Single-item lot sizing problem with carbon emission under the cap-and-trade policy

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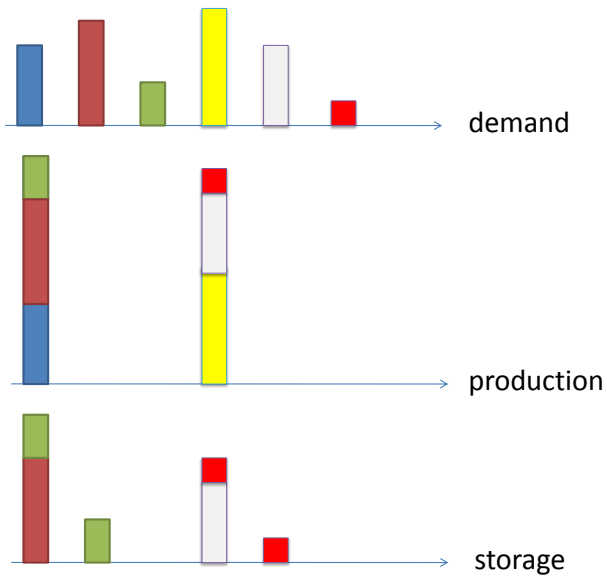
Problem overview

- Uncapacitated single-item lot sizing problem (ULSP)
- Carbon emission constraints under the cap-and-trade policy
- Unlimited budget case & 2 limited budget cases
- Another extension : multi-technology green ULSP

Outline

What is ULSP - Uncapacitated lot sizing problem ?

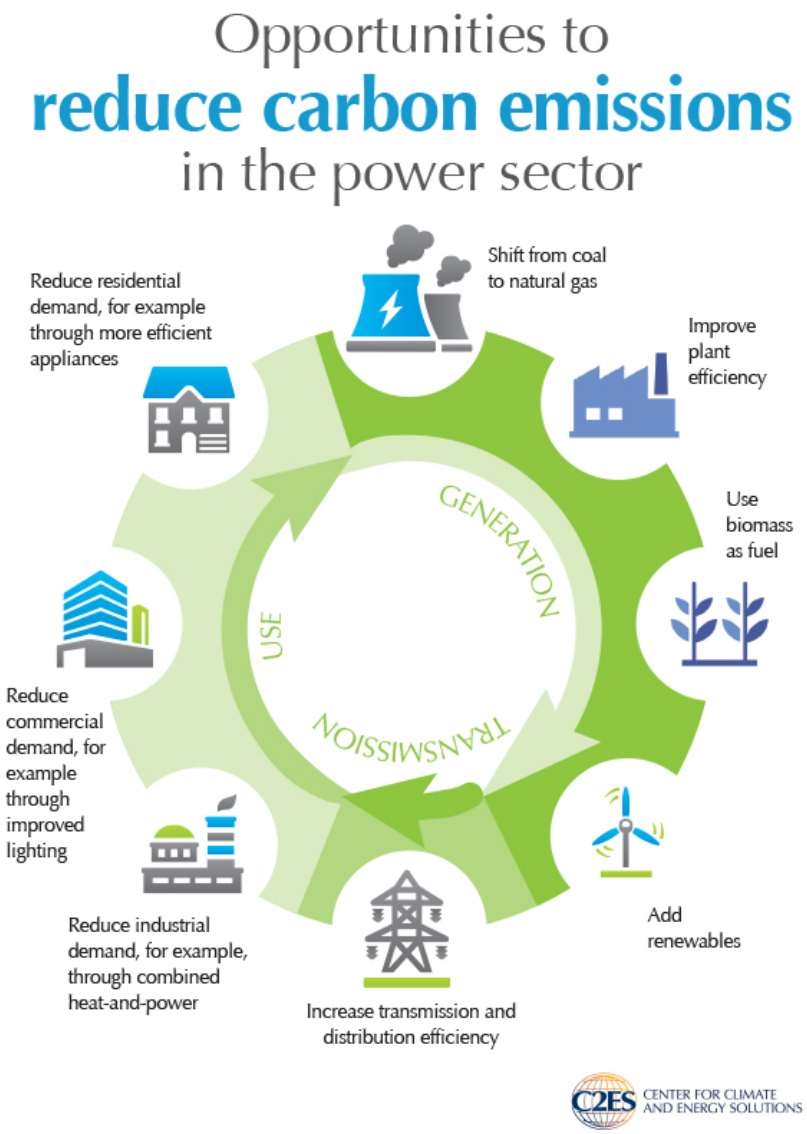
- Demands d are known over T periods, to satisfy without backlog
- Setup cost f_t, unit production cost p_t, unit holding cost h_t are known
- We have to decide on the quantity x_t to produce in each period t



