# Multi-level Networked Knowledge Base: DDL-Reasoning

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**Abstract.** This paper describes a new formalism based on multi-level networked knowledge (MLNK), a combination of different ontologies describing heterogeneous and complementary domains aligned with semantic correspondences. Ontology alignments make explicit the correspondences between terms from different ontologies and must be taken into account in reasoning, where two explicit form of correspondences are given: mappings represent predefined relations such as subsumption, equivalence, or disjointness, that have a fixed semantics in all interpretations; as well as links that can relate complementary ontologies by introducing terms defined by experts, and their semantics varies according to interpretations. The proposed MLNK formalism can be transformed into a Distributed System capable of supporting DDL semantics. It permits to apply a contextual reasoning where ontologies and alignments by pairs of ontologies are developed in different and incompatible contexts. The semantic of the proposed formalism is extensively described along with an illustrative example.

**Keywords:** Multi-Level Networked Knowledge Base; ontologies; ontology-alignment; DDL-reasoning.

## 1 Introduction

In information systems, and more recently in the Semantic Web, a number of heterogeneous, independently developed ontologies may be exploited in a single application that needs to share some knowledge. These ontologies are developed in different contexts and may well cover complementary domains.

In order to overcome the heterogeneity problem, complementary knowledge may be introduced in order to describe correspondences between ontologies to be exploited. These correspondences, represent relations between entities (terms or formulas) belonging to different ontologies. This set of correspondences is named ontology alignment.

In order to exploit, during reasoning, a number of heterogeneous ontologies as well as correspondences, a simple solution consists in viewing the ontology system as a unique global ontology. Therefore, each local ontology as well as each alignment, is then considered as a knowledge complement over a larger domain. Taking into account all this knowledge, namely ontologies and alignments, may be performed using a fusion process obtaining a centralized system, or using distributed reasoning algorithms based on classical logic, (as shown in: SomeRDFS [2] and SomeOWL [1]). Such approaches consider ontologies and alignments describing a unique global theory, however, presenting inconvenient if the ontologies to be combined are highly heterogeneous. They will, therefore, describe different contexts and points of view, potentially incompatible.

The other possible solution consists in managing the set of ontologies as well as the corresponding alignments as a complex semantic network, where each node is represented by an ontology formalizing a given domain, with a different context than the ones given by the rest of the ontologies within the network. Such an approach, needs strong formalism modeling the already aligned ontology network, offering specific algorithms and techniques for contextual reasoning. In that sense, a number of formalisms have been proposed to model an aligned ontology network for contextual reasoning. These formalisms may be divided into two categories based on definition and application purposes. Alignment is a major concern, as it constitutes an important element of the complete system, distinguishing two major types of correspondences in order to define them.

The first type is for instance given by Distributed Description Logic [5] and Iqntegrated Distributed Description Logics [11] that define relations, named *mappings*, in order to reduce semantic heterogeneity problems between terms and entities belonging to different ontologies. These correspondences are associated with a predefined set of relations such as subsumption, equivalence, disjunction where the given semantic is fixed for all interpretations (*e.g.*, $\texttt{mt:belong} \xleftrightarrow{\equiv} \texttt{eq:belong}$).

The second type of alignments is used to link ontologies covering complementary domains. It is the case of $\mathcal{E}$-connection [8]. It is represented by inter-ontological links between entities, termed simply *links* (*e.g.*, $\texttt{pr:T}_1 \overset{\textbf{compose}}{\longleftrightarrow} \texttt{eq:FD}_1$). This type of relations is defined by experts in the context of domain ontology combination, as well as semantic representation of context links.

However, a number of other works on contextual reasoning do not use corrspondences as defined here. Indeed, knowledge from multiple contexts are jointly exploited via a meta description of the contexts themselves. Established relations between contexts play a similar role, ensured by alignments between ontologies, (We can quote the recent work: [6]).

In this paper, we focus on proposing a formalism which supports both types of correspondences in order to permit heterogeneous ontology combination associated with different contexts covering complementary domains as is the case of [10]. Regarding the second point of difference, which is the application or treatment of alignments, the majority of proposed formalisms, such as Distributed Description Logic [5], $\mathcal{E}$-connection [8] and Package-based Description Logic [3]), consist in integrating alignment as external knowledge for the corresponding target ontology. The alignment is then defined and exploited following the target ontology point of view. In order to ensure reasoning over the ontology network, each ontology must be enriched with a reasoning mechanism which supports external knowledge.

The other way of looking at the problem is to consider alignments to be exploited at a higher level independent from local ontologies and termed global level [11]. In this approach, the alignment language may be more expressive than the languages defining local ontologies, allowing better alignment reuse. However, only mappings are considered in this work, and no proposition concerning the integration of links at the global level is made.

Links are, supposedly, introduced by experts. However, this may be unfeasible if experts covering all ontologies domain cannot be found. Therefore, if distinct pairs of ontologies are aligned by different experts with different terms and points of view, then it is likely that the heterogeneity problem will need to be considered this time between links.

The proposed contribution consists in defining and giving a semantics to a formalism named *Multi Level Networked Knowledge* (or MLNK for short). Here, we extend the work of [7] and provide an alternative, more distributed semantics for such networks.

The organization of the rest of the article is as follows: Section 2 describes a scenario representing an ontology network aligned using heterogeneous pairs ontology alignments. Section 3, describes the syntax formalism of the MLNK components. In Section 4, we recall the basic concepts of DDL. In Section 5, a possible interpretation using DDL semantics for automating reasoning tasks is proposed. Section 6 finishes the article with a general conclusion.

