Multi-level capacitated lot-sizing problem with energy consideration

Industrial Systems Optimisation Laboratory, LOSI, UTT

Friday, June 26th 2015

Sustainable production planning, GT P2LS

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Outline

- Introduction
- Bibliographic synthesis
- Proposed model
- Proposed heuristics and results
- Conclusions and perspectives

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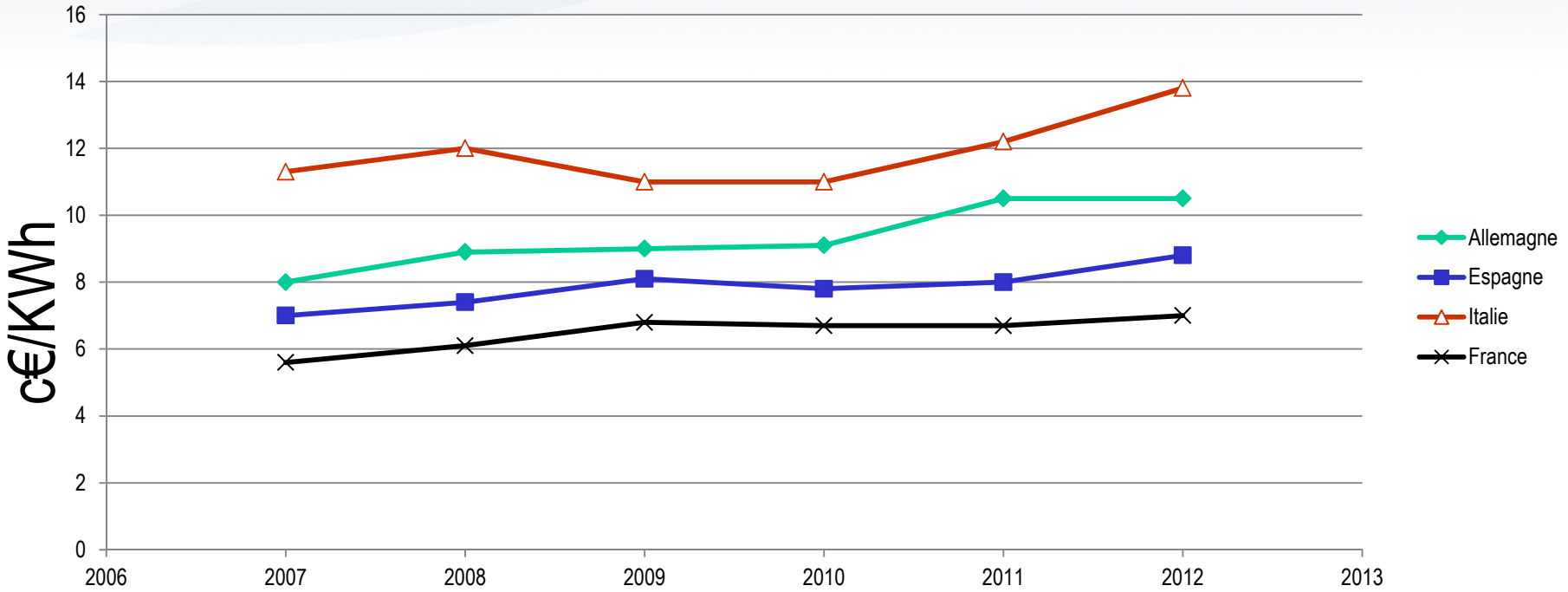
Introduction

- **To reduce the environmental impacts in manufacturing systems :**
  - **Minimize carbon emission**
  - **Minimize water consumption**
  - **Minimize wastes**
  - **Minimize energy consumption**

Figure : Relationship between generated power – carbon emission [1]

[1] Wang and Li, 2013

Introduction



Price of electricity between 2007 and 2012 for manufacturing

Source: http://www.observatoire-electricite.fr/Evolution-des-prix-de-l

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Tactical level

- **Absi *et al.*, 2013, Yu *et al.*, 2013, Benjaafar *et al.*, 2011 :**
  - Lot sizing problem with carbon emission constraints
- **Heck and Shmidt, 2010 :**
  - Integrate ecological aspects in lot-sizing problem
    - Carbon emission
    - Energy consumption
    - Water consumption

Operational level

- **Bayram and Mouzon, 2012:**
  - Minimize the energy consumption
  - System with single machine

- **Sun et *al., 2015*, Bego *et al.*, 2014, Fernandez *et al.*, 2013, Wang and Li, 2013:**
  - Flow-shop scheduling
  - Variation of price of electricity according to periods

- **Artigues and Lopez, 2015, Bruzzone *et al.*, 2012, Artigues *et al.*, 2013 :**
  - Energy and power limitation

Lot-sizing in flow-shop systems

- **Sahling et al., 2009 :**
  - Minimize setup, inventory and extra hours costs
- **Ramezanian et al., 2013, Mohammadi et al., 2010 :**
  - Minimize production, setup and inventory costs
- **Babaei et al., 2014 :**
  - Minimize production, setup, inventory and shortage costs

The energetic aspect is not considered for this type of system

Sequential/Integrated Approach

Sequential Approach

Establish the production plan
Take into account only the production constraints in manufacturing unit

Estimate energy demands
Calculations made for utility demands for each period

Utility system planning
Operational planning of the utility system to fulfill the demands

Integrated Approach

Simultaneously determine the production plan and the utility system

Globally taking into acount the constraints of production and the constraints of energy

