parameters for the strength problem

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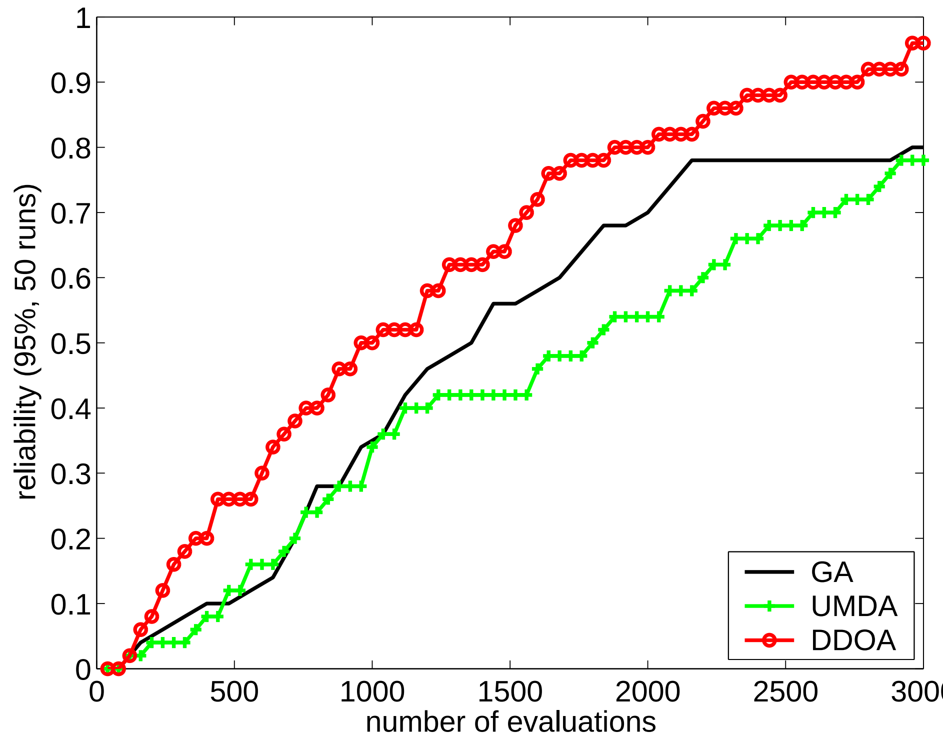

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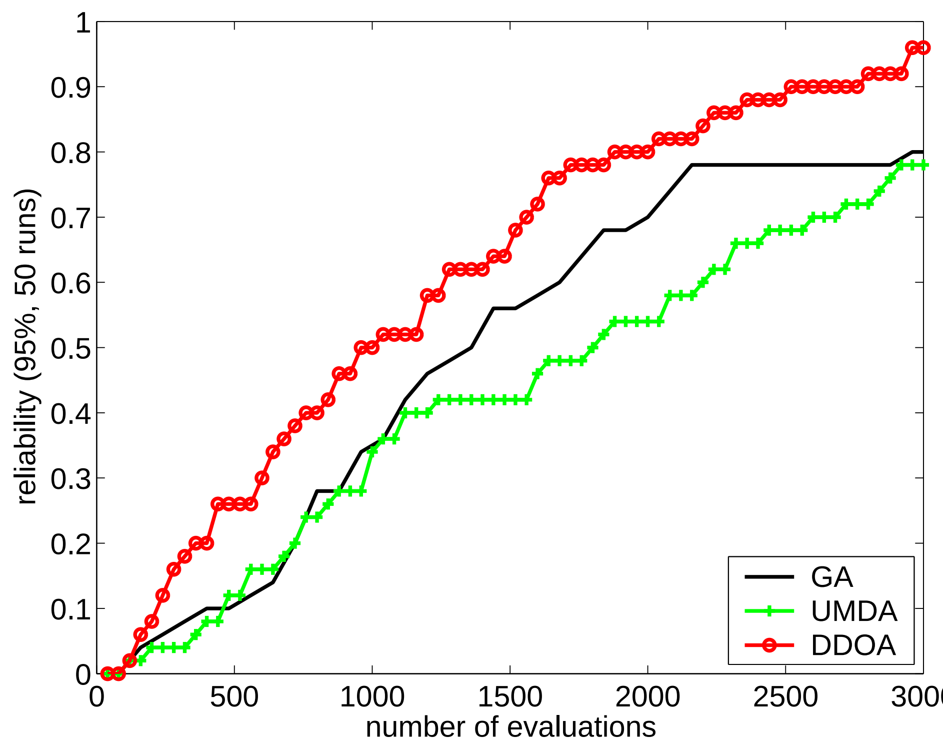