## 2 Motivating Example

In this section, a real life application example of gas turbine ontological representation is presented. Due to their wide usage in electricity production, gas turbine are often found in the center of large power systems that need to be managed in terms of knowledge and maintenance. Four ontologies describing gas turbine have been developed for the purpose of this example, namely:

- an ontology for equipment (`eq`) that describes turbine components, such as the concept `flame-detector` given by instance $\mathtt{FD}_1$. A set of equipments forms the group `instrumentation`;
- an ontology termed (`Pr`), modeling spare parts, such as the concept `trim` given by the instance $\mathtt{T}_1$. They compose the equipment and they can be replaced;
- an ontology for modeling the position of the equipment in the turbine hierarchy (`zn`);
- an ontology created from an existing database `mt`, using a semi automatic approach, covering maintenance operations (both preventive and after breakdown), defining the symptoms, the defects, the causes and the remedies for each case. The `mt` ontology exploits the first ontologies (`eq`), `Pr` and `zn`) in order to provide details on equipments and spare parts concerned by maintenance operations.

Exploiting these ontologies requires their alignment and the integration of the latter in all reasoning or search strategies. For this purpose a number of alignment tools have been applied in order to provide *mappings* such as: $\mathtt{mt{:}belong} \xleftrightarrow{\perp} \mathtt{eq{:}belong}$ between $(\mathtt{mt}, \mathtt{eq})$ ontologies pair and $\mathtt{pr{:}trim} \xleftrightarrow{\sqsubseteq} \mathtt{eq{:}instrumentation}$ between

$(\texttt{pr}, \texttt{eq})$. These sets of correspondences (or *mappings*) are enriched in a semi-automatic manner using *links* as well as consulting domain experts, an expert for a pair of ontologies. This operation revealed the existence of semantic heterogeneity problems between ontology alignments, and more precisely between links. As an example of heterogeneity, the links $A_{\texttt{pr-eq}}$:**compose** and $A_{\texttt{eq-zn}}$:**part-of** have similar semantics but appear in different alignments, namely $A_{\texttt{pr-eq}}$ and $A_{\texttt{eq-zn}}$. It is clear that reasoning on the set of ontologies and their existing alignments, semantic heterogeneity problem between *links* need to be solved. For the previous case, inserting an equivalence relation between *links* $A_{\texttt{pr-eq}}$:**compose** and $A_{\texttt{eq-zn}}$:**part-of** becomes necessary.

*Example 1.* An excerpt of ontologies and associated alignments are presented in Table 1.

| Ontologies | Axioms |
|---|---|
| eq: | flame-detector$(\texttt{FD}_1)$ |
| | flame-detector $\sqsubseteq \exists$belong.instrumentation |
| pr: | trim$(\texttt{T}_1)$ |
| zn: | zone(ANNA1TG01) |
| mt: | intervention$(\texttt{I}_1)$ |
| | team$(\texttt{TE}_1)$ |
| | intervene$(\texttt{TE}_1, \texttt{I}_1)$ |
| | member $\sqsubseteq \exists$belong.team |
| **Alignments** | |
| $A_{\texttt{eq-zn}}$: | eq:$\texttt{FD}_1 \overset{\textbf{part-of}}{\longleftrightarrow}$ zn:ANNA1TG01 |
| $A_{\texttt{pr-eq}}$: | pr:trim $\overset{\sqsubseteq}{\longleftrightarrow}$ eq:instrumentation |
| | pr:$\texttt{T}_1 \overset{\textbf{compose}}{\longleftrightarrow}$ eq:$\texttt{FD}_1$ |
| $A_{\texttt{mt-eq}}$: | mt:$\texttt{I}_1 \overset{\textbf{concern}}{\longleftrightarrow}$ eq:$\texttt{FD}_1$ |
| | eq:belong $\overset{\perp}{\longleftrightarrow}$ mt:belong |
| $A_{\texttt{A}_{\texttt{pr-eq}}\texttt{-A}_{\texttt{eq-zn}}}$: | pr-eq:**compose** $\overset{\equiv}{\longleftrightarrow}$ eq-zn:**part-of** |

**Table 1.** an excerpt of ontologies and associated alignments

In order to solve semantic heterogeneity problem between links, it is essential to represent and understand the semantics of the MLNK. Further than semantic connections of a MLNK representing local knowledge, we may find semantic connections between alignments themselves, Figure 1 represents the turbine example showing alignment levels.

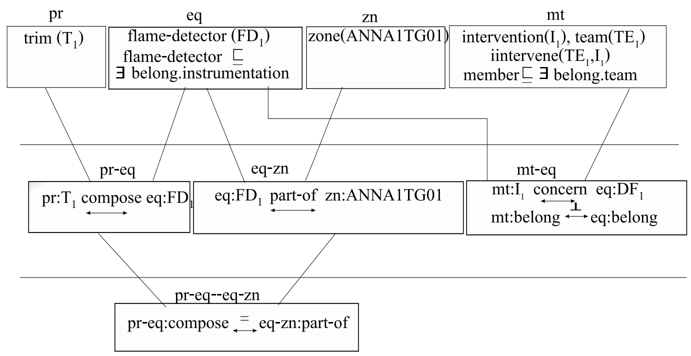


**Fig. 1.** Knowledge representation levels.

# 3 Multi-level Networked Knowledge Base syntax

Representation and formalization of MLNK implies on the one hand, represent each component of this network and formalize the semantics and on the other hand, that the relationships between these components can be represented. This section, is dedicated to the syntactic representation of a multi-level networked knowledge components, such as ontologies, alignments and knowledge nodes.

