Evolutionary Approach to Multiobjective Optimization of Dimensionally Stable Composites

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Abstract

The design of dimensionally stable composite laminates is formulated as a multiobjective optimization problem. Design is performed by a niched Pareto evolutionary algorithm. The result is the set of all possible laminates associated to all possible trade-offs between design criteria. This should be opposed to single objective optimization where design is achieved with respect to a particular (arbitrary) combination of criteria. Results are given for the stacking sequence optimization of organic matrix composites. Pareto optimal designs are presented where a set of designs trades off thermal stability against hygral stability.

1 Introduction

Many space structures, such as telescopes or antennas, need dimensionally stable materials. Structural requirements on long term stability are increasingly stringent while manufacturing costs are tightened. There is therefore a need for numerical tools providing optimized materials under changing, complex, and contradictory criteria : time, temperature, low pressure, mechanical stresses, structural weight, manufacturing costs (see [1] for example).

One of the most flimsy decision a designer has to make within the context of traditional optimization is how to balance the different criteria. The difficulty might become an impossibility when a criterion has no valid model (like cost or aging).

Multicriterion optimization circumvents this difficult trade-off by providing a set of solutions for all possible criteria associations [2]. The final design can more freely be chosen among the proposed solutions based on expert or late information.

In this paper, an evolutionary niched Pareto algorithm [3] is the multiobjective design technique applied to dimensionally stable laminates. Laminate analysis is first presented. Then, the niched Pareto evolutionary algorithm is introduced. The discussion compares laminates located in different regions of optimality sets. In particular, the trade off between thermal stability and hygral stability is studied.

2 Analysis and design formulation

2.1 A versatile composite design program

Although a particular design problem is presented here, it should be mentionned that calculations have been carried out in an existing C++ object oriented program. The program has 3 main modules, an analysis module, an optimization module, and the interface between the modules, called `Simulation_environment`. The analysis and optimization modules are composed of stable routines with restricted access. The right routines within those modules are accessed after reading input files through dynamic pointer inheritance and the virtual function mechanism (see [4]). The `Simulation_environment` class has priviledged access to the analysis and optimization classes. It fetches information on design variables from the optimizer, starts the analysis, retrieves physical information relevant to the current design problem. Finally, objective functions calculation is performed by `Simulation_environment`. Pseudocode for the main program is given in Figure 1 and overall program organization is sketched in Figure 2. It is easier to add new analysis capabilities (homogenization methods, other types of structures, other categories of physical problems like buckling or dynamics), new optimizers and new design problems with this type of program than with other non-object oriented languages.

2.2 Homogenization and plate analysis

Ply hygral, thermal, and mechanical properties are calculated from micromechanical data according to Chamis relations [5]. Fibers are assumed to be transversely isotropic.

Classical Lamination Theory is used to calculate laminate properties, i.e., we apply Kirchhoff plate theory, and neglect through-the-thickness stresses and strains. The twice homogenized (once from fiber/matrix to layer and once from layers to plates) stiffness matrix is composed of the 3×3 extensional, coupling and flexural stiffness matrices, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{D}$, respectively. They are functions of the material properties, the fiber orientations in each ply, and the position and number of ply interfaces. Introducing the compliance matrix,

$$\begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} = \begin{bmatrix} a & b \\ b^T & d \end{bmatrix} . \tag{1}$$

the effective Young's moduli of the laminate in the longitudinal, and transverse direction, E_x, E_y, and shear modulus G_{xy}, respectively, are expressed as,

$$E_x = \frac{1}{a_{11}h} \ , \ E_y = \frac{1}{a_{22}h} \ , \ G_{xy} = \frac{1}{a_{66}h} \ , \tag{2}$$

where the index 6 points to the third row/column of the relevant matrix. For a laminate under general loading conditions, the forces and moments result from the superposition of the mechanical, thermal, and hygral forces and moments, $\mathbf{N}^m$, $\mathbf{N}^T$, $\mathbf{N}^H$, $\mathbf{M}^m$, $\mathbf{M}^T$, and $\mathbf{M}^H$, respectively. The stress-strain relationship is,

$$\left\{ \begin{array}{c} \varepsilon \\ \kappa \end{array} \right\} = \begin{bmatrix} a & b \\ b^T & d \end{bmatrix} \left\{ \begin{array}{c} N^m + N^T + N^H \\ M^m + M^T + M^H \end{array} \right\} \tag{3}$$