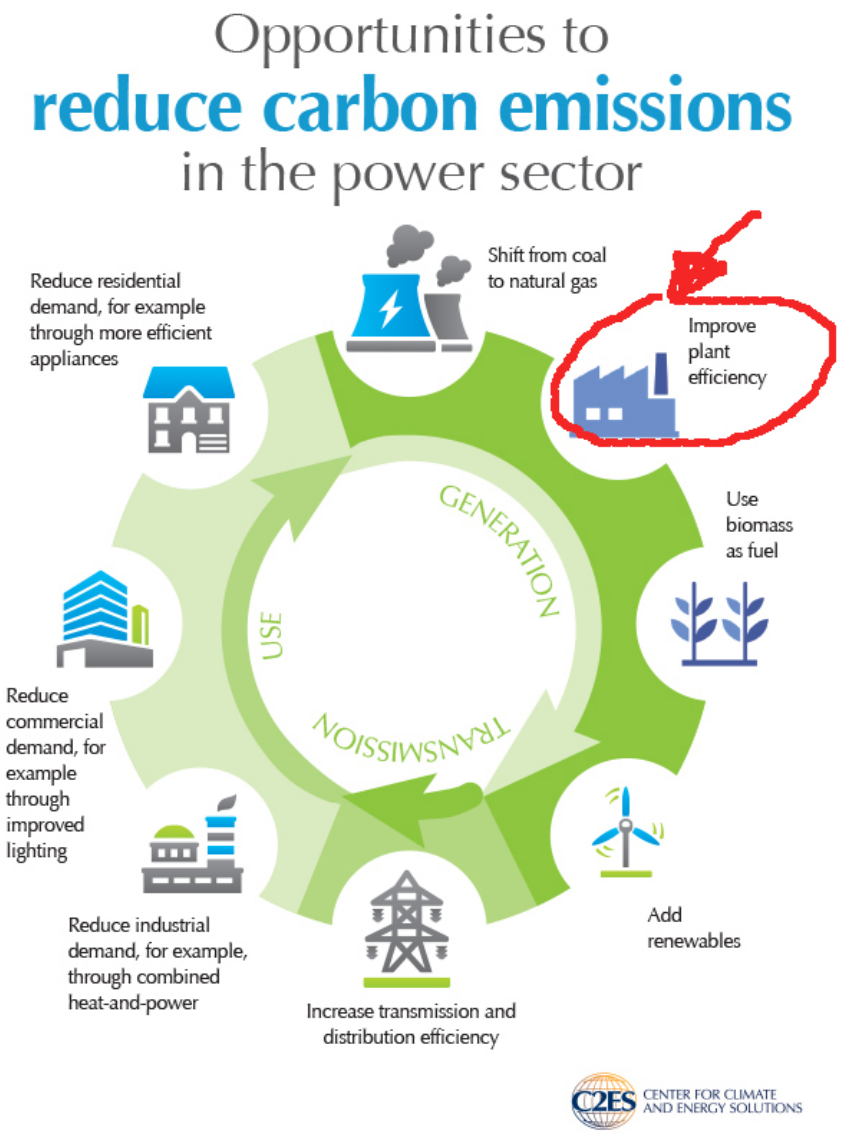
What is Green Logistics ?