➪ Significant improvement over UMDA (variable dependencies)

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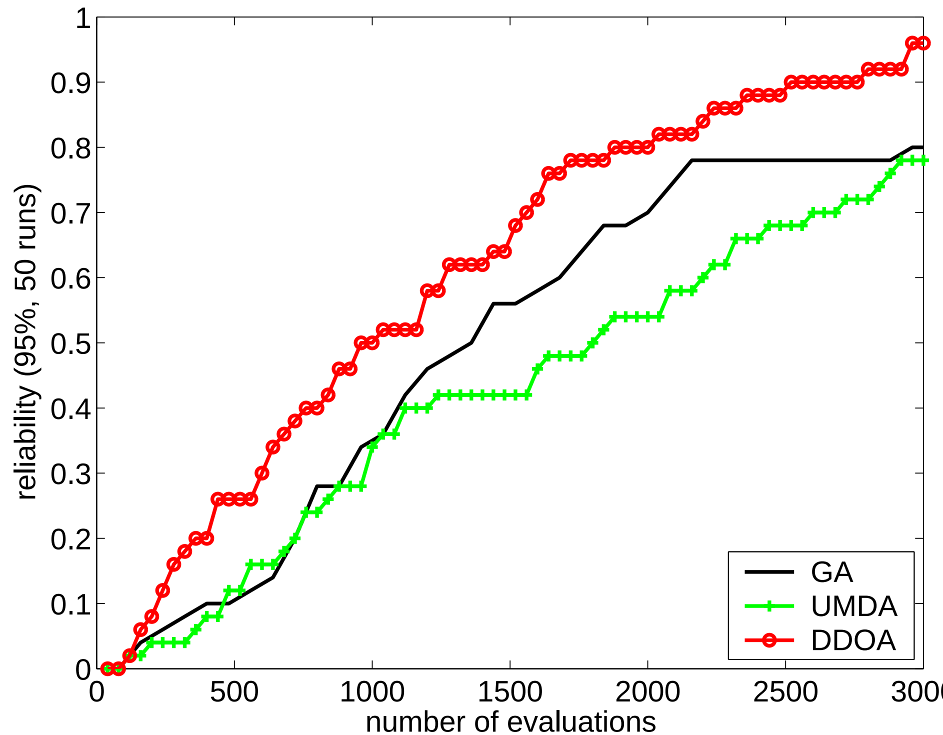

- ➩ Significant improvement over UMDA (variable dependencies)
- ➩ DDOA more efficient than GA even without mutation

General Conclusions

Concluding Remarks

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- A study of the diversity was conducted: proposed direct control of the distribution diversity

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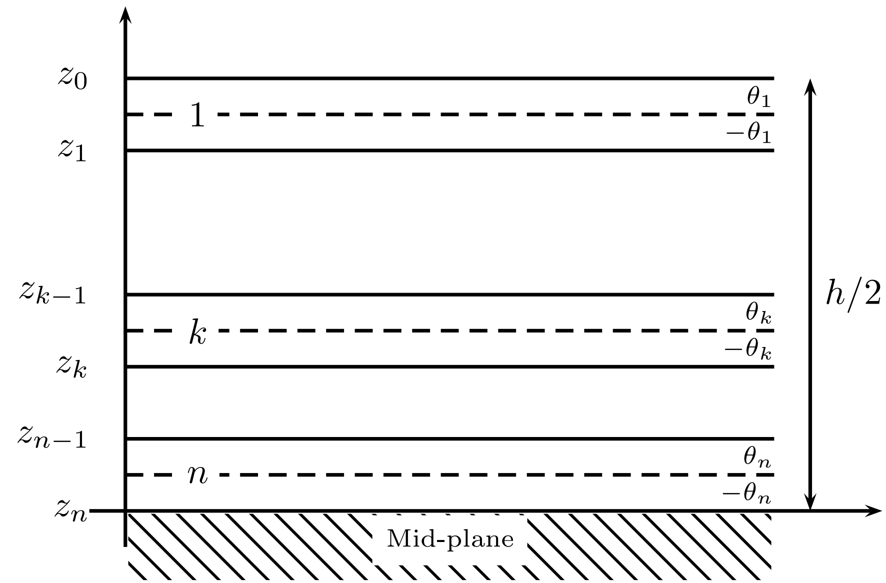
- Detection of failure situations: how to check the validity of $p(\mathbf{x})$ and what to do when failure is detected?