The thermal and hygral forces and moments are defined as,

$$\begin{aligned}
\mathbf{N}^T &= \sum_{k=1}^{n} \bar{\mathbf{Q}}_k \bar{\alpha}_k^T (z_k - z_{k-1}) \Delta T \\
\mathbf{M}^T &= \frac{1}{2} \sum_{k=1}^{n} \bar{\mathbf{Q}}_k \bar{\alpha}_k^T (z_k - z_{k-1})^2 \Delta T \\
\mathbf{N}^H &= \sum_{k=1}^{n} \bar{\mathbf{Q}}_k \bar{\alpha}_k^H (z_k - z_{k-1}) \Delta H \\
\mathbf{M}^H &= \frac{1}{2} \sum_{k=1}^{n} \bar{\mathbf{Q}}_k \bar{\alpha}_k^H (z_k - z_{k-1})^2 \Delta H
\end{aligned}$$

where n is the total number of layers, $\bar{\mathbf{Q}}$ is the reduced ply stiffness matrix in laminate coordinates, $\bar{\alpha}^T$ and $\bar{\alpha}^H$ are the ply coefficients of thermal and hygral expansion in the laminate coordinates, respectively, and z_k is the position of the k-th ply interface. ΔT is the temperature variation and ΔH is the moisture variation. The strains and curvatures obtained from equation (3) when $\mathbf{N}^m = \mathbf{M}^m = \mathbf{N}^H = \mathbf{M}^H = 0$ and $\Delta T = 1°$ are the coefficients of thermal expansion (CTE) and curvature, α^T and $\mathbf{W}^T$. Identically, when mechanical and thermal loads are set to zero and $\Delta H = 1$ %M, the strains and curvatures of equation (3) are the coefficients of moisture expansion (CME) and bending α^H (or more commonly β) and $\mathbf{W}^H$.

Further details on composite plate analysis can be found in [1].

2.3 Optimization problem formulation

The objective of the design is,

$$\begin{array}{lrcl} \text{Minimize} & f_1 & = & \frac{|\alpha_x^T|}{\alpha_x^{T,allow}} \\ \text{and} & f_2 & = & \frac{|\beta_x|}{\beta_x^{allow}} \\ \text{and} & f_3 & = & \max(0, 1 - \frac{E_x}{E_x^{min}}) \\ \text{and} & f_4 & = & \max(0, 1 - \frac{E_y}{E_y^{min}}) \\ \text{and} & f_5 & = & \max(0, \frac{|\varepsilon_x|}{\varepsilon_x^{allow}} - 1) \\ \text{and} & f_6 & = & \max(0, \frac{|\varepsilon_y|}{\varepsilon_y^{allow}} - 1) \end{array}$$

by changing θ_i, $i = 1, n/2$

where θ_i's are the continuous ply angles and n is the total number of plies ($n/2$ angles are to be determined because we consider symmetric laminates). The objectives f_3 to f_6 express in fact optimization constraints. Because the $\max(0, x)$ function has flat regions when $x < 0$, such a formulation would be inefficient for mathematical programming algorithms. In our case however, we avoid constrained optimization (and the difficulty of handling constraints) by reformulating the problem as a multiobjective optimization problem. This strategy is made possible by the use of evolutionary algorithms and niching (cf. Section 3) which can handle flat functions.

3 Multiobjective optimization through the niched pareto evolutionary algorithm

The multiobjective optimization method used in this study is a steady state evolutionary algorithm [6] with a niched Pareto selection procedure [3]. The evolutionary algorithm is a stochastic optimization method that manages a set of points and the associated objective functions. The set of points is often referred to as population. A new point is generated through the combined effect of *(i)* a selection procedure which chooses two points from the current set, *(ii)* a crossover operator which randomly generates a new point in the region spanned by the previously selected couple of points, *(iii)* a mutation operator which randomly perturbates the point produced by the crossover. The perturbation follows a normal distribution, *(iv)* a replacement procedure, where a point of the current set is discarded in favour of the new point. The objective functions of the new point are calculated. The process is repeated until a maximum number of objective functions calculations.