- Growing concern about the global warming
- New regulations on the environment (Government, companies, Conference Climat Paris 2015, etc.)
- Friendlier products to the environment
- Carbon footprints on the final product
- Our concern : reduce the total carbon emissions due to production & storage

An example from the power sector



An example from the power sector

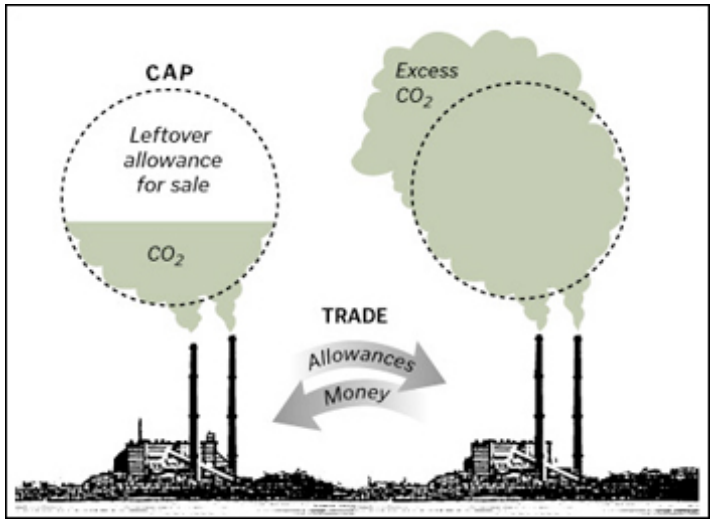


Which policies ? [1]

- Carbon cap policy : limit on the carbon emission
- Carbon tax policy : a tax paid per carbon units emitted
- Carbon offset policy : carbon units can be purchased from independent suppliers and/or investing into projects
- **Carbon cap-and-trade**

[1] Benjafaar *et al.* (2013), Arslan and Turkay (2013), Absi *et al.* (2013)

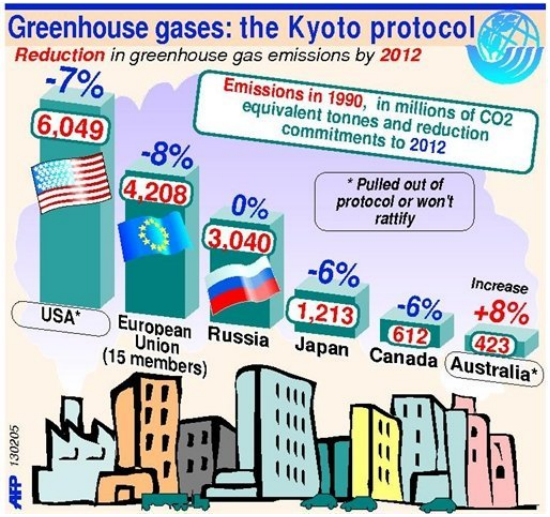
What is the cap-and-trade policy ?



- Carbon cap on the total carbon emissions
- Possibility to trade carbon units [a], a firm can
  - emit more than its carbon cap by purchasing additional carbon units
  - sell its surplus carbon units to earn money

[a] (1 carbon unit is equivalent to 1 ton CO_2)

What is the cap-and-trade policy ? (Cont.)



- One of the mechanism established by the Kyoto Protocol
- European Union has implemented a carbon market for its most important industries
- But, the price of 1 ton of CO_2 decreases... In 2008, the unit price was 36€, in 2011 it was 20€ and in 2014 it is 6€.
- According to "Les Echos", in order to spurr the companies to pollute less, this price should be at least 24€.

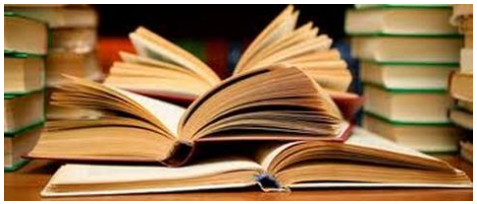
The EU Emissions Trading System (EU ETS)

Some information from European Commission...