## 3.1 Knowledge representation languages and ontologies

A knowledge representation language $L$, is defined by a syntax (how formulas are expressed) and a semantic (the meaning and sense of formulas). We then speak of signatures or vocabulary in order to design structured terms which are subsets of a given language symbols. Each signature permits the definition of a set of formulas defined by the used language, and a set of formulas constructed from a common signature form an ontology. Local ontologies or knowledge sources in a multi-level networked knowledge are linked using alignments.

## 3.2 Alignment language

An alignment $L_A$ language permits the description of correspondences between two vocabularies. It is also caracterised by a syntax (how correspondences are expressed) and a semantic (how correspondences are interpreted). The syntax of $L_A$ is defined by:

- a set of terms , called links, specific to the alignment language noted $V(L_A)$;
- a function $E(L_A)$, which associate to each signature of a representation language $L$, a set of entities that can be aligned;
- a set of relation's symbols $R(L_A)$.

Thus, the syntax of an alignment language $L_A$ is defined by the triple $\langle V(L_A), E(L_A), R(L_A)\rangle$, noted $\langle V, E, R\rangle$ when no ambiguity exists. Two types of correspondences might be defined as *mapping* and *link* correspondences.

**Definition 1** **(*mapping* correspondence).** *Let $V_1$ and $V_2$ two aligned vocabularies and $L_A\langle V, E, R\rangle$ an alignment language. A* mapping *correspondence is a triple $\langle e_1, e_2, r\rangle$ noted $e_1 \stackrel{r}{\longleftrightarrow} e_2$ where:*

- *$e_1 \in E(V_1)$ and $e_2 \in E(V_2)$ are matchable entities;*
- *$r \in R$ denotes an existing relation between $e_1$ and $e_2$.*

Referring to Example 1, `eq:belong` $\stackrel{\perp}{\longleftrightarrow}$ `mt:belong` is a *mapping* correspondence. The term `belong` can be found in both ontologies vocabulary, for instance `eq` and `mt`, formalized in description logic, with different meanings. *Mappings* are constructed using the set of relations $R = \{\sqsubseteq, \equiv \perp, \in =\}$.

**Definition 2** **(*link* correspondence).** *Let $V_1$ and $V_2$ be two aligned vocabularies and $L_A\langle V, E, R\rangle$ an alignment language. A* link *correspondence is a formula in the form $e_1 \stackrel{l}{\longleftrightarrow} e_2$ where:*

- *$e_1 \in E(V_1)$ and $e_2 \in E(V_2)$ are matchable entities;*
- *$l \in V$ denotes an existing relation between $e_1$ and $e_2$.*

Again referring to Example 1, `eq:FD`$_1$ $\stackrel{\textbf{part-of}}{\longleftrightarrow}$ `zn:ANNA1TG01` and `mt:I`$_1$ $\stackrel{\textbf{concern}}{\longleftrightarrow}$ `eq:FD`$_1$ are *link* correspondences. Terms $\textbf{part-of}$ and $\textbf{concern}$ do not appear in the ontologies vocabularies, they were introduced at the alignment level in order to link different vocabularies entities. The alignment, now, possesses its own vocabulary and therefore may be aligned with other vocabularies in order to avoid heterogeneity problems.

**Definition 3** **(Alignment).** *Let $V_1$ and $V_2$ be two vocabularies. An* alignment of $V_1$ and $V_2$ *is a tuple $\Lambda = \langle V, \kappa, \lambda\rangle$ where:*

- *$V$ is an alignment vocabulary;*
- *$\kappa$ is a set of* mapping *correspondences, $e_1 \stackrel{r}{\longleftrightarrow} e_2$ where $e_1 \in E(V_1)$, $e_2 \in E(V_2)$ et $r \in R$;*
- *$\lambda$ is a set of* link *correspondences, $e_1 \stackrel{l}{\longleftrightarrow} e_2$ where $e_1 \in E(V_1)$, $e_2 \in E(V_2)$ and $l \in V$;*

*Example 2.* In DDL or in IDDL, alignments are between the ontologies signatures and the sets $V$, $\lambda$ are all empty. In $\mathcal{E}$-connections, cross-ontology knowledge can involve terms from more than two ontologies. However, if one restricts to $\mathcal{E}$-connection axioms of the form $\langle E_i\rangle^j(a_i, b_j)$, where $E_i$ is a link relation, $a_i$ is an individual in ontology $O_i$ and $b_j$ is an individual in ontology $O_j$, then this can be represented as a correspondence in $\lambda$, with $l$ being a term in the alignment vocabulary (the set $\kappa$ is empty).