Source : Agha et *al.*, 2010

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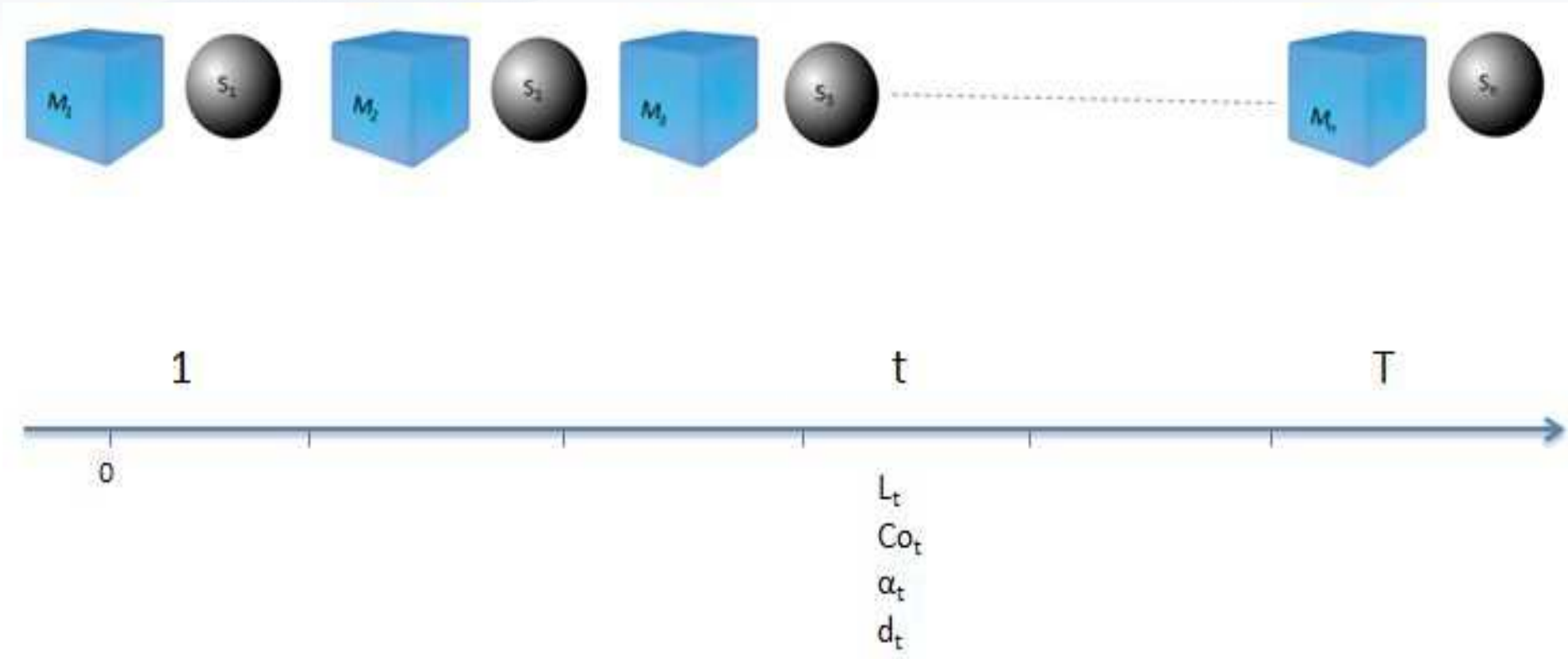
Model



L_t : Length of period *t*

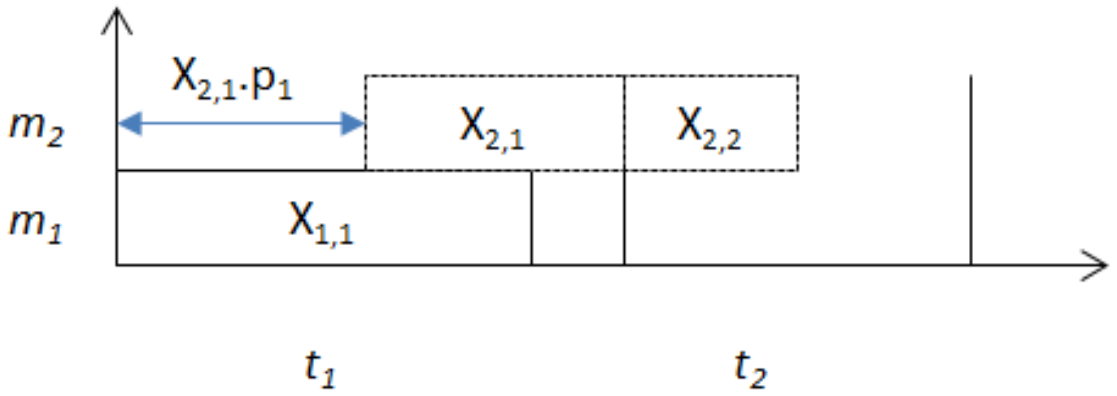
Co_t : Price of electricity during period *t*

α_t : Allowed maximal power in period *t*

d_t : External demand at period *t*

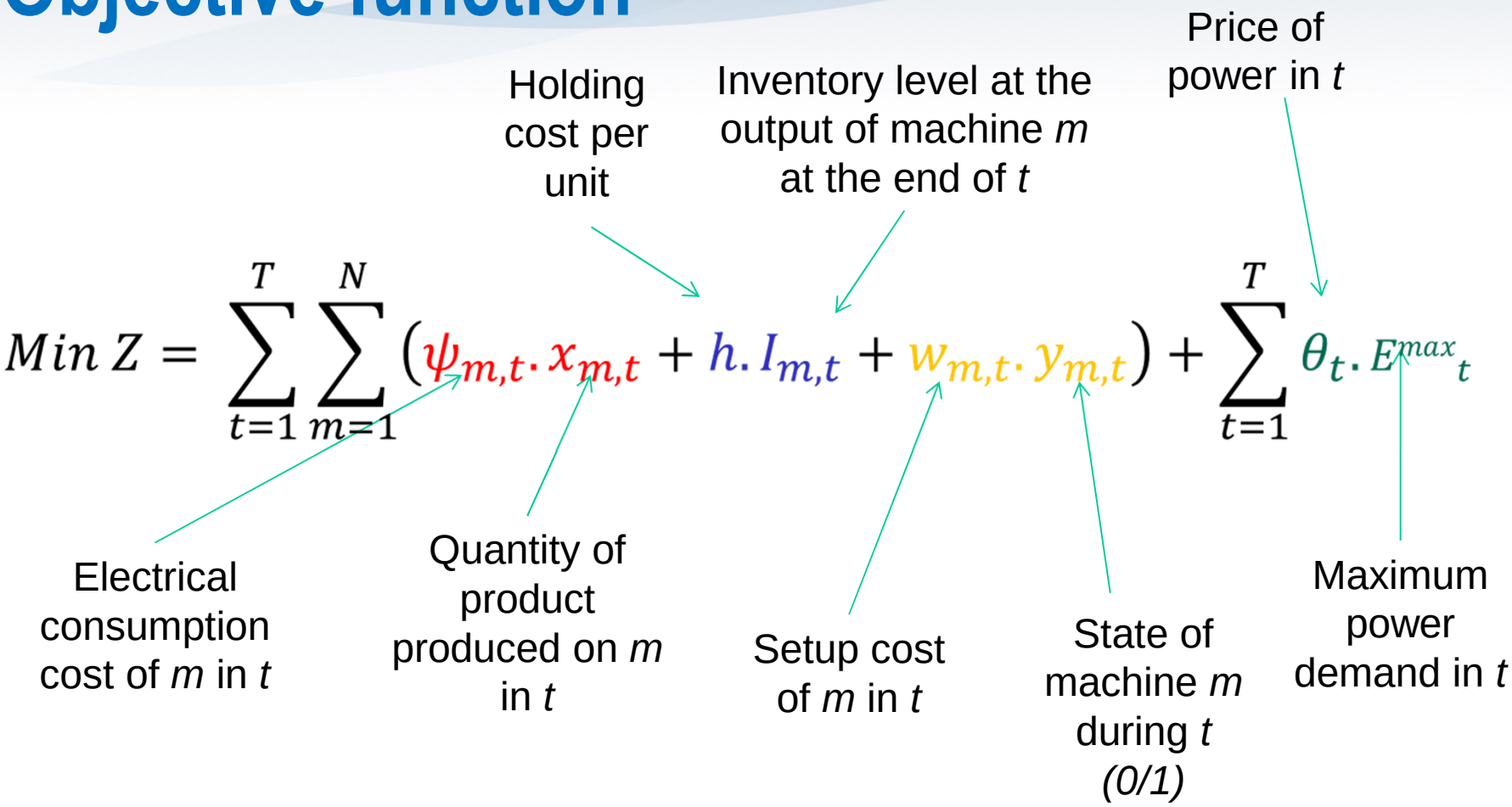
Model

- **Assumptions :**
  - Demand satisfied at each period (mono product).
  - Lot transfer constraint (vertical interaction).



