Statistical Estimation of Aircraft InfraRed Signature Dispersion

S. Lefebvre – A. Roblin – G. Durand – S. Varet

sidonie.lefebvre@onera.fr
Context

• Optimization of optronics sensor
  Objectives: high detection probability – low false alarms rate
• Computer program to calculate aircraft IRS according to aircraft properties
  weather conditions
  attack profiles

The Infrared & Electro-Optical Systems Handbook, Vol 7, Countermeasure Systems

Uncertainty on input data
Uncertainty on input data

• Several data types:

  Fixed data or parameters defined by scenario

  Uncertain data:
  • bounds min.-max. (coating, engine, aircraft orientation…)
  • statistical (weather conditions…)
  • qualitative (season, flight above: sea-land,…)

• Correlations

Take IRS dispersion into account to estimate optronics sensor properties
scalar response: difference between target and background irradiance
• Identification of uncertainty associated to each input: variations range - correlations

• Sensitivity analysis: **identify most important inputs**

• \( P (\text{IRS} < \text{threshold}) \) by Quasi Monte Carlo

• Metamodel (neural network): faster – enables sensor properties optimization
Sensitivity Analysis

5000 experiments max. – 28 factors – must account for interactions → design of experiments

Procedure:  
- two levels (min. - max.) for each input data – functions of scenario

  - factors are assumed to be independent

  - standardization: -1 and +1

  - choice of underlying model (depending on accuracy, interactions level…)

\[
Y = Cste + \sum_{i} c_i X_i + \sum_{i<j} c_{ij} X_i X_j + \sum_{i<j<k} c_{ijk} X_i X_j X_k + ... 
\]

Response  \quad \text{Main effects}  \quad \text{Two-factors interactions}

- choice of experimental design (DOE)

- collect response data for all numerical experiments prescribed by matrix:

\[
X_i^j \quad i = 1..n_{\text{fact}} \quad j = 1..n_{\text{calc}}
\]

- estimation of main effects and factors interactions (stepwise – Student’s test)
Factorial Designs

Each level of each factor is combined with each value of each and every other factor

\[ N = 2^n \] experiments, \( n \) factors

Too expensive \( \Rightarrow \) fraction of this design \textbf{Fractional factorial design}

Fractional factorial design: \( N = 2^{n-p} \) experiments

Can’t estimate all coefficients, but \textit{set of aliased coefficients}

Aim: study main effects and two-factors interactions

+ take into account three-factors interactions - not negligible

Design property: resolution

For a resolution R design, main effects are aliased with interactions involving at least (R-1) factors

\[ Y = \text{cste} + \sum_i c_i X_i + \sum_{i,j} c_{ij} X_i X_j + \varepsilon \]

28 input data – 2048 exp – resolution VI
Application to a typical air-to-ground attack example

Daytime air-to-ground attack, in France, at low altitude

Pareto plot: factors sorting / main effects only

B II

- 7 related to atmosphere background
- 5 to flight conditions - 1 to characteristics of aircraft

B III

- 7 related to atmosphere background
- 4 to flight conditions - 2 to characteristics of aircraft

13 Factors that mostly contribute to IRS variability
Quasi Monte Carlo estimation of the IRS dispersion
Estimation of $P$ (IRS < threshold)

$$P(IRS(X_1,\ldots,X_n) < \alpha) = \int_{\mathbb{R}^n} 1_{IRS(x_1,\ldots,x_n) < \alpha}(t) p_{X_1,\ldots,X_n}(t) dt$$

- **Monte Carlo:**
  $$P(IRS < \alpha) \approx \frac{1}{N} \sum_{i=1}^{N} I(IRS(u_i) < \alpha)$$
  
  $u_i = (X_i^1, X_i^2, \ldots, X_i^n)$, $n$ = number of input factors
  
  $u_i$ independent – uniform law, cv rate $O(1/\sqrt{N})$

- **Alternative: Quasi Monte Carlo**
  
  $u_i$ independent, determinist low discrepancy sequence
  
  5-10 times faster / MC dimension 10 (Lapeyre et al. 1990)

  discrepancy = characterizes uniformity of sequence distribution
Low discrepancy sequence

\[ \Gamma = (\xi^j)_{j \in \mathbb{N}} \text{ sequence n-dim} \]