Crossover and mutation operators induce pseudo-random transitions in the space of possible designs. Selection and replacement are the componenst of the algorithm that bias odds towards designs of decreasing objective function. Further general details on evolutionary optimizers are given in [6]. We focuse here on the selection and replacement methods which characterize the multiobjective approach.

Since there are many objective functions that are not related to each other through penalization, it cannot be said whether a vector of design variables has a lower objective function than another vector. Comparison between points is based instead on Pareto domination [2] where a point is said to be better than another point if it is better for all objectives.

Definition 1 (Pareto Domination) *The variables x dominate the variables y $\Longleftrightarrow$ $f_i(x) \leq f_i(y)$, $\forall i = 1, no$, where no is the number of objectives.*

The purpose of the multiobjective evolutionary algorithm is to find the set of all non-dominated points of the design space (called Pareto front).

The selection operator works as follows:

1. Choose candidate 1 and candidate 2 at random. Choose a set of nc comparison points from current set.

2. If candidate 1 dominates the comparison set and not candidate 2, return candidate 1.

3. Else if candidate 2 dominates the comparison set and not candidate 1, return candidate 2.

4. Else do niching: return candidate that has the least neighbours.

It is applied twice to yield two "parent" designs for crossover. One of the candidates that is not selected is replaced by the new design resulting from subsequent crossover and mutation. When no selection candidate is better than the other in terms of Pareto domination, niching is applied as a way to preserve diversity in the set of points. Niching selects the point that has the least neighbours. Without niching, early good design points, under the combined effect of selection and replacement, would spread in the current design set and only part of the Pareto front would be determined. A point y is said to be in the neighbourhood of x if the distance from x to y in the objective function space with respect to Holder's norm with $p = 1$ is less than σ_{share},

$$\sum_i (|f_i(x) - f_i(y)|^p)^{(1/p)} < \sigma_{share} . \quad (4)$$

4 Optimization results

Results are obtained for a high modulus carbon fiber (P75 from Amoco) and an organic matrix (934 from Fiberite) with properties given in Table 1. The plate is $0.3 \times 0.3\ m$, submitted to the loads $N_x^m = 20000N$ and $N_y^m = 20000N$, with design values $E_x^{min} = 100GPa$, $E_x^{min} = 10GPa$, $\varepsilon_x^{allow} = 0.05$, $\varepsilon_y^{allow} = 0.05$, $\alpha_x^{allow} = 1.10^{-9}/^oC$, $\beta_x^{allow} = 10^{-1}/\%M$. Unless otherwise stated, the evolutionary algorithm has a set size of 400 points, a probability of applying crossover of 40%, a probability of applying mutation of 20% per design variable (variance equal to $(\theta_{max} - \theta_{min})^2/16 = 2025$), it stops after 9000 laminate analyses and the comparison set used in selection has 50 points.

The size of the neighbourhood, σ_{share}, is a rather sensitive parameter. For the problem at hand, it is possible to obtain a reasonable value for σ_{share} because it is possible to have bounds on the criteria f_i. For no criteria, the Pareto front is an $no - 1$ dimensional surface. Its area is bounded from above by,

$$A_{Pareto}^{max} = \sum_{k=1}^{no} \{ \frac{1}{no-1} \prod_{i\,1}^{i\,(no-1)} \Delta f_{i\,1} \ldots \Delta f_{i\,(no-1)} \} \quad (5)$$

where $i\,j \neq i\,l$, $i\,j \neq k$, $j, l = i\,1, i\,(no{-}1)$, and $\Delta f_i = f_{i,max} - f_{i,min}$. For two criteria, this yields,

$$A_{Pareto}^{max} = \Delta f_1 + \Delta f_2 . \quad (6)$$

The slice of the neighbourhood that belongs to the Pareto set is approximated by σ_{share}^{no-1}, so that, with N points in the current set that share the Pareto surface, σ_{share} should satisfy,

$$\sigma_{share}^{no-1} \approx \frac{A_{Pareto}^{max}}{N} . \quad (7)$$

It should be mentionned that, eventhough the primary purpose of multi-objective optimization is to remove all comparison of values of objectives, introducing niching is a step backwards: values of the different objectives are mixed in Equation (4) to define the neighbourhood. If the objectives are not properly scaled, the Pareto front will be more spread along large objectives than along small objectives. This is not, however, as critical a problem as obtaining the right penalization factors.