### 3.3 Knowledge node

It is now possible to introduce the notion of knowledge node, which generalize the notion of ontology. Informally, an ontology is a level 0 knowledge node, while all knowledge node of level $m > 0$ is constructed from a number of nodes with inferior levels, linked using alignment (figure 2). Formally the node is defined as:

**Definition 4 (Knowledge node).** *A* knowledge node *is a pair* $K = \langle V_K, A_K \rangle$ *where* $V_K$ *is a vocabulary, also written* $\mathrm{Voc}(K)$ *and both* $V_K$ *and* $A_K$ *are defined recursively:*

- *an ontology* $O$ *is a knowledge node with vocabulary* $\mathrm{Voc}(O) = \mathrm{Sig}(O)$ *and* $A_K$ *is the set of axioms;*
- *for* $n \geq 1$*, if* $K_1, \ldots, K_n$ *are knowledge nodes with vocabularies* $\mathrm{Voc}(K_1), \ldots,$ $\mathrm{Voc}(K_n)$*, and for all* $i, j \in [1, n]$*,* $\Lambda_{ij}$ *is an alignment of* $\mathrm{Voc}(K_i)$ *and* $\mathrm{Voc}(K_j)$*, then* $K = \langle V_K, A_K \rangle$ *is a knowledge node with the vocabulary:*

$$V_K = \bigcup_{i,j \in [1,n]} \{ij : l \mid l \in \mathrm{Voc}(\Lambda_{ij})\} \cup \bigcup_{i \in [1,n]} \{i : e \mid e \in \mathrm{Voc}(K_i)\}$$

*and* $A_K = \langle (K_i)_{i \in [1,n]}, (\Lambda_{ij})_{i,j \in [1,n]} \rangle$.

If a knowledge node includes only ontologies and ontology alignments, we call it a *network of aligned ontologies*. If a knowledge node is neither a single ontology, nor a network of aligned ontologies, we call it a *multi-level networked knowledge base* (see Figure 2).

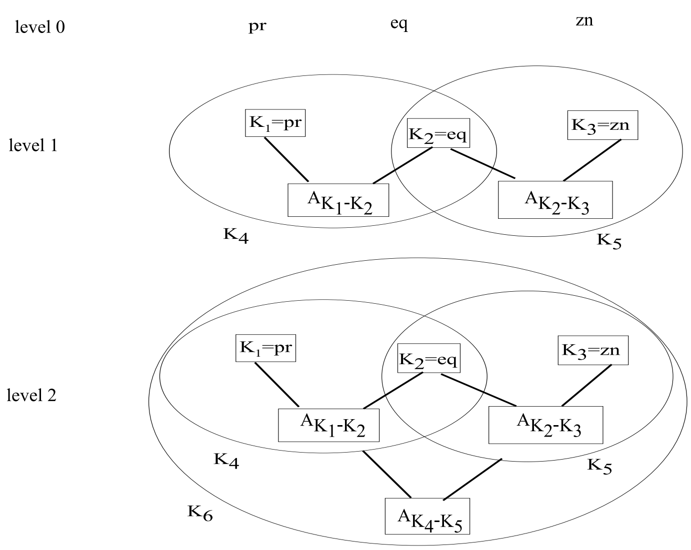


**Fig. 2.** Recursive representation of nodes.

## 4 Distributed description logics: Preliminaries

[4] introduces *Distributed Description Logics* (DDL) to connect and reason with multiple existing ontologies formalized in DL with different contexts. Ontologies are interconnected through (*Bridge rules*) which are the relationships between concepts, roles

and individuals belonging to different ontologies. The bridge rules are oriented because they are established between a source ontology and a target one with the viewpoint of the target ontology. To represent distributed description logics system, a concrete syntax based on OWL has been defined.

### 4.1 Syntax

Ontologies in DDL are formalized in DL and the syntax of bridge rule is defined as follows.

**Definition 5 (Bridge rules).** *Let $O_i$ et $O_j$ two ontologies. Bridge rule from $O_i$ to $O_j$ ($i \neq j$) is an expression of one of the following forms: $i{:}X \xrightarrow{\sqsubseteq} j{:}Y$ is an into bridge rule; $i{:}X \xrightarrow{\sqsupseteq} j{:}Y$ is an onto bridge rule; $i{:}a \xrightarrow{=} j{:}b$ is an individual correspondence where $X$ (respectively $Y$) is either a concept or a role of $O_i$ (respectively $O_j$) and $a$ (respectively $b$) is an individual of $O_i$ (respectively $O_j$). The notation $i{:}X \xrightarrow{\equiv} j{:}Y$ is expressing the combination of $i{:}X \xrightarrow{\sqsubseteq} j{:}Y$ and $i{:}X \xrightarrow{\sqsupseteq} j{:}Y$ and denotes the equivalence bridge rule.*

DDL knowledge system is composed of a family of local and prefixed ontologies and a set of bridge rules expressing mappings between entities belonging to these ontologies.

**Definition 6 (DDL distributed system).** *A DDL* distributed system *is a pair $\langle (O_i)_{i \in [1,n]}, (\mathcal{B}_{ij})_{i,j \in [1,n], i \neq j} \rangle$ where $(O_i)$ is a family of local ontologies and $\mathcal{B}_{ij}$ is a set of bridge rules between $O_i$ and $O_j$ for all $i, j \in [1, n]$ et $i \neq j$.*[3]

### 4.2 Semantics

Interpreting DDL distributed system by assigning to each ontology a local interpretation, then defining relationships between different local interpretations. A domain relation $r_{ij}$ represents a relationship between $O_i$ domain interpretation and $O_j$ domain interpretation.

**Definition 7 (Distributed interpretation).** *Let $S$ be a distributed system $S = \langle \{O_i\}, \{\mathcal{B}_{ij}\} \rangle$. A* distributed interpretation *of $S$ is a pair $\mathcal{I} = \langle \{I_i\}, \{r_{ij}\} \rangle$, for all $i \in [1, n]$, $I_i = \langle \Delta^{I_i}, \cdot^{I_i} \rangle$ is an ontology interpretation $O_i$ for $i, j \in [1, n]$ and $i \neq j$, $r_{ij} \subseteq \Delta^{I_i} \times \Delta^{I_j}$ is a relation domain.*

Distributed interpretation is a model of the DDL distributed system if every local interpretation is a model of its associated ontology, and if the domain relation satisfies all the bridge rules.