Model

Objective function



Model

- **Traditional constraints of lot sizing problem in flow-shop systems :**

$$x_{N,t} + I_{N,t} = d_t + I_{N,t} \qquad \forall t=2,\ldots, T \qquad (2)$$

$$x_{m,t} + I_{m,t-1} = I_{m,t} + x_{m+1,t} \qquad \forall m=1,\ldots, N\text{-}1,\ t=2,\ldots, T \qquad (3)$$

$$x_{m,t} \leq y_{m,t} \cdot \sum_{\tau=t}^{T} d_\tau \qquad \forall m=1,\ldots, N,\ t=1,\ldots, T \qquad (4)$$

Model

Precedence constraints :

A binary variable, equal to 1 if $x_{m,t}$ available in buffer *m-1* at the beguing of *t*, *0* otherwise

$$I_{m-1,t-1} \leq x_{m,t} + M.v_{m,t} \qquad \forall m=2,\dots, N,\ t=2,\dots, T \quad (5)$$

$$x_{m,t} \leq I_{m-1,t-1} + M.(1 - v_{m,t}) \quad \forall m=2,\dots, N,\ t=2,\dots, T \quad (6)$$

$$C_{m,t} - x_{m,t}.p_m \geq C_{m-1,t} - x_{m-1,t}.p_{m-1} + \left(x_{m,t} - I_{m-1,t-1}\right).p_{m-1} - M.v_{m,t} \quad \forall m=2,\dots, N,\ t=2,\dots, T \quad (7)$$

$$C_{m,1} - x_{m,1}.p_m \geq C_{m-1,1} - x_{m-1,1}.p_{m-1} + x_{m,1}.p_{m-1} \qquad \forall m=2,\dots, N \quad (8)$$

Completion time of machine *m* in *t*

Model

- **Power constraints :**

A binary variable, equal to 1 if $C_{r,t} \geq C_{m,t} - x_{m,t} \cdot p_m$, *0* otherwise

$$C_{r,t} - C_{m,t} + x_{m,t} \cdot p_m \leq M \cdot f_{m,r,t} \quad \forall m,r=1,\ldots, N,\ m \neq r,\ t=1,\ldots, T \qquad (9)$$

$$C_{m,t} - C_{r,t} \leq M \cdot g_{m,r,t} - 1 \quad \forall m,r=1,\ldots, N,\ m \neq r,\ t=1,\ldots, T \qquad (10)$$

A binary variable, equal to 1 if $C_{m,t} \geq C_{r,t}$, *0* otherwise

$$E^{max}{}_t \geq \emptyset_m \cdot y_{m,t} + \sum_{r=1, r\neq m}^{N} (f_{m,r,t} + g_{m,r,t} - 1) \cdot \emptyset_r \quad \forall m=1,\ldots, N,\ t=1,\ldots, T \qquad (11)$$

$$E^{max}{}_t \leq \alpha_t \quad \forall t=1,\ldots, T \qquad (12)$$

The allowed maximal power in period *t*

Model

The initial conditions

$$I_{m,1} = x_{m,1} - x_{m+1,1} \qquad \forall m=1,\ldots, N\text{-}1 \qquad (13)$$

$$I_{N,1} = x_{N,1} - d_1 \qquad (14)$$

$$C_{1,1} - x_{1,1}.p_1 = 0 \qquad (15)$$

$$C_{m,t} - x_{m,t}.p_m \geq 0 \qquad \forall m=1,\ldots, N,\ t=1,\ldots, T \qquad (16)$$

$$C_{m,t} - x_{m,t}.p_m \leq L_t.y_{m,t} \qquad \forall m=1,\ldots,N,\ t=1,\ldots, T \qquad (17)$$

$$C_{m,t} \leq L_t \qquad \forall m=1,\ldots, N,\ t=1,\ldots, T \qquad (18)$$

$$f_{m,m,t} = g_{m,m,t} = 0 \qquad \forall m=1,\ldots, N,\ t=1,\ldots, T \qquad (19)$$

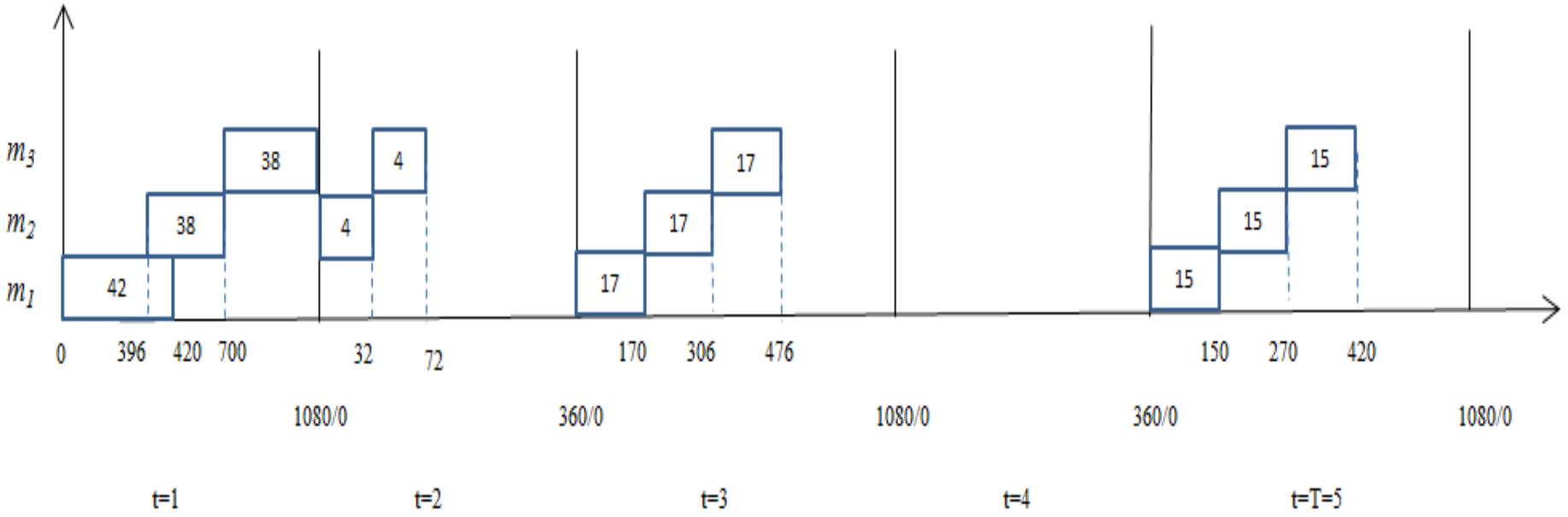
Example

t	1	2	3	4	5
d_t	37	5	6	11	15
$w_{1,t}$ ($)	14	22	22	12	22
$w_{2,t}$ ($)	25	16	14	28	15
$w_{3,t}$ ($)	19	21	18	19	23
α_t (KW)	17	14	11	15	12