**Discrepancy:**

\[ D_N^* (\Gamma) = \sup_{I \in \Gamma} \left| \frac{A_N(I, \Gamma)}{N} - \mu(I) \right| \]

\[ I^* = \left\{ \prod_{i=1}^{n} [0, \alpha_i); \alpha_i \in [0,1] \right\} \]

\[ \mu(I) = \text{volume of I (theoretical measure)} \]

\[ A_N(I, \Gamma) = \text{nb points } \xi^i \text{ in I among N first} \]

\[ \text{(empirical measure of volume of I)} \]

**Low discrepancy**

\[ D_N^* = O \left( \frac{(\log(N))^n}{N} \right) \]

**Koksma-Hlawka theorem:**

\[ \frac{1}{N} \sum_{j=1}^{N} f(\xi_j) - \int_{[0,1]^n} f(u) du \leq V(f) D_N^*(\Gamma) \]

**Estimation of** \( D_N^* \) **and** \( V(f) \) **difficult**

Large dimension: theoretically cv rate MC better / QMC

but practical studies show better results with QMC / MC (Caflisch et al. 1997 – dimension 360)

\[ \Rightarrow \text{Effective dimension} \]

\[ \Rightarrow \text{PhD S. Varet – effective discrepancy: joint property of sequence + integrand} \]
Results: IRS dispersion

10240 computations

- 8 Variables related to weather conditions: bootstrap in statistics database [web site meteo.infospace.ru]
- Other variables: 14 unimportant: constant

Randomization methods modify the decomposition on prime number basis used in sequence building.

They preserve the low discrepancy, add randomness, useful to estimate CI, and decrease projection irregularities on small dimension subspaces.

Empirical cumulative distribution function \(\Rightarrow\) estimation of \(P(\text{IRS} < \text{threshold})\)
Results: Empirical probability density function

**B II**

**B III**
Realistic thresholds for non-detection probabilities depend a lot on the optronics sensor we want to size

=> estimation of three quantiles $\beta$, which correspond to typical non-detection probabilities

1 %, 5 % and 25 %

Empirical estimator: $\inf\{y, F_N(y) > \beta\} = y_{[\beta N]}$ after IRS reordering ($F_N$ ecdf)

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>1 %</th>
<th>5 %</th>
<th>25 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>[1.63, 1.71]</td>
<td>[11.36, 11.56]</td>
<td>[54.8, 55.2]</td>
</tr>
<tr>
<td>500</td>
<td>[1.57, 1.62]</td>
<td>[11.44, 11.6]</td>
<td>55</td>
</tr>
<tr>
<td>1000</td>
<td>[1.34, 1.38]</td>
<td>[11.12, 11.24]</td>
<td>55</td>
</tr>
<tr>
<td>2000</td>
<td>[1.23, 1.26]</td>
<td>[11, 11.08]</td>
<td>55</td>
</tr>
<tr>
<td>5000</td>
<td>[1.18, 1.2]</td>
<td>[10.94, 10.98]</td>
<td>55</td>
</tr>
<tr>
<td>10000</td>
<td>[1.16, 1.17]</td>
<td>10.92</td>
<td>55</td>
</tr>
</tbody>
</table>

95% confidence level bootstrap estimations based on 5000 draws among the 10240 IRS values, for different sample sizes

Good evaluation of 5 % and 25 % quantiles with 2000 values
Set-up of a Neural Network Metamodel
Neural Network

Linear Regression => poor predictions in our case
Multilayer feedforward neural network = universal approximator (Hornik et al. 1989)

Hidden layer \( n_c \) neurons

Sigmoid activation function: \( \theta(\cdot) \)