**Definition 8 (satisfaction relation).**
*Let $\mathcal{I}$ a distributed interpretation $\mathcal{I} = \langle \{I_i\}, \{r_{ij}\} \rangle$. We define the satisfaction relation in DDL $\models_d$ as follows: if $\alpha$ is an axiom then $\mathcal{I} \models_d i : \alpha$ if and only if $I_i \models \alpha$; if*

[3] In the rest of the paper, for readability, we omit the set of indices by writing, e.g., $(O_i)$ instead of $(O_i)_{i \in [1,n]}$.

*$X$, $Y$ are concepts or roles then $I \models_d i{:}X \stackrel{\sqsubseteq}{\rightarrow} j{:}Y$ if and only if $r_{ij}(X^{I_i}) \subseteq Y^{I_j}$; if $X$, $Y$ are concepts or roles then $I \models_d i{:}X \stackrel{\sqsupseteq}{\rightarrow} j{:}Y$ if and only if $r_{ij}(X^{I_i}) \supseteq Y^{I_j}$; if $a$ and $b$ are individuals then $I \models_d i{:}a \stackrel{\equiv}{\rightarrow} j{:}b$ if and only if $r_{ij}(a^{I_i}) = \{b^{I_j}\}$; $\mathcal{I} \models_d B_{ij}$ if and only if for all $\beta \in B_{ij}$, $\mathcal{I} \models_d \beta$.*

$\mathcal{I}$ is a model of $S$ if and only if: for all $i \in [1, n]$ and all axioms $\alpha$ in $O_i$, $\mathcal{I} \models_d i : \alpha$ and for all $i, j \in [1, n]$, $i \neq j$, $\mathcal{I} \models B_{ij}$. After recalling the fundamentals of description logics, we will, in the next section use DDL to interpret multi-level networked knowledge semantics.

## 5 Multi-level networked knowledge semantics

The representation of MLNK was defined independently of any language and can support multiple semantics. In this paper, we adopt distributed description logics (DDL) which is one of the most popular and contextual semantics to interpret the multi-level knowledge network.

The presented approach, consists in transforming the multi-level networked knowledge into a DDL distributed system. An ontology in description logic (called alignment-ontology) is created from the alignment then bridge rules between local ontologies and alignment-ontologies are generated. All local ontologies, alignment-ontologies and the set of bridge rules constitute the DDL distributed system. We will start by detailing the alignment-ontology building process.

**Generating alignment-ontology.** The alignment-ontology is a DL-ontology generated from the transformation of the alignment between a pair of nodes which is composed of a signature and a set of axioms. The signature of this ontology is made from terms (concepts, roles or individuals) contained in the left and right of mappings belonging to local aligned vocabularies. It also contains the links terms that belong to the alignment vocabularies. Before defining the new notions of alignement-ontology signature and formulas, a complementary function permitting the indexing of the ontology elements is firstly defined.

**Definition 9 (Index the element of ontology).** *Let $i$ be an indice. We define the function* prefix *on the terms, axioms and ontologies, such that* $\mathsf{prefix}(X, i) = \{i{:}X\}$ *when $X$ is an atomic concept, atomic role or an individual, and if $X$ is a formula,* $\mathsf{prefix}(X, i)$ *is a formula where all terms are prefixed by $i$.*

**Definition 10 (Alignment-ontology signature).** *Let $K$ a multi-level knowledge node, alignment-ontology signature $\Sigma_A$ is defined as follows according to the case:*

- *if $K$ is an ontology then $\Sigma_A = \emptyset$;*
- *if $K$ a multi-level knowledge node composed of sub nodes $K_1, \ldots, K_n$ and $A_{ij}$ which is alignment between $K_i$ and $K_j$ for $i, j \in [1, n]$, then:*

$$\Sigma_A(K) = \bigcup_{i,j \in [1,n]} \{\mathsf{prefix}(X, i), \mathsf{prefix}(Y, j) \mid i{:}X \stackrel{r}{\longleftrightarrow} j{:}Y \in A_{ij}\} \cup \bigcup_{i,j \in [1,n]} \mathrm{Voc}(A_{ij})$$

*where $X$ and $Y$ are the concepts, roles or individuals and $r \in \{\sqsubseteq, \equiv, \bot, \in, =\}$, and* $\mathrm{Voc}(A_{ij})$ *means the alignment vocabulary, the* links *of $A_{ij}$.*

Alignment-ontology formulas is the set of generated formulas from correspondences. Firstly, the function associating each correspondence in to an axiom is defined.