Backup Slides

Balanced symmetric laminate

- Particular case: balanced symmetric laminates $[\pm\theta_1, \pm\theta_2, \ldots, \pm\theta_n]_s$



Constraint enforcement through a penalty approach

- Consider the following optimization problem:

$$\begin{array}{ll}\textbf{maximize} & F(\mathbf{x}) \\ \textbf{such that} & g_j(\mathbf{x}) \geq 0,\ j = 1, \ldots, m\end{array}$$

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- The penalty approach transforms this constrained problem into an unconstrained problem by decreasing the objective function proportionally to the constraint violation:

$$F_p(\mathbf{x}) = \begin{cases} F(\mathbf{x}) & \text{if } g_j(\mathbf{x}) \geq 0,\ j = 1, \ldots, m \\ F(\mathbf{x}) + p \, \min_{j=1}^{m} \left(g_j(\mathbf{x}) \right) & \text{if } \exists\, k \in \{1, \ldots, m\} \text{ s.t. } g_k(\mathbf{x}) < 0 \end{cases}$$

Kernel density estimate: principle

- $f(V)$ is continuous, low dimensional (2D or 4D in our problems) ➩A kernel density estimation approach is adopted:

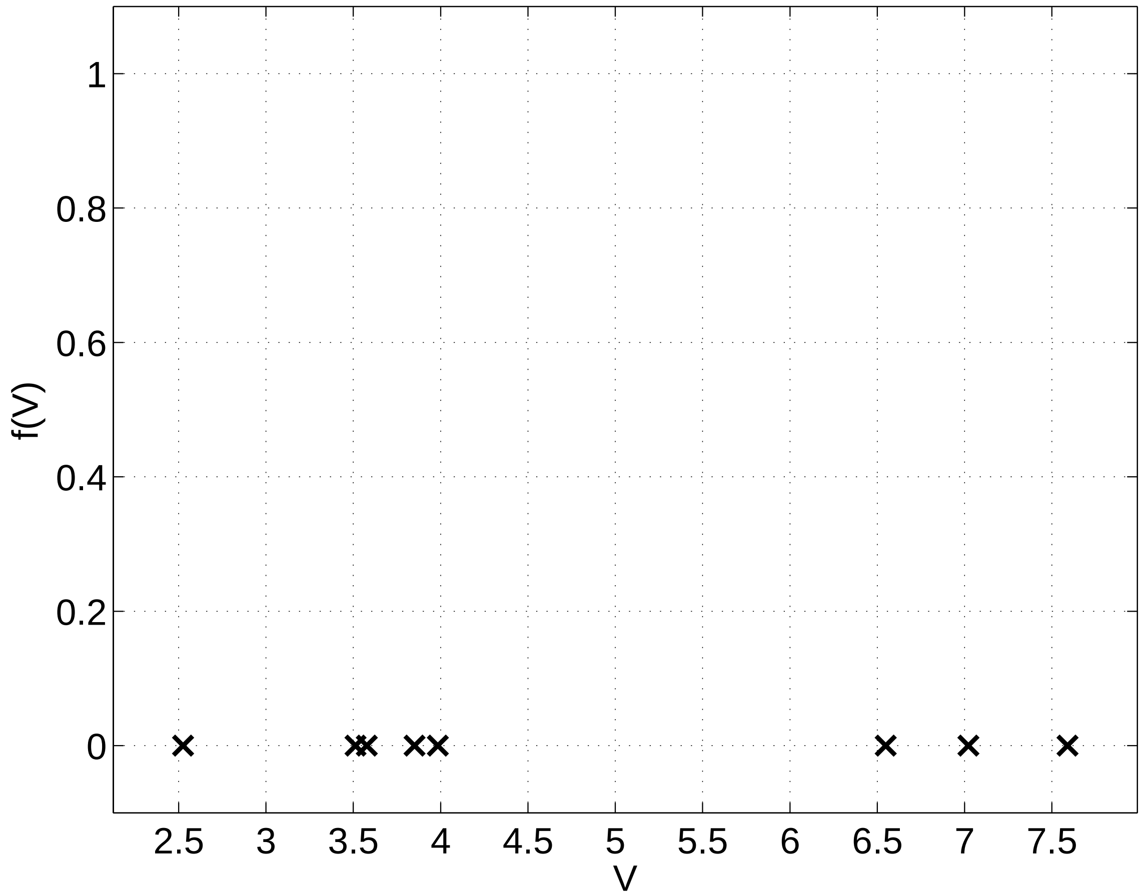
$$f(\mathbf{V}) = \frac{1}{\mu} \sum_{i=1}^{\mu} K\left(\mathbf{V} - \mathbf{V}_i\right)$$