m	1	2	3
Φ_m (KW)	7	5	8
P_m (min)	10	8	10

Time of day	Co_t ($/KWh)	Θ_t ($/KW)
7pm-1pm	0,08	1
1pm-7pm	0,16	19,8

Example


m_3
m_2
m_1
42
38
38
4
4
17
17
17
15
15
15
0
396
420
700
32
72
170
306
476
150
270
420
1080/0
360/0
1080/0
360/0
1080/0
t=1
t=2
t=3
t=4
t=T=5

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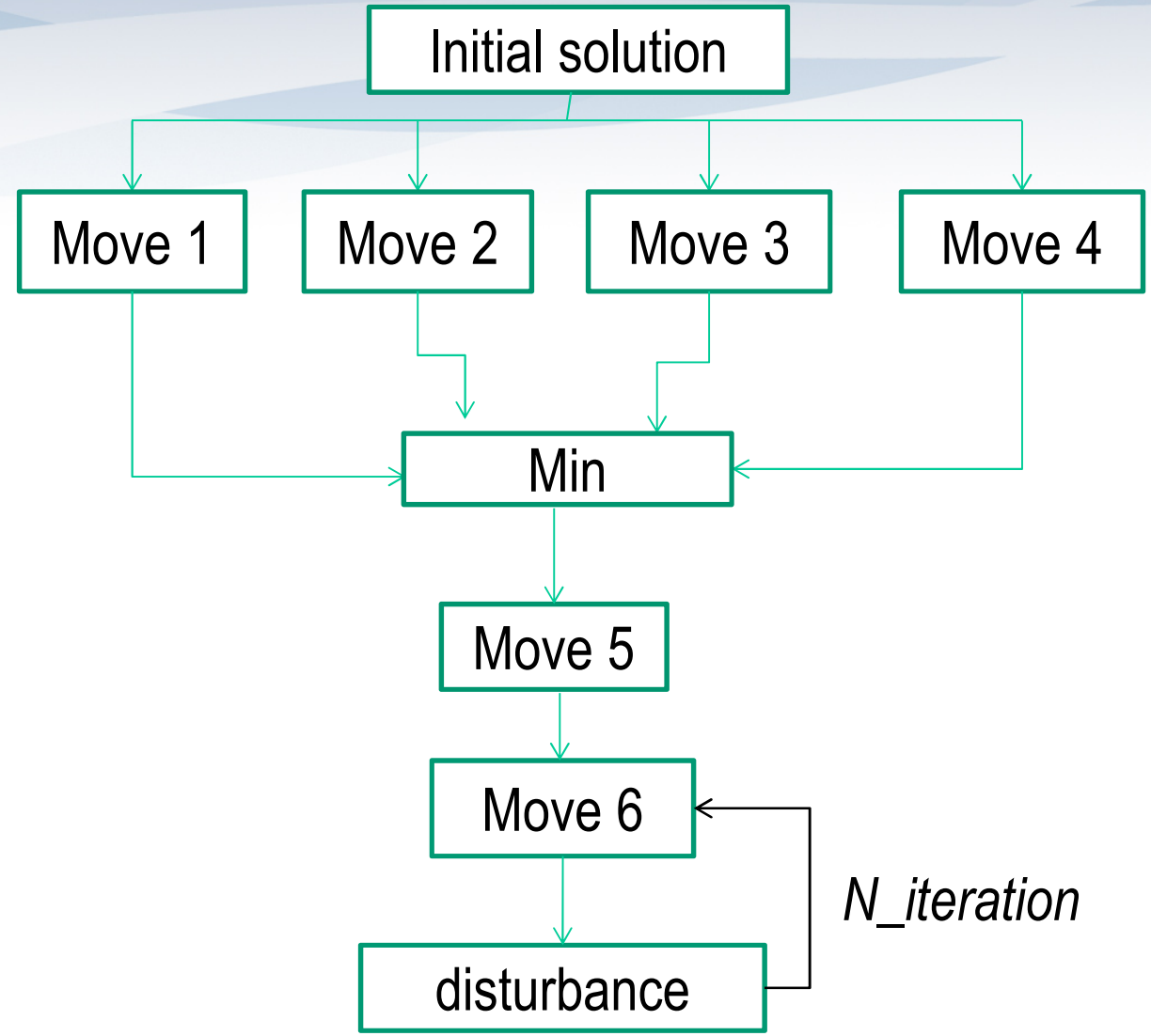
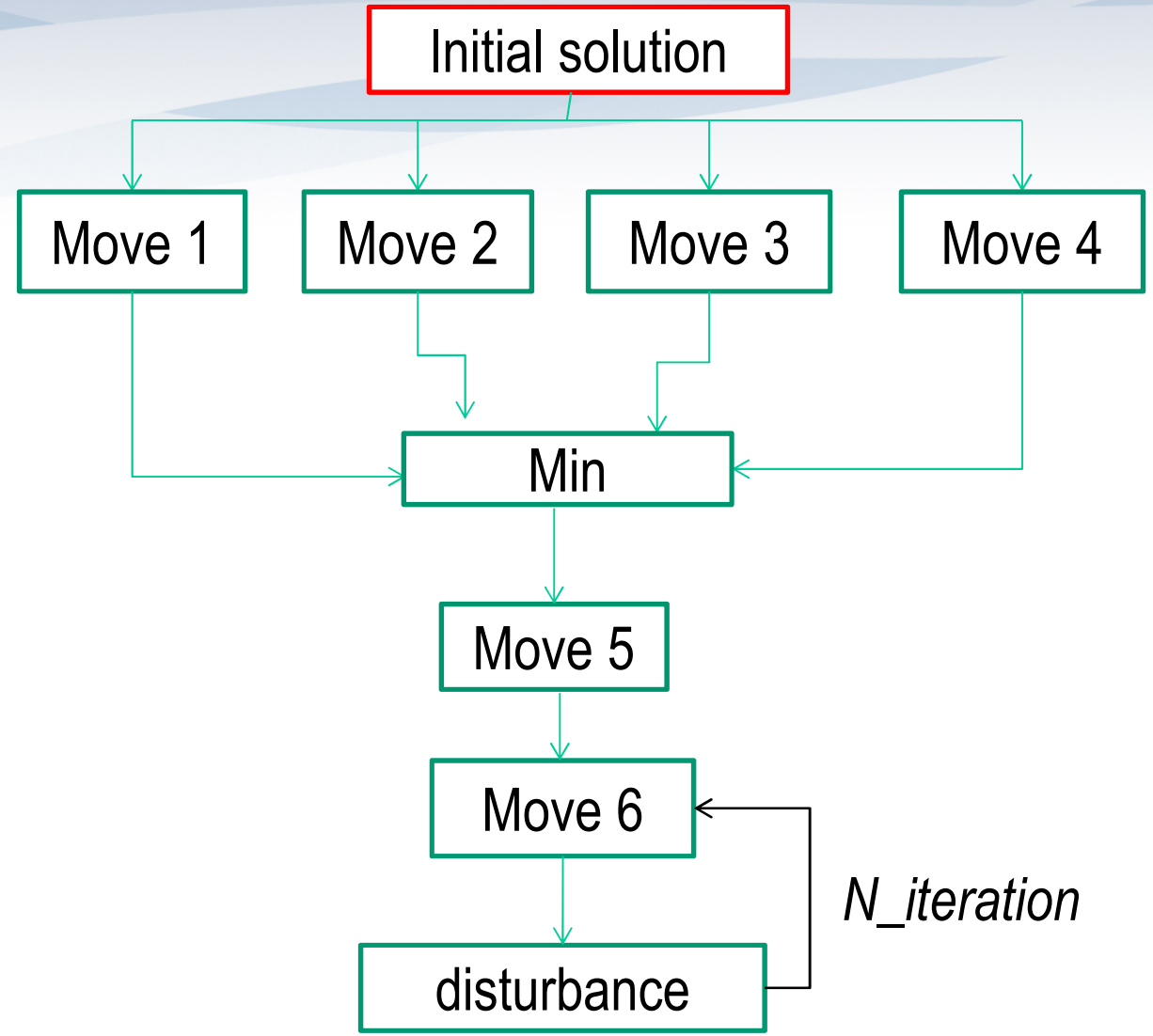
Heuristic H1