- EU ETS covers more than 11,000 power stations and industrial plants in 31 countries, as well as airlines : around 45% of the EU's greenhouse gas emissions
- 2020, emissions from sectors covered by the EU ETS will be 21% lower than in 2005 and by 2030, 43% lower
- BUT...

Our study

A short literature review



- Survey green logistics & OR :
  Dekker *et al.* (2011), Sbihi and Eglese (2007), Srivastava (2007)
- Green lot sizing & EOQ :
  Hua *et al.* (2011), Arslan and Turkay (2013), Bouchery *et al.* (2012)
- Cap & Trade and EOQ :
  Zhou *et al.* (2015), He *et al.* (2015), Du *et al.* (2014)
- **Green lot sizing** :
  van den Heuvel et al. (2011, 2012), Hong (2013), Absi et al. (2013), Palak et al. (2014), Helmrich et al. (2014)

Single-item, single-mode green lot sizing

- **Helmrich et al. (2014)** : The single-item ULSP with a global carbon cap constraint. NP-hardness result for the case where only production emits pollutants and these emissions are linear in the quantity produced. Lagrangean heuristic for feasible solutions and lower bounds ; Pseudo-polynomial time algorithm for the cases with valid ZIO property ; FPTAS.
- **van den Heuvel et al. (2011, 2012)** : The single-item ULSP with rolling carbon cap constraint (derived from the bi-objective function). NP-Hard and Polynomial cases.

Single-item, multi-mode green lot sizing

- **Absi et al. (2013)** : The multi-mode ULSP problem is studied under different emission constraints : periodic, cumulative, global and rolling horizon.
- **Palak et al. (2014)** : The multi-mode (or multi-supplier) ULSP problem is studied under four carbon regulatory mechanisms : carbon cap, carbon tax, carbon cap-and-trade and carbon offset.
- **Helmrich et al. (2014)** : The multi-mode problem for T periods and m modes can be transformed into a single-mode problem over Tm periods. The methods proposed for the single-mode case can handle the extended multi-mode case.
- **Hong (2013)** : Two technology choice ULSP under the cap-and-trade scheme : green technology less polutant but costly ; regular technology more pollutant but less costly. Polynomial time algorithm.

The contribution of our study

- No study on the single-mode single-item ULSP with carbon cap & trade scheme
- Impact of two budget limitations on the carbon trade of the firm
- Both problems have the same optimal value and are NP-Hard to solve
- Equivalence to the ULSP studied in Helmrich *et al.* (2014) [a]
- The methods proposed by Helmrich *et al.* can be used to deal with a cap-and-trade policy with limited budget.

[a] Helmrich, M.J.R., R. Jans, W. van den Heuvel, and A.P.M. Wagelmans, The economic lot-sizing problem with an emission capacity constraint, EJOR,

Outline

MILP formulation

$$\min \sum_{t=1}^{T} (f_t y_t + p_t x_t + h_t s_t + \alpha(e_t^+ - e_t^-))$$

$$s.t.$$

$$\sum_{t=1}^{T} (\hat{f}_t y_t + \hat{p}_t x_t + \hat{h}_t s_t + e_t^-) \leq C + \sum_{t=1}^{T} e_t^+ \quad (1)$$

$$s_t + x_t = d_t + s_{t+1}, \ \forall t \in \{1..T\} \quad (2)$$

$$x_t \leq (\sum_{j=t}^{T} d_j) y_t, \ \forall t \in \{1..T\} \quad (3)$$

$$x_t, s_t \geq 0; y_t \in \{0, 1\}, \ \forall t \in \{1..T\} \quad (4)$$

$$e_t^+, e_t^- \geq 0, \ \forall t \in \{1..T\} \quad (5)$$

Notations

We use the following notations to let the model more compact :

$E^+ = \sum_{t=1}^{T} e_t^+$: the total carbon units to purchase over the entire horizon