4.1 Two criteria

4.1.1 Two design variables

We first look at the dimensional stability problem with the two objectives f_1 and f_2 and two design variables θ_1 and θ_2 because graphical representation in both the space of ply angles and the space of objectives is possible. The laminates considered are of the form $[\theta_1/\theta_2]_s$. The first and the last set of designs are shown in the ply angle and properties spaces in Figures 3 and 4, respectively. The last population and the Pareto front are represented in Figure 5 in the objective functions space and ply angle space. All laminates of the form $[\theta_1/\theta_2]_s$ are located along a branched line in the α_x β_x space. The lower left-most of these branches is the Pareto front. The mapping between the fiber angle space and the objectives space is drawn in Figure 5. Optimal laminates with respect to α_x are comprised in the interval $-35^o \leq \theta_1 \leq -25^o$, $1^o \leq \theta_2 \leq 5^o$. Optimal laminates with respect to α_x are located in the region $-18^o \leq \theta_1 \leq -12^o$, $-15^o \leq \theta_2 \leq -12^o$. More generally, points in the $-35^o \leq \theta_1 \leq -10^o$, $-11^o \leq \theta_2 \leq 0^o$ region achieve various degrees of compromise beween thermal and hygral dimensional stability.

4.1.2 Six design variables

In order to test the multicriterion approach, while involving all six objective functions f_1 to f_6 and keeping clear graphical representations, f_1 and f_6 have been included in two criteria, $C1$ and $C2$, defined as,

$$C1 = \frac{|\beta_x|}{\beta_x^{allow}} + p_{11}\max(0, 1 - \frac{E_x}{E_x^{min}}) + p_{12}\max(0, \frac{|\varepsilon_x|}{\varepsilon_x^{allow}} - 1) \quad (8)$$

$$C2 = \frac{|\alpha_x^T|}{\alpha_x^{T,allow}} + p_{21}\max(0, 1 - \frac{E_y}{E_y^{min}}) + p_{22}\max(0, \frac{|\varepsilon_y|}{\varepsilon_y^{allow}} - 1) \quad (9)$$

where all p_{ij}'s are taken equal to 5. The formulation of $C1$ and $C2$ is guided by the similarities between their terms. In $C1$, β_x, E_x and ε_x have good values for fibers along or near the x-axis. In $C2$, α_x^T, E_y and ε_y have good values for disoriented fibers. In this sense, $C1$ and $C2$ are two competing features. The laminates have $n = 12$ plies (6 design variables).

Figure 6 represents the initial population of designs and the Pareto front obtained after 9000 laminate analyses. New points along the Pareto front have appeared, especially in border regions where one criterion is clearly favored. Figure 7 shows the last population and its Pareto front. It is checked that most of the population is distributed along the front. In Figure 8, the results of five independent optimizations are compared. The algorithm consistently finds the middle of the Pareto front, but determination of extreme designs is subject to noise. The results of the five independent runs are summed up in Figure 9 where the Pareto front of the five Pareto fronts of Figure 8 has been extracted. This constitutes a reference list of all compromises that can be achieved for the $C1$ and $C2$ criteria. Five characteristic designs, noted Point 1 to 5 in Figure 9 have been detailed in Table . Point 1 has high performance with respect to criterion $C1$ and low performance with respect to $C2$, Point 5 has low performance with respect to $C1$ and high performance with respect to $C2$. Other points are graded tradeoffs. It should be noted that the rule-of-thumb used to group objectives f_1 to f_6 into $C1$ and $C2$ are checked: designs that have good $C2$ (α_x,E_y, ε_y) have fibers globally more disoriented than desings with good $C1$ (β_x,E_x, ε_x). The couplings between α_x and E_y (ε_y) and between β_x and E_x (ε_x) are respected in design points 1 to 5. In other terms, for the designing dimensionally stable and stiff laminates, on x-axis and off x-axis properties are competing. The laminate exchanges hygral stability and x-axis stiffness against thermal stability and y-axis stiffness. The Pareto front is a description of the principal trade-offs.