**Definition 11 (Correspondence transformation in to axiom).** *Let $A_{ij}$ for $i, j \in [1, n]$ an alignment between a node $i$ and a node $j$. We define* trans *a function which assigns to each correspondence of $A_{ij}$ a DL axiom:* $\mathsf{trans}(\{i{:}A \xleftrightarrow{\sqsubseteq} j{:}B\}) = \{\mathsf{prefix}(A, i) \sqsubseteq \mathsf{prefix}(B, j)\}$*;* $\mathsf{trans}(\{i{:}A \xleftrightarrow{\equiv} j{:}B\}) = \{\mathsf{prefix}(A, i) \equiv \mathsf{prefix}(B, j)\}$*;* $\mathsf{trans}(\{i{:}A \xleftrightarrow{\bot} j{:}B\}) = \{\mathsf{prefix}(A, i) \sqsubseteq \neg\mathsf{prefix}(B, j)\}$*;* $\mathsf{trans}(\{i{:}u \xleftrightarrow{\in} j{:}A\}) = \{\mathsf{prefix}(A, j)(i{:}u)\}$*;* $\mathsf{trans}(\{i{:}u \xleftrightarrow{=} j{:}u'\}) = \{i{:}u = j{:}u'\}$*;* $\mathsf{trans}(\{i{:}u \xleftrightarrow{l} j{:}u'\}) = \{\mathsf{role}(l)(i{:}u, j{:}u')\}$*;* $\mathsf{trans}(\{i{:}A \xleftrightarrow{l} j{:}B\}) = \{\mathsf{prefix}(A, i) \sqsubseteq \exists\mathsf{role}(l).\mathsf{prefix}(B, j)\}$*, where $A$, $B$, $u$ et $u'$ are the matchable entities and $l$ is a* link.

**Definition 12 (Alignment-ontology formulas).** *Let $K$ a multi-level knowledge node, the set of alignment-ontology formulas $F_A$ is defined as follows according to the cases:*

- *if $K$ is an ontology then $F_A = \emptyset$;*
- *if $K$ a multi-level knowledge node composed of sub nodes $K_1, \ldots, K_n$ and alignments $A_{ij}$ between $K_i$ and $K_j$ for $i, j \in [1, n]$ and* trans *is the function that associates to any correspondence of $A_{ij}$ a DL-axiom (see definition 11) and alignment-ontology-formula set $F_A(K) = \{f \mid f \in \mathsf{trans}(A_{ij})\}$.*

**Definition 13 (Alignment-ontology).** *Let a node $K = \langle\{K_i\}, \{A_{ij}\}\rangle$ for $i, j \in [1, n]$, $K_i$ are local nodes and $A_{ij}$ is an alignment between $K_i$ and $K_j$. We define* OntoAlign *the alignment-ontology generated from $A_{ij}$ of $K$,* $\mathsf{OntoAlign}(K) = \langle \Sigma_A(K), F_A(K)\rangle$.

The bridge rules of multi-level knowledge node represent the equivalence correspondences established between the terms of alignment-ontology and terms belonging to the corresponding local ontologies.

**Definition 14 (Bridge rules toward alignment-ontology).** *Let $K$ be a knowledge node. We define the bridge rules oriented towards the alignment-ontology (noted $B(K)$) as follows depending on the cases:*

- *if $K$ is an ontology then $B(K) = \emptyset$;*
- *if $K$ a multi-level knowledge node composed of sub nodes $K_1, \ldots, K_n$ and of $A_{ij}$ which is alignment between $K_i$ and $K_j$ for $i, j \in [1, n]$ then $B(K)$ contains a bridge rules defined as follows, for $i \in [1, n]$:*
  - *if $K_i$ is an ontology and $X$ is a concept or a role of $K_i$ then* $i{:}X \xrightarrow{\equiv} \mathsf{OntoAlign}(K){:}i{:}X \in B(K)$*;*
  - *if $K_i$ is an ontology $a$ is an individual of $K_i$ then* $i{:}a \xrightarrow{\equiv} \mathsf{OntoAlign}(K){:}i{:}a \in B(K)$*;*
  - *if $K_i$ is a composed node and $X$ a concept or role of* $\mathsf{OntoAlign}(K_i)$ *then* $\mathsf{OntoAlign}(K_i){:}X \xrightarrow{\equiv} \mathsf{OntoAlign}(K){:}k_i{:}X \in B(K)$*;*

- *if $K_i$ is a composed node and $a$ an individual of* $\mathsf{OntoAlign}(K_i)$ *then* $\mathsf{OntoAlign}(K_i){:}a \stackrel{\equiv}{\rightarrow} \mathsf{OntoAlign}(K){:}k_i{:}a \in B(K)$.

The MLNK interpreted as a DDL system is composed of several local nodes connected to their alignment-ontology through a family on bridge rules.

**Definition 15 (MLNK in DDL form).** *Let $K$ a knowledge node.* SystDis *is a DDL system of $K$,* $\mathsf{SystDis}(K) = \langle \mathsf{Onto}(K), \mathsf{Bridge}(K) \rangle$ *with* $\mathsf{Onto}(K)$ *a family of local ontologies which is recursively defined as follows*

- $\mathsf{Onto}(K) = \{K\}$, *if $K$ is a DL-ontology;*
- $\mathsf{Onto}(K) = \mathsf{Onto}(K_1) \cup \mathsf{Onto}(K_2) \cup \ldots \cup \mathsf{Onto}(K_n) \cup \mathsf{OntoAlign}(K)$ *if $K$ is a node with $K_i$ local nodes.*

$\mathsf{Bridge}(K)$ *is a family of bridge rules of $K$ recursively defined as follows:*

- $\mathsf{Bridge}(K) = \emptyset$ *if $K$ is an ontology;*
- $\mathsf{Bridge}(K) = \mathsf{Bridge}(K_1) \cup \ldots \cup \mathsf{Bridge}(K_n) \cup B(K)$.