$E^- = \sum_{t=1}^{T} e_t^-$: the total carbon units to sell over the entire horizon

$\phi(x) = \sum_{t=1}^{T}(f_t y_t + p_t x_t + h_t s_t)$: classical lot sizing cost without carbon emission

$\hat{\phi}(x) = \sum_{t=1}^{T}(\hat{f}_t y_t + \hat{p}_t x_t + \hat{h}_t s_t)$: total carbon amount emitted

More compact MILP

With the new notations the model ULSP-CT studied thus becomes :

$$
\text{(ULSP-CT)} \qquad
\begin{aligned}
& \min(\phi(x) + \alpha(E^+ - E^-)) \\
& s.t. \\
& \hat{\phi}(x) - C \leq E^+ - E^- && (6) \\
& E^+, E^- \geq 0 && (7) \\
& (2), (3), (4)
\end{aligned}
$$

Some properties

Property

In an optimal solution, constraint (6) is tight (Benjaafar et al., (2013)).

Property

The model ULSP-CT can be reduced to a classical ULSP without constraint (6) on the carbon emission limit and with a modified objective function.

How to solve it ?

- Under the non-speculative (or Wagner-Whitin (WW)) cost structure the ZIO (Zero-Inventory-Ordering) policy is still dominant.
- Wagner and Whitin, (1958) : the best known with an $\mathcal{O}(T^2)$ time complexity.
- $\mathcal{O}(T \log T)$ time for linear cost structures and no backlogging
- $\mathcal{O}(T)$ time under non-speculative cost assumptions (see Aggarwal and Park, (1993), Federgruen and Tzur, (1991), Wagelmans *et al.*, (1992)).

Outline

Budget constraints

- Budget constraint 1 : $\alpha E^+ \leq B$
  The total carbon units the firm can buy over the entire horizon is limited.
- Budget constraint 2 : $\alpha(E^+ - E^-) \leq B$
  The initial budget can be increased from benefices obtained by selling the unused carbon units.

Budget constraints

- ULSP-B1CT : Problem ULSP-CT with the budget constraint 1
- ULSP-B2CT : Problem ULSP-CT with the budget constraint 2

Complexity result

Theorem

The ULSP-B1CT is $\mathcal{NP}$-hard even with a null budget $B = 0$.

Polynomial time reduction from the lot sizing problem with an emission constraint stated in Helmrich *et al.*, (2014) to an instance of the ULSP-B1CT.

Some properties

Property

In an optimal solution for the problem ULSP-B2CT, the constraint $\hat{\phi}(x) + E^- \leq C + E^+$ *is tight.*

- In the objective function $(E^+ - E^-)$ is minimized.
- In the constraint $\hat{\phi}(x) - C \leq E^+ - E^-$, the term $E^+ - E^-$ takes its lower bound value.
- The other budget constraint $(E^+ - E^-) \leq \frac{B}{\alpha}$ ensures only an upper bound on $E^+ - E^-$

Relaxation of ULSP-B2CT

Combining constraints $\hat{\phi}(x) + E^- \leq C + E^+$ and $\alpha(E^+ - E^-) \leq B$, one can write the following valid inequality :

$$\hat{\phi}(x) - C \leq E^+ - E^- \leq \frac{B}{\alpha}$$

The upper limit on the total carbon emission $\hat{\phi}(x)$ is thus :

$$\hat{\phi}(x) \leq C + \frac{B}{\alpha}$$

The relaxation R is...

The relaxation R

$$
\text{(R)} \qquad \begin{array}{l}
\min(\phi(x) + \alpha(\hat{\phi}(x) - C)) \\
s.t. \\
\hat{\phi}(x) \leq C + \dfrac{B}{\alpha} \\
(2), (3), (4)
\end{array}
$$

Relaxation of ULSP-B1CT

Property

R is also a relaxation of ULSP-B1CT.