In this work, we used Gaussian kernels:

$$K(\mathbf{u}) = \frac{1}{(2\pi)^{d/2}\sigma^d} \exp\left(-\frac{\mathbf{u}^T\mathbf{u}}{\sigma^2}\right)$$

where σ determines the smoothness of the estimate

Kernel density estimate: illustration



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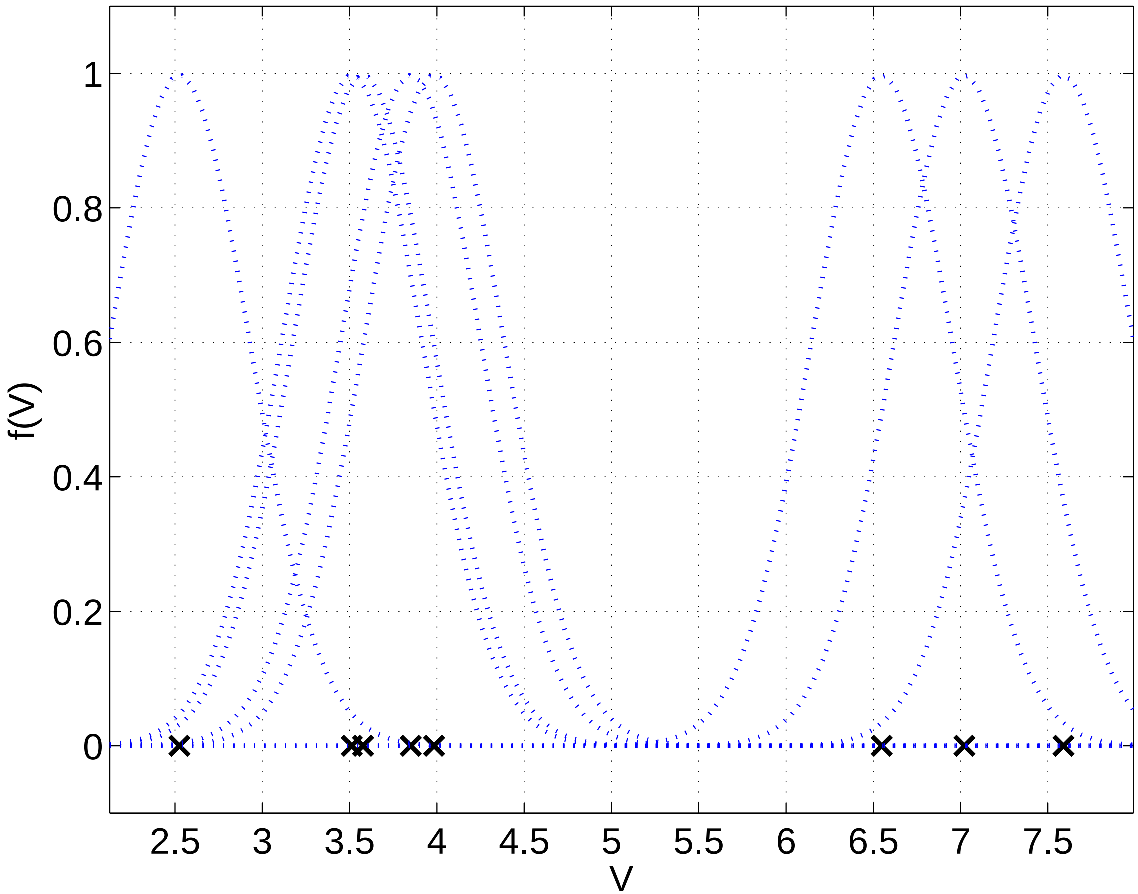

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