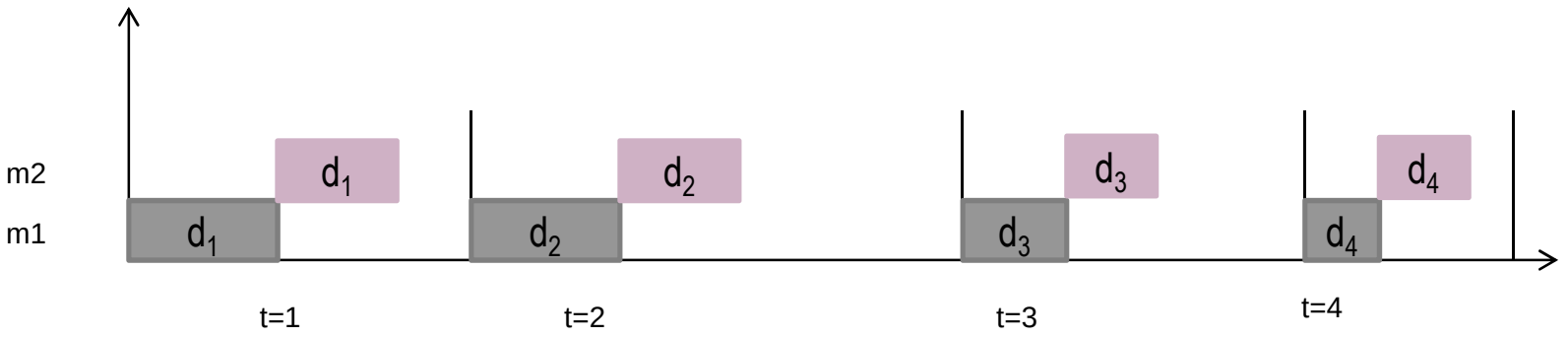


Heuristic H1

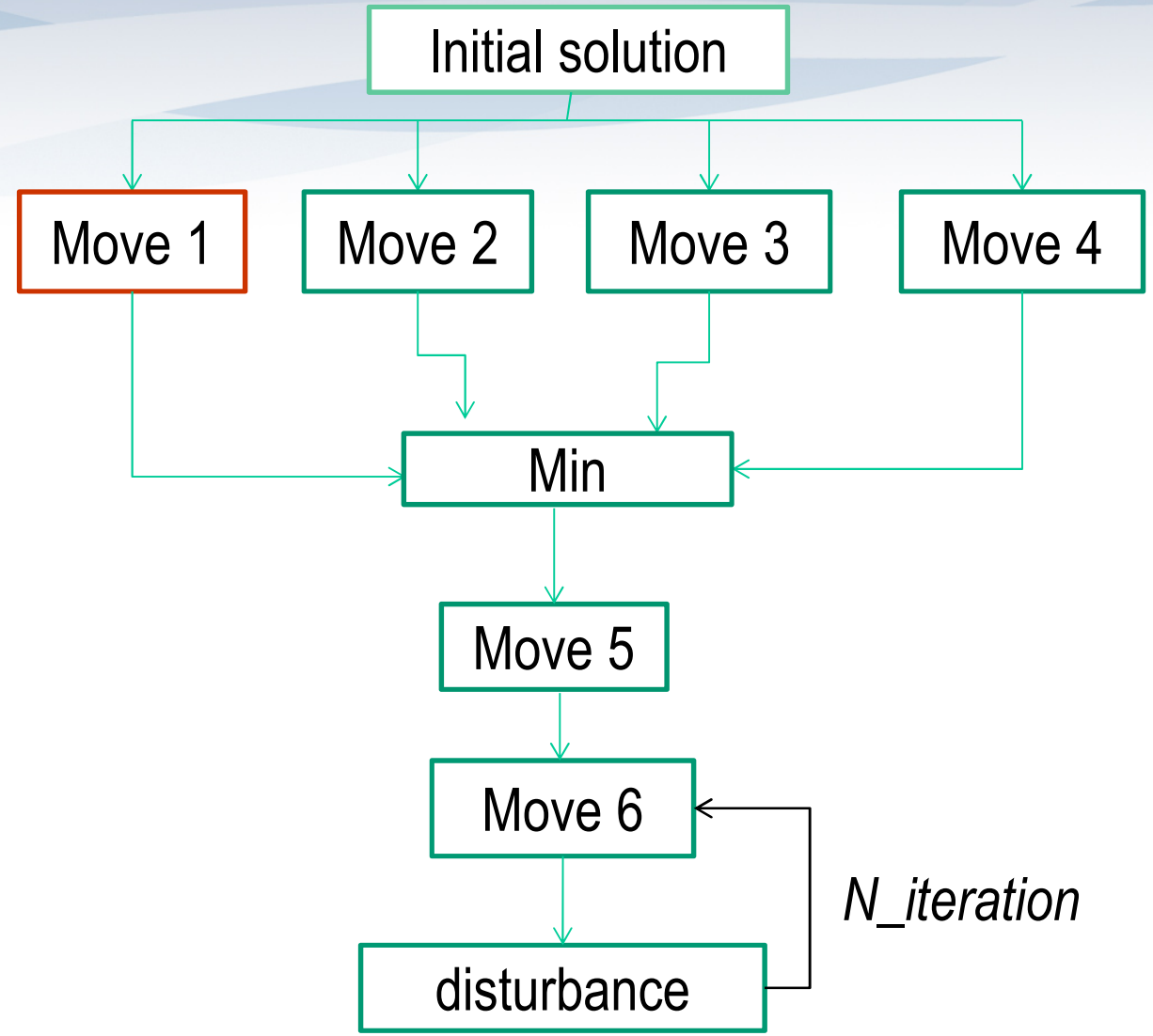


Heuristic H1

Initial solution

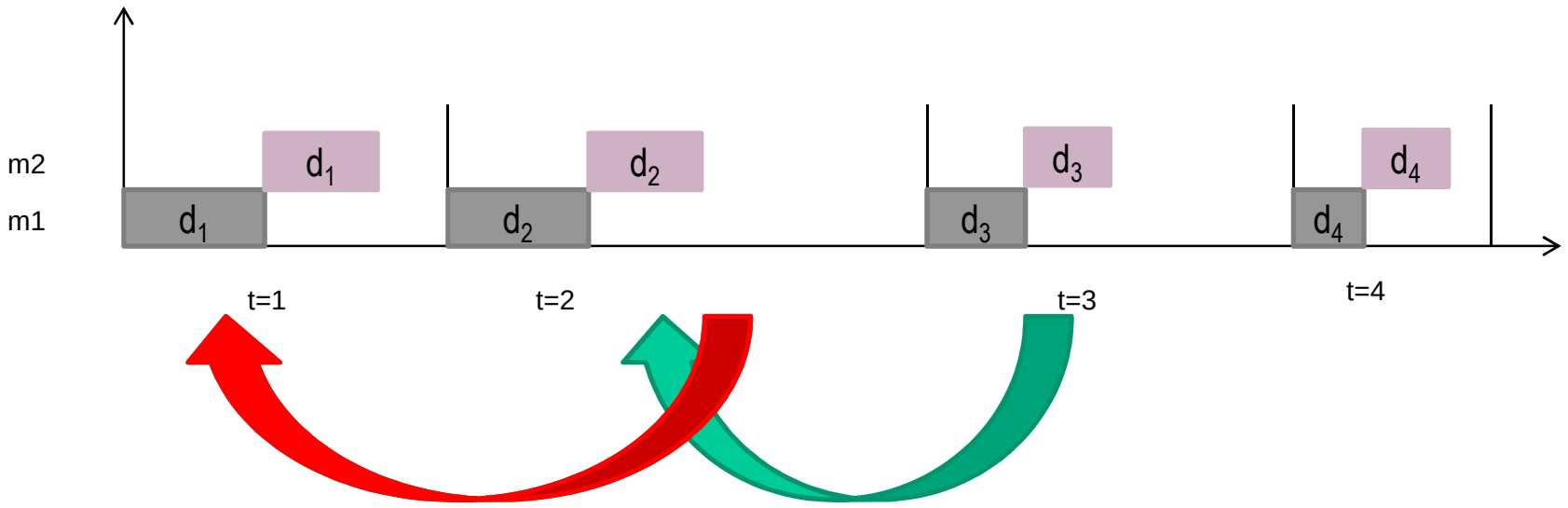