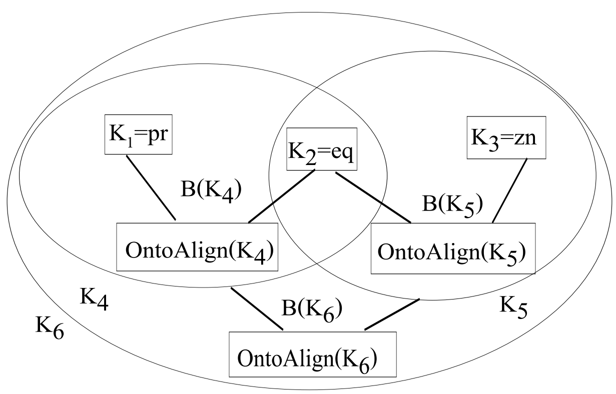


**Fig. 3.** Example of an MLNK in DDL form.

*Example 3.* Ontologies and alignments of Example 1 are used to build a DDL system. Figure 3 shows the structured knowledge nodes. Table 2 details the contents of those nodes.

**Semantics.** In this section, a multi-level knowledge base $X$ is interpret in the same way as the DDL distributed system $\mathsf{SystDis}(X)$ built out of it using the previous definitions.

**Definition 16 (Knowledge node DDL-interpretation).** *A DDL-interpretation of knowledge node $X$ is an interpretation in DDL formalism of the distributed system (*$\mathsf{SystDis}(X)$*).*

| Node | Distributed system |
|---|---|
| level 0 | |
| $K_1 = \texttt{pr}$ | $B(K_1) = \emptyset, \mathsf{Onto}(K_1) = \{K_1\}, \mathsf{Bridge}(K_1) = \emptyset, \mathsf{SystDis}(K_1) = \{\{K_1\}, \emptyset\}$ |
| $K_2 = \texttt{eq}$ | $B(K_2) = \emptyset, \mathsf{Onto}(K_2) = \{K_2\}, \mathsf{Bridge}(K_2) = \emptyset, \mathsf{SystDis}(K_2) = \{\{K_2\}, \emptyset\}$ |
| $K_3 = \texttt{zn}$ | $B(K_3) = \emptyset, \mathsf{Onto}(K_3) = \{K_3\}, \mathsf{Bridge}(K_3) = \emptyset, \mathsf{SystDis}(K_3) = \{\{K_3\}, \emptyset\}$ |
| niveau 1 | |
| $K_4 = \{K_1, K_2, A_{\texttt{K}_1\text{-}\texttt{K}_2}\}$ | $\mathsf{OntoAlign}(K_4) = \mathsf{oa}_4 = \langle \Sigma_4, F_4 \rangle$,<br>where $\Sigma_4 = \{k_1{:}\texttt{G}_1, k_2{:}\texttt{DF}_1, \textbf{compose}\}$<br>and $F_4 = \{\textbf{compose}(k_1{:}\texttt{G}_1, k_2{:}\texttt{DF}_1)\}$<br>$B(K_4) = \{k_1{:}\texttt{G}_1 \stackrel{\equiv}{\rightarrow} \mathsf{oa}_4{:}k_1{:}\texttt{G}_1,$<br>$k_2{:}\texttt{DF}_1 \stackrel{\equiv}{\rightarrow} \mathsf{oa}_4{:}k_2{:}\texttt{DF}_1\};$<br>$\mathsf{Onto}(K_4) = \{K_1, K_2, \mathsf{oa}_4\}$<br>$\mathsf{Bridge}(K_4) = B(K_4);$<br>$\mathsf{SystDis}(K_4) = \langle \mathsf{Onto}(K_4), \mathsf{Bridge}(K_4) \rangle$ |
| $K_5 = \{K_2, K_3, A_{\texttt{K}_2\text{-}\texttt{K}_3}\}$ | $\mathsf{OntoAlign}(K_5) = \mathsf{oa}_5 = \langle \Sigma_5, F_5 \rangle$,<br>where $\Sigma_5 = \{k_2{:}\texttt{DF}_1, k_3{:}\texttt{ANNA1TG01}, \textbf{part-of}\}$<br>and $F_5 = \{\textbf{part-of}(k_2{:}\texttt{DF}_1, k_3{:}\texttt{ANNA1TG01})\}$<br>$B(K_5) = \{k_2{:}\texttt{DF}_1 \stackrel{\equiv}{\rightarrow} \mathsf{oa}_5{:}k_2{:}\texttt{DF}_1,$<br>$k_3{:}\texttt{ANNA1TG01} \stackrel{\equiv}{\rightarrow} \mathsf{oa}_5{:}k_3{:}\texttt{ANNA1TG01}\};$<br>$\mathsf{Onto}(K_5) = \{K_2, K_3, \mathsf{oa}_5\}$<br>$\mathsf{Bridge}(K_5) = B(K_5)$<br>$\mathsf{SystDis}(K_5) = \langle \mathsf{Onto}(K_5), \mathsf{Bridge}(K_5) \rangle$ |
| niveau 2 | |
| $K_6 = \{K_4, K_5, A_{\texttt{K}_4\text{-}\texttt{K}_5}\}$ | $\mathsf{OntoAlign}(K_6) = \mathsf{oa}_6 = \langle \Sigma_6, F_6 \rangle$<br>where $\Sigma_6 = \{\mathsf{oa}_4{:}\textbf{compose}, \mathsf{oa}_5{:}\textbf{part-of}\}$<br>and $F_6 = \{\mathsf{oa}_4{:}\textbf{compose} \equiv \mathsf{oa}_5{:}\textbf{part-of}\}$<br>$B(K_6) = \{\mathsf{oa}_4{:}\textbf{compose} \stackrel{\equiv}{\rightarrow} \mathsf{oa}_6{:}\textbf{compose},$<br>$\mathsf{oa}_5{:}\textbf{part-of} \stackrel{\equiv}{\rightarrow} \mathsf{oa}_6{:}\textbf{part-of}$<br>$\mathsf{Onto}(K_6) = \mathsf{Onto}(K_4) \cup \mathsf{Onto}(K_5) \cup \{\mathsf{oa}_6\}$<br>$= \{K_1, K_2, K_3, \mathsf{oa}_4, \mathsf{oa}_5, \mathsf{oa}_6\}$<br>$\mathsf{Bridge}(K_6) = \mathsf{Bridge}(K_4) \cup \mathsf{Bridge}(K_5) \cup B(K_6)$<br>$= B(K_4) \cup B(K_5) \cup B(K_6)$<br>$\mathsf{SystDis}(K_6) = \langle \mathsf{Onto}(K_6), \mathsf{Bridge}(K_6) \rangle$ |