4.2 Design with uncertainties

5 Concluding remarks

This study proposes to consider the design of composite laminates as a multi-objectives problem. A specific method for multiobjective and nonconvex optimization is implemented, the niched Pareto evolutionary algorithm. Results show that the observation of Pareto set of optimal laminates is meaningfull to

```
int main()
{
  SIMULATION_ENV composite;
  OPTIMIZER* p_opt;

  composite.load();
  p_opt = OPTIMIZER::make_optimizer();
  p_opt->load(composite);
  p_opt->optimize();
}
```

Figure 1: Pseudo-code for main program.

the designer. It is found in particular, when optimizing laminates for dimensional stability and stiffness, that designs exchanges hygral stability and x-axis stiffness against thermal stability and y-axis stiffness. More complex situations might arise when considering other objectives such as strength and property dispersions.

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property	**fiber P75**	**org. matrix 934**
ρ (ton/m^3)	1.9	1.3
E_L (GPa)	550.20	4.34
E_T (GPa)	9.52	4.34
G_{LT} (GPa)	6.90	1.59
ν_{LT}	0.20	0.37
α_L $(10^{-6}/\ ^\circ C)$	-1.35	43.92
α_T $(10^{-6}/\ ^\circ C)$	6.84	43.92
β_L $(10^{-6}/\ \%\ M)$	/	2000.0
β_T $(10^{-6}/\ \%\ M)$	/	2000.0

Table 1: Basic material properties.

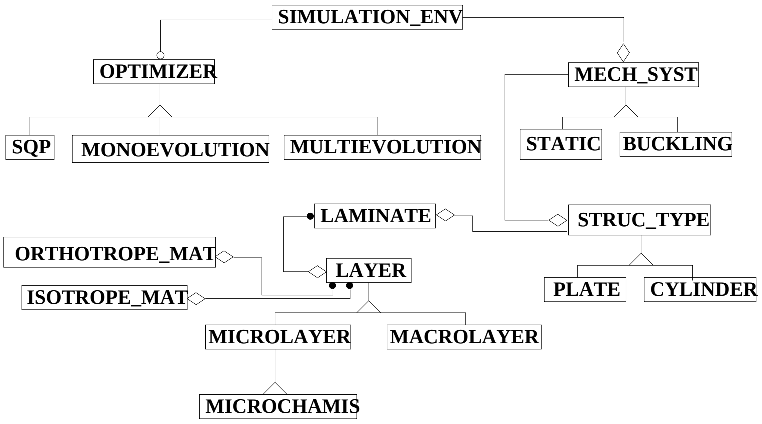

Figure 2: Connectivity of basic classes, notations from [4]

Criterion	Point 1	Point 2	Point 3	Point 4	Point 5
$f1$ $p_{11} = 5$ $p_{12} = 5$	$1.24e-09$	$9.3e-06$	$2.07e-04$	$3.7e-04$	1.97
$f2$ $p_{21} = 5$ $p_{22} = 5$	18.8	10.22	3.47	$6.1e-04$	$2.61e-04$
Stacking sequence (^{o})	$[-24/2/-52/1/28/4]_s$	$[-20/20/-23/43/56/-43]_s$	$[-53/83/24/-16/0/-60]_s$	$[24/78/80/88/34/-15]_s$	$[-15/60/-24/67/89/-39]_s$
E_x (MPa)	$2.1e11$	$1.01e11$	$1.43e11$	$1e11$	$6.04e10$
E_y (MPa)	$2.39e10$	$3.62e10$	$9.34e10$	$1.38e11$	$9.62e10$
α_x $(/^oC)$	$-1.5e-06$	$-1.5e-06$	$-6.9e-07$	$-1.23e-10$	$-5.2e-11$
β $(/\%M^oC)$	$-1.24e-10$	$9.36e-07$	$2.07e-05$	$3.78e-05$	$3.78e-05$
ε_x	$-2.5e-05$	$-1.43e-05$	$2.1e-04$	$4.2e-04$	$4.2e-04$
ε_y	$1.6e-03$	$7.7e-04$	$3.8e-04$	$3.e-04$	$1.4e-04$