Heuristic H1



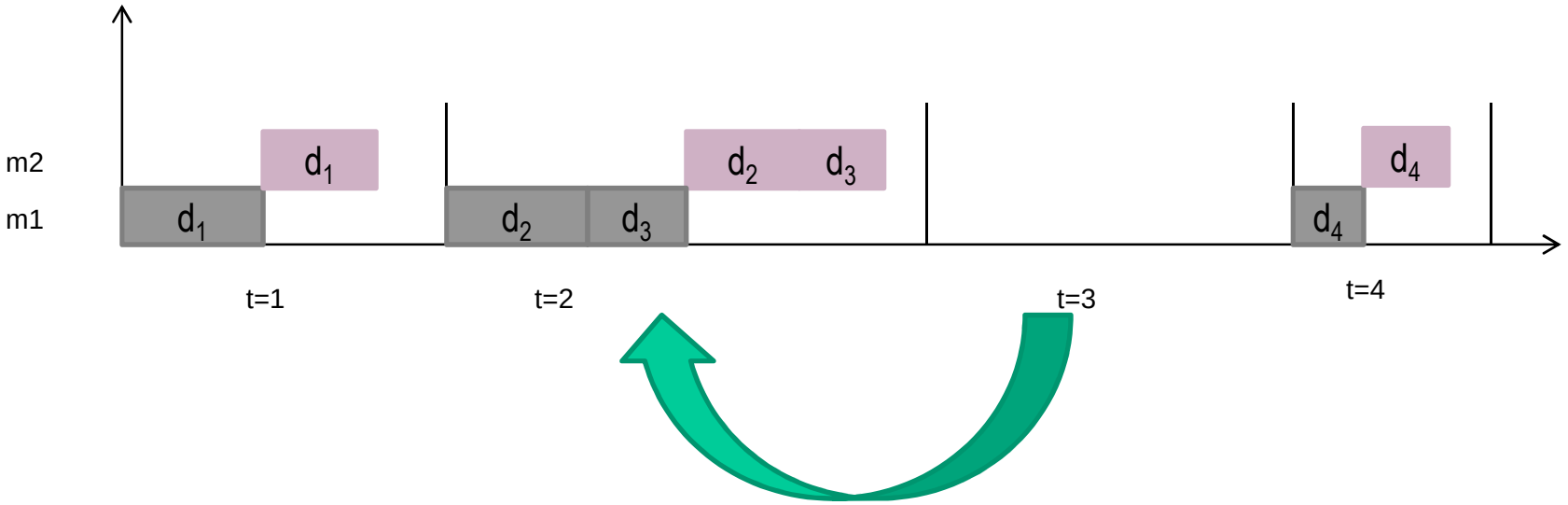
Heuristic H1

Move 1



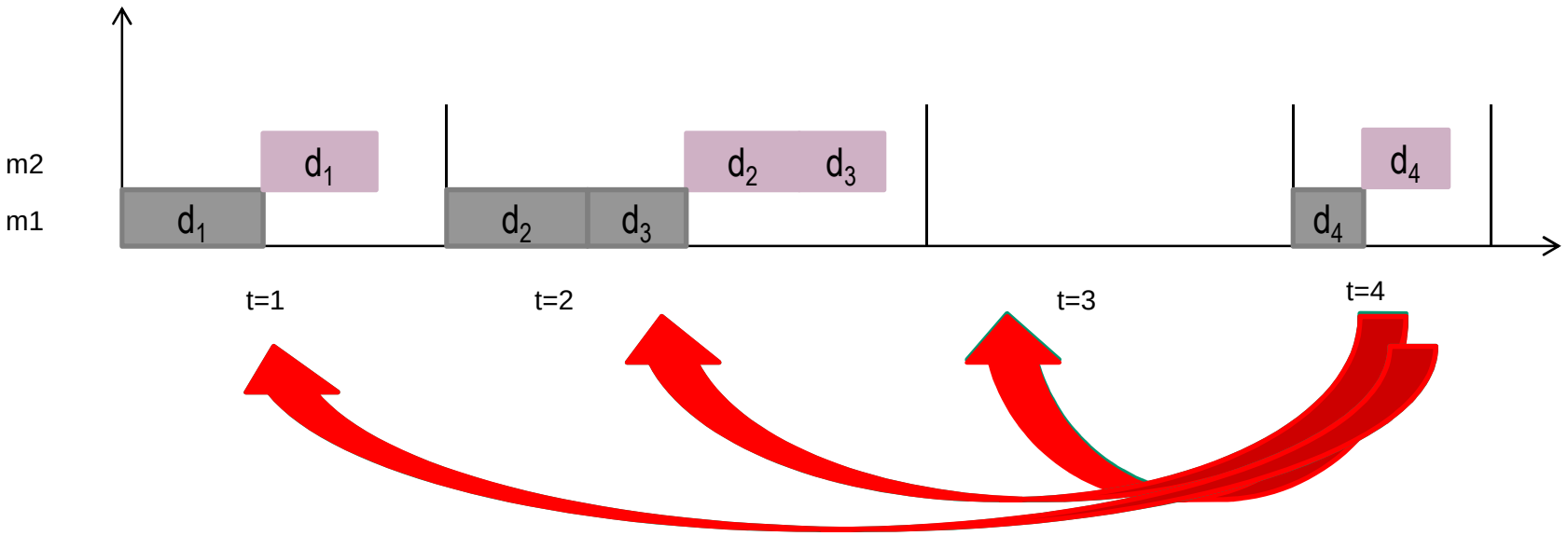
Heuristic H1

Move 1

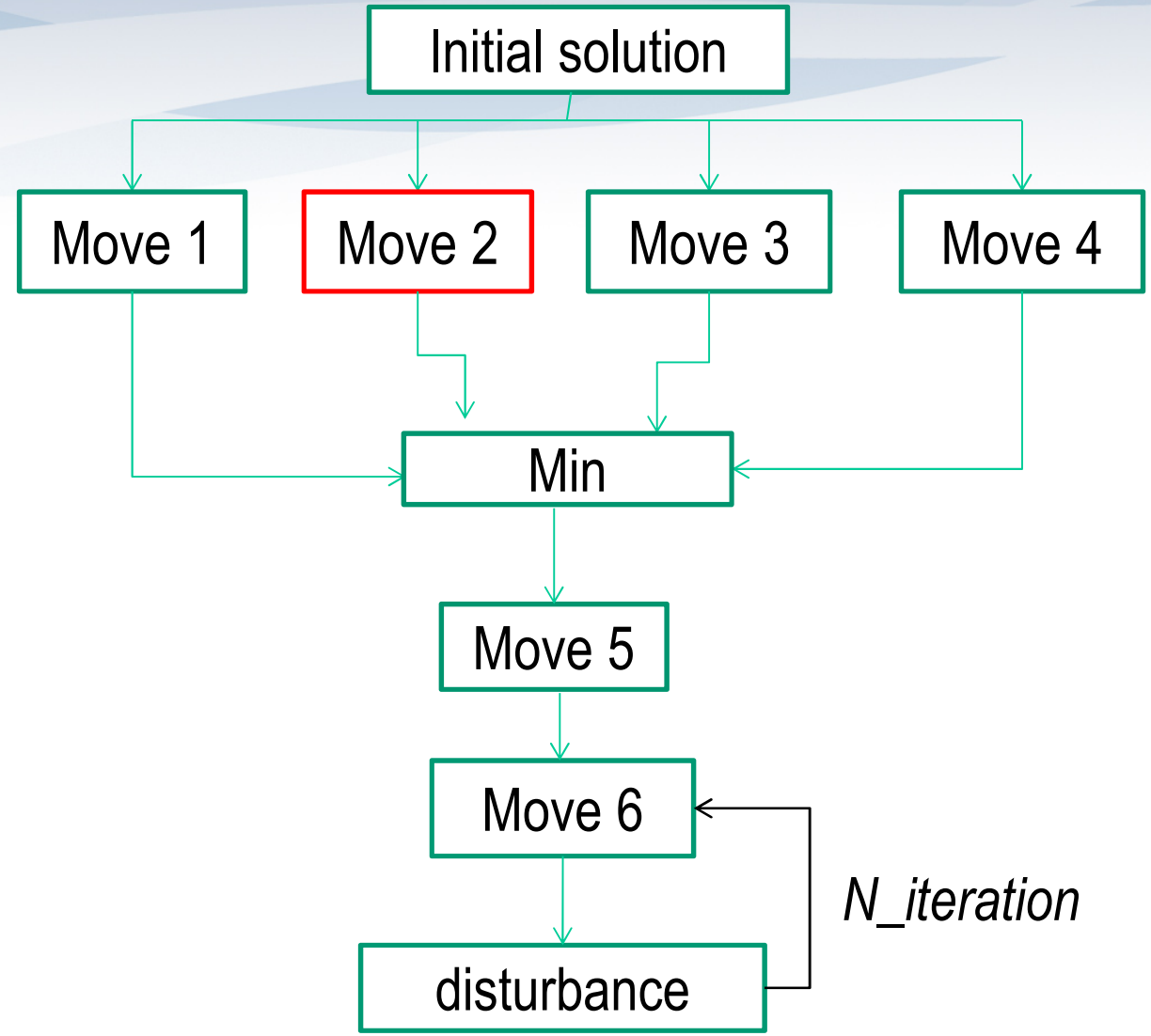