One can write :

$$\hat{\phi}(x) - C \leq E^+ - E^-, \text{ and using that } E^+ \leq \frac{B}{\alpha}$$
$$\Rightarrow \hat{\phi}(x) - C \leq \frac{B}{\alpha} - E^-, \text{ and using that } E^- \geq 0$$
$$\Rightarrow \hat{\phi}(x) \leq C + \frac{B}{\alpha}$$

Feasibility

$\bar{x}$ is a feasible solution for the relaxation R.
Question : Is it also feasible for problems ULSP-B1CT and ULSP-B2CT ?

The answer is yes !

For $\bar{x}$ a solution of R, define

- $x = \bar{x}$,
- $E^+ = (\hat{\phi}(x) - C)^+$,
- $E^- = (C - \hat{\phi}(x))^+$

Then, (x, E^+, E^-) is feasible for both problems, and has the same value.

Consequence

Theorem

Problems ULSP-B1CT, ULSP-B2CT and R have the same optimal solution.

- R is exactly the model studied in Helmrich *et al.* (2014),
- Any algorithm for the relaxation R can be applied to solve problems ULSP-B1CT and ULSP-B2CT
- Helmrich *et al.* (2014) propose a Lagrangean heuristic, a pseudo-polynomial time algorithm for the case where ZIO property is dominant and different FPTAS

Conclusion and perspectives for the single-mode problem

Conclusions

- Carbon emission parameters both in the objective function and in the constraints.
- ULSP-CT can be reduced to the classical ULSP and thus remains polynomial
- ULSP-B1CT is proven to be NP-hard
- Both problems can be reduced to LSP with carbon cap but without trade.

Perspectives

- Multi-technology options in ULSP
- Multi-mode replenishment with green aspects
- Other carbon emission constraints

A preliminary study on the multi-technology selection (with O.N. Attila)

In the literature...

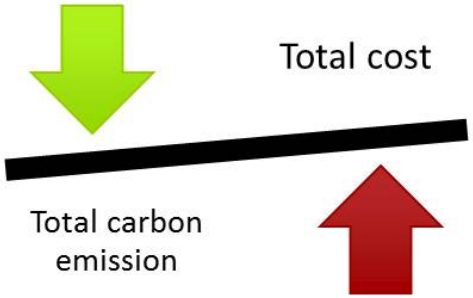
Ph.D thesis of Hong (2013) :

- Single-item ULSP under the carbon cap scheme, with two technologies available, green and regular.
- Exact polynomial time algorithm in $O(T^6)$ to solve it.
- Hypotheses : only the production activity causes carbon emissions, classical lot sizing costs related to both technologies in the objective function.

Our originality & contribution

We consider a more general setting by extending this model to the multi-technology case under the carbon cap policy. We also consider emissions related to both setup and production activities.

General assumptions



- A certain number of different technology types are available.
- For all technology options, there is a tradeoff between production and setup costs vs. total carbon emission caused.
- Total carbon emission is limited by a periodical emission cap constraint.
- Budget constraints and investment options are not included in the model.

Mathematical model

$$\min \sum_{t=1}^{T}\left(\sum_{i=1}^{I}(f_{it}y_{it} + p_{it}x_{it})\right) + h_t s_t$$

$$s.t.$$

$$\sum_{i=1}^{I}(\hat{f}_i y_{it} + \hat{p}_i x_{it}) \leq C,\ \forall t \in \{1..T\}$$

$$s_t + \sum_{i=1}^{I} x_{it} = d_t + s_{t+1},\ \forall t \in \{1..T\}$$

$$x_{it} \leq \left(\sum_{j=t}^{T} d_j\right) y_{it},\ \forall i \in \{1..I\}, \forall t \in \{1..T\}$$