Single output

Linear activation function

\[
\phi(x) = \sum_{i=1}^{n} w_{n_c+1,i} \theta\left( \sum_{j=1}^{n} w_{ij} x_j + w_{i0} \right) + w_{n_c+1,0}
\]

\( q = n*n_c + 2*n_c + 1 \) parameters

n input data selected thanks to Fractional Factorial Design sensitivity analysis

Weight decay: cost function

\[
J(w) = \frac{1}{2} \sum_{i=1}^{N} \| y_i - \phi(x_i, w) \|^2 + \frac{\alpha}{2} \sum_{j=1}^{q} w_j^2
\]
Neural Network Metamodel – Band II

13 input data  7 hidden neurons  decay 0.01

Learning: first 500/2000/4000 points among 10240 QMC + bootstrap  Test: 10000 MC + bootstrap

Very good agreement between real and predicted cdf and pdf

4000 pts neural network metamodel
Neural Network Metamodel – quantiles

13 input data    7 hidden neurons

Learning: random sampling of 4000 points among 10240 QMC + bootstrap
Test: 10240 MC + bootstrap

Bootstrap estimations of three quantiles of the metamodel

<table>
<thead>
<tr>
<th></th>
<th>1 %</th>
<th>5 %</th>
<th>25 %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[1.42, 1.44]</td>
<td>[13.5, 13.6]</td>
<td>[53.9, 54]</td>
</tr>
</tbody>
</table>

The metamodel gives a very good prediction of non-detection probability:
difference < 1 %

Best choice of learning points?
Downsize learning database?
=> Adaptive metamodel
Adaptive construction (Gazut, Martinez et al. 2008)

1. $N_0 = 500$ first pts Faure sequence
2. 100 bootstrap on set of $N_n$ pts $\Rightarrow$ learning of neural network 7 hidden neuron
3. estimation of mean and variance of the 100 predictions for a 50000 pts database
4. add 100 pts largest variance to $N_n$ pts

2. with $N_{n+1} = N_n + 100$

Very good metamodel with 1500 pts

More stable: small impact of choice of $N_0$ first pts
Adaptive construction

### CI 95% Quantiles

<table>
<thead>
<tr>
<th></th>
<th>1 %</th>
<th>5 %</th>
<th>25 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metamodel 4000 pts</td>
<td>[1.34, 1.38]</td>
<td>[13.5, 13.6]</td>
<td>54</td>
</tr>
<tr>
<td>Metamodel adaptive 1500 pts</td>
<td>[1.42, 1.46]</td>
<td>[14.42, 14.58]</td>
<td>[54.23, 54.43]</td>
</tr>
<tr>
<td>Metamodel adaptive 2000 pts</td>
<td>[1.72, 1.74]</td>
<td>[13.32, 13.44]</td>
<td>[54.95, 55.17]</td>
</tr>
<tr>
<td>10000 CRIRA</td>
<td>[1.16, 1.17]</td>
<td>10.92</td>
<td>55</td>
</tr>
</tbody>
</table>
Concluding remarks

✓ Sensitivity analysis =&gt; Factors that mostly contribute to IRS variability

✓ P(IRS < threshold) by Quasi Monte Carlo

✓ Metamodel (neural network) =&gt; IRS approximation
  =&gt; allows much faster sensor properties optimization

✓ Efficient methodology: predicts simulated IRS dispersion of poorly known aircraft can be extended to IRS models of other military objects
Concluding remarks

- **Infer the joint density probability of meteorological factors from the database**
  - variance reduction methods => quantiles estimation
  - space filling designs
  - adaptive metamodels

- **Use of S. Varet PhD results: designs which minimize effective discrepancy**
  - estimation of non-detection probabilities
  - metamodel

- **Aircraft spatially resolved: vectorial output (picture 10x10 pixels)**
  - new detection and classification algorithms
  - sensitivity analysis for a vectorial output ?
  - characterization of IRS dispersion ?
  - classification algorithm which accounts for IRS dispersion

  Different aspect angles of aircraft => very dissimilar pictures
Thanks

sidonie.lefebvre@onera.fr