Heuristic H1

Move 1

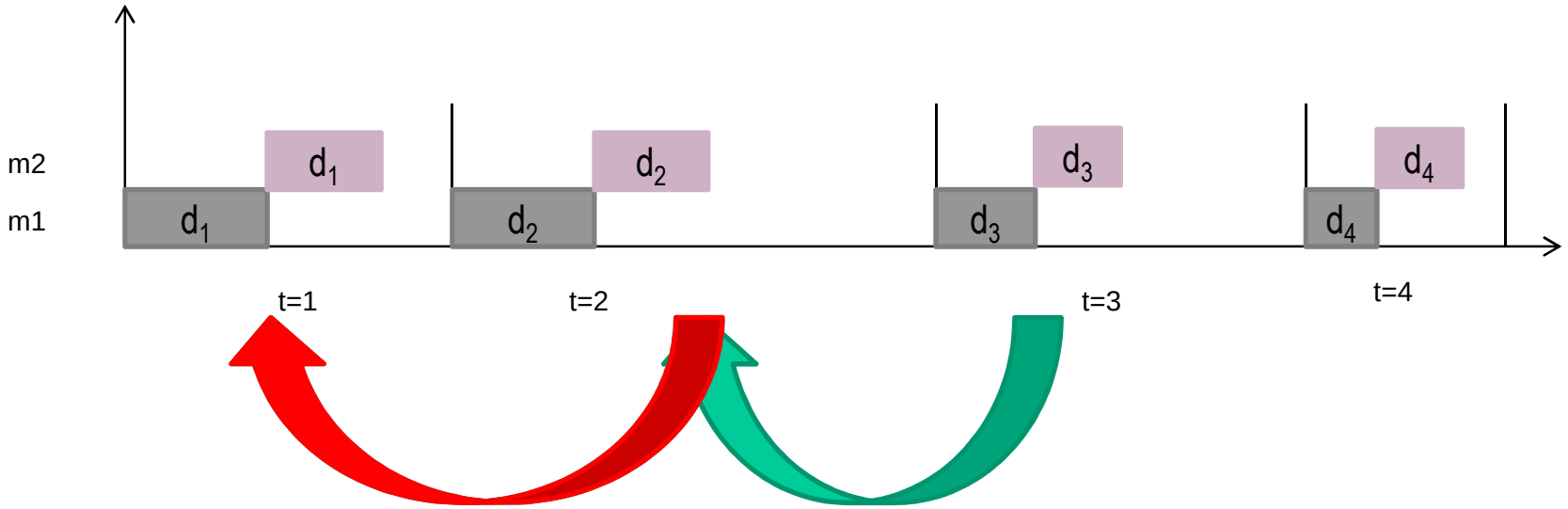


Heuristic H1



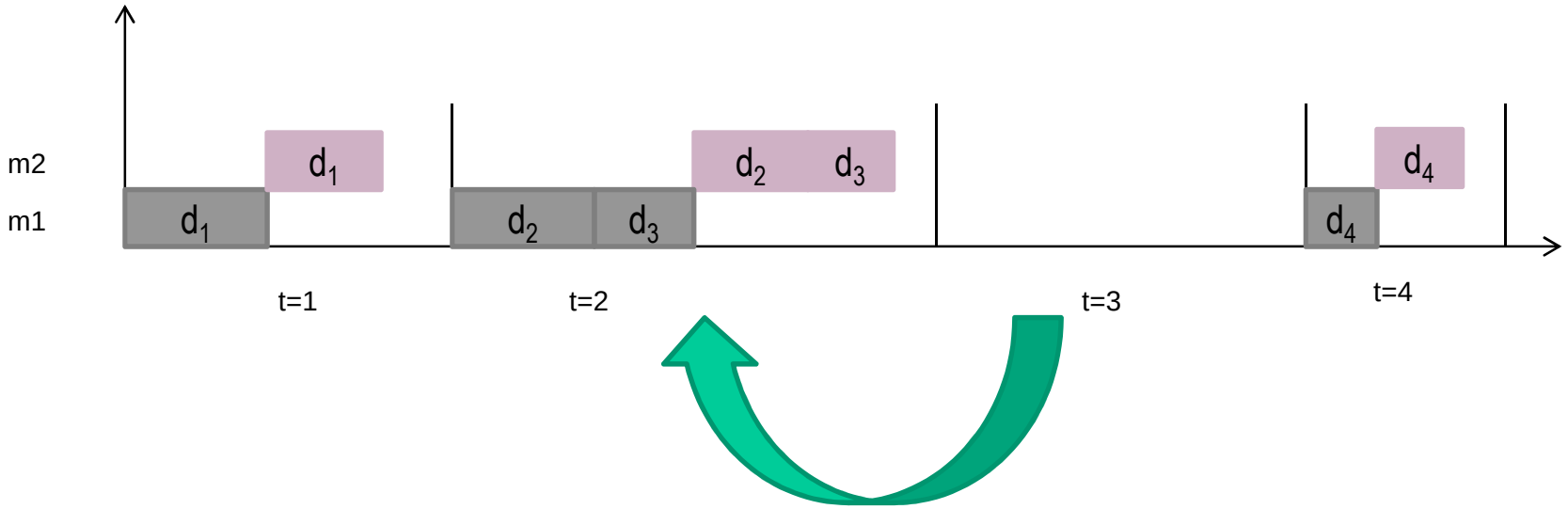
Heuristic H1

Move 2



Heuristic H1