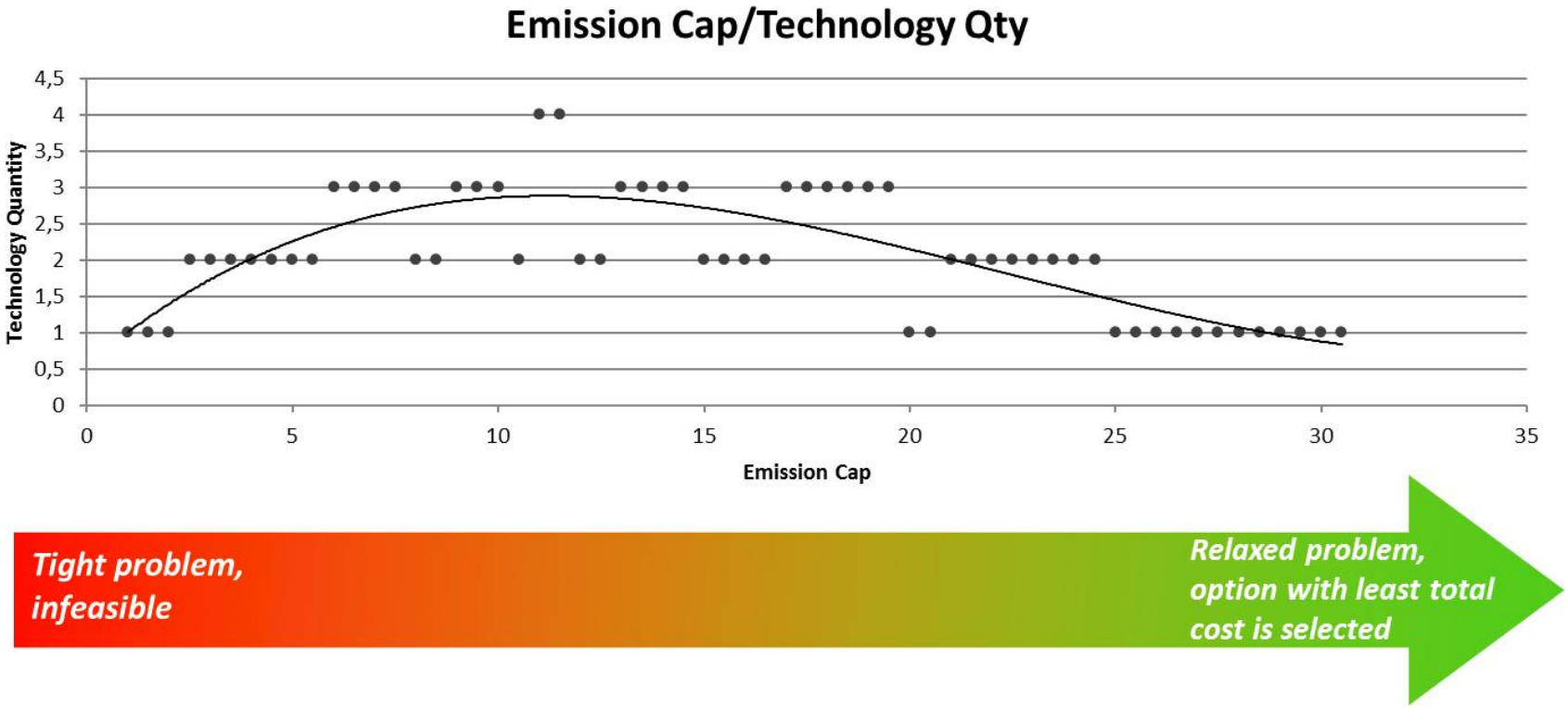
$$x_{it}, s_t \geq 0; y_{it} \in \{0,1\},\ \forall i \in \{1..I\}, \forall t \in \{1..T\}$$

Runs & Results

Set of instances	Number of periods (T)	Number of Technologies (I)	Demand vector	Cost parameters (over time)
1	1	2	Invariant	Invariant
2	10	2	Variant	Invariant
3	10	2	Variant	Variant
4	10	10	Variant	Variant
5	50	10	Variant	Variant
6	10	30	Variant	Variant

Runs & Results

In order to observe the impact of the emission cap on the number of different technologies selected in the optimal solution, 100 different models were solved where the emission cap (C) was gradually increased.



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