Table 2: Typical designs from Pareto front

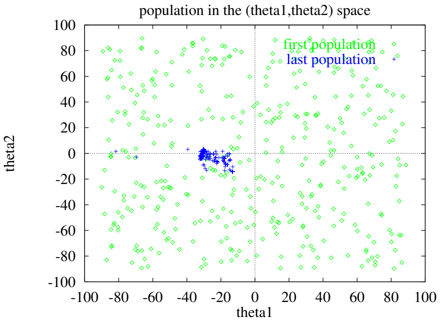

Figure 3: First and last set of designs in the ply angle space

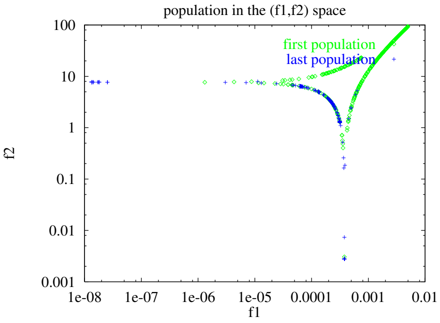

Figure 4: First and last set of designs in the α_x - β_x objectives space

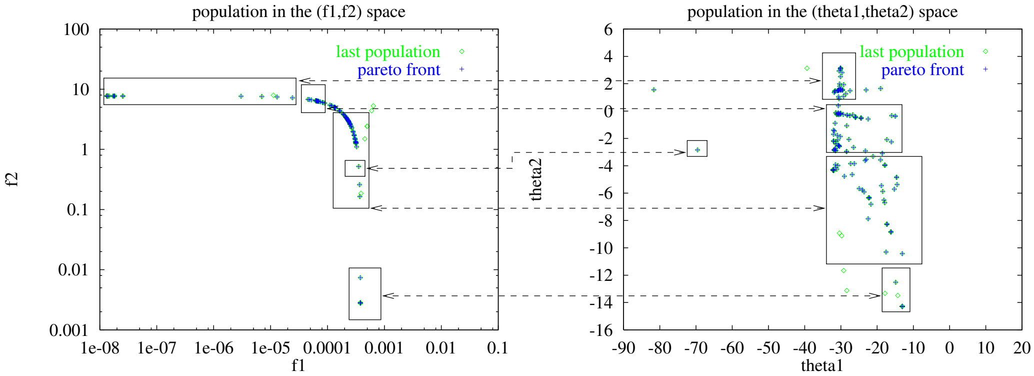

Figure 5: Representation of the last population (and Pareto set) in the ply angle and objectives spaces

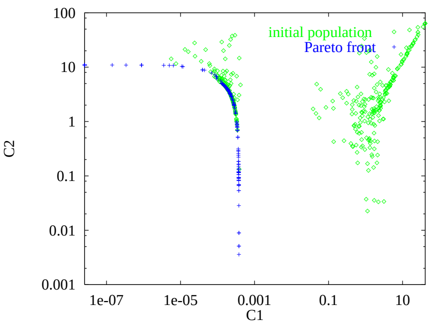

Figure 6: Pareto front of the last population of designs and initial population

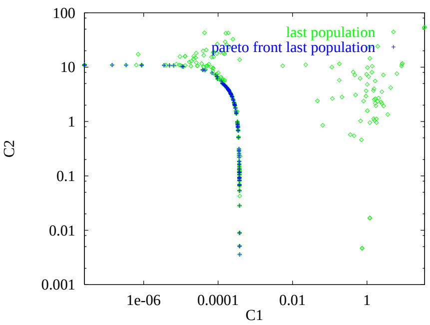

Figure 7: Pareto front and last population of designs

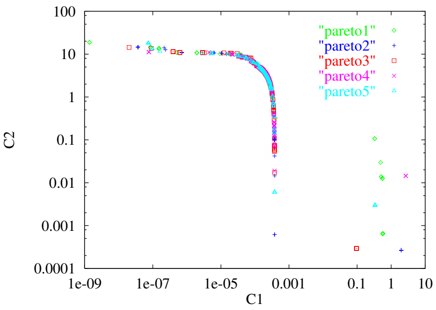

Figure 8: Comparison of results of 5 independent runs in terms of Pareto front

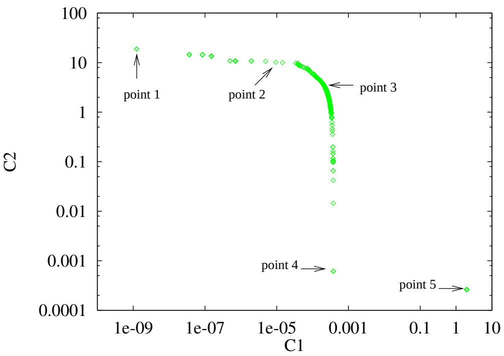

Figure 9: Pareto front of five independent Pareto fronts