Move 2



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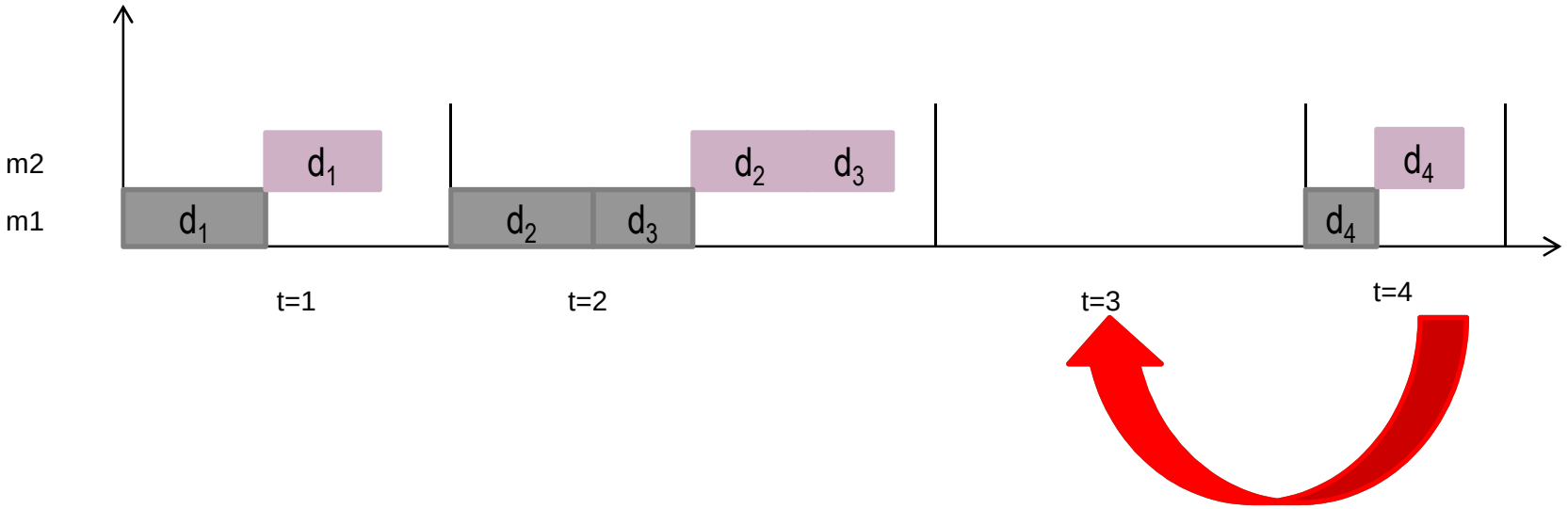
Move 2



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