**Table 2.** Example of an MLNK in DDL form. We rename $\mathsf{OntoAlign}(K_i)$ in $\mathsf{oa}_i$ for $i \in [4, 6]$.

Distributed interpretation is a model of multi-level knowledge base if, each local interpretation is a model of associated local ontology and if domain relation satisfies the bridge rules.

**Definition 17 (DDL-based satisfaction relation of an MLNK).** *Let $X$ be a multi-level knowledge node, $I$ is DDL-interpretation of $X$, then $I$ DDL-satisfies $X$ (noted $I \models_{N-DDL} X$) if and only if $I$ satisfies* $\mathsf{SystDis}(X)$ *in DDL meaning (noted $I \models_{DDL}$* $\mathsf{SystDis}(X)$*). In this case, we say that $I$ is a DDL-model of $X$.*

Additionally, $X$ DDL-entails $Y$ if and only if all DDL-models of $X$ are also DDL-models of $Y$.

**Property 1** *Let $X$ be a multi-level knowledge node, composed of local nodes $N_1, \ldots, N_k$, then $X$ DDL-entails all local nodes ($X \models_{N-DDL} N_i$) for $i \in [1, k]$).*

*Proof.* Let a node $X = \langle\{N_i\}, \{A_{ij}\}\rangle$. $X \models_{N-DDL} N_i$ if it exists an interpretation $I$ where $I$ is a DDL-model of $X$ then $I$ is also DDL-model of $N_i$. We suppose that $I \models_{N-DDL} X$. This implies that $I \models_{DDL} \mathsf{SystDis}(X)$ (definition 17) and $I \models_{DDL} \langle \mathsf{Onto}(X), \mathsf{Bridge}(X)\rangle$ (definition 15). Then $I \models_{DDL} \mathsf{Onto}(N_1) \cup \mathsf{Onto}(N_2) \cup \ldots \cup \mathsf{Onto}(N_n) \cup \mathsf{OntoAlign}(X), \mathsf{Bridge}(N_1), \mathsf{Bridge}(N_2) \cup \ldots \cup \mathsf{Bridge}(N_n)$ and $I \models_{DDL} \mathsf{SystDis}(N_i)$ for $i = [1..n]$ consequently $I$ is a DDL-model of $N_i$

*Example 4.* With reference to Example 3, $\mathsf{SystDis}(K_6) = \langle\{K_1, K_2, K_3, \mathtt{oa}_4, \mathtt{oa}_5, \mathtt{oa}_6\}, \mathsf{Bridge}(K_6)\}\rangle$. We note that $I \models_{DDL} K_1, K_2, K_4$, $I \models_{DDL} k_1{:}\mathtt{G_1} \stackrel{=}{\rightarrow} \mathtt{oa}_4{:}k_1{:}\mathtt{G_1}$ and $I \models_{DDL} k_2{:}\mathtt{DF_1} \stackrel{=}{\rightarrow} \mathtt{oa}_4{:}k_2{:}\mathtt{DF_1}$ then $I$ satisfies $\mathsf{SystDis}(K_4)$ and it implies that $I$ a model of $K_4$. $I$ satisfies also $\mathsf{SystDis}(K_5)$ then $I$ is a model of $K_5$. So, we can conclude that $K_6$ implies a local nodes $K_4$ and $K_5$.

The transformation of a multi-level knowledge node allows us to relate it to distributed description logics which has an operational reasoning tool able to reason on the node, named DRAGO [9]. Therefore, the transformation provides us with an effective way of reasoning over multi-level networked knowledge.

## 6 Conclusions

In this paper a formalism capable of reasoning on a network of heterogeneous, complementary aligned ontologies, is presented. The alignments have proper vocabularies and necessitate, sometimes, to be aligned at different levels which represent the novelty of the presented formalism called MLNK. The semantic interpretation of the formalism is based on an existing paradigm, having complete reasoning procedures, along with operational tools such as a distribute description logics DDL, used in this case. An implementation of the approach as well as experimentation and tests on significant examples will be presented in our future work, but preliminary experiments on a variant semantics of MLNK was provided in [7]. We have chosen in this work to use an existing paradigm but it would be interesting to think about other ways of interpreting multi-level networked knowledge semantics by defining a formal semantic built directly into the network structure and then propose a correct and complete reasoning algorithm better adapted in this structure. Finally, we consider of paramount importance the implementation of a system able to integrate knowledge of such networks and respond to queries in a formalism question that remains to be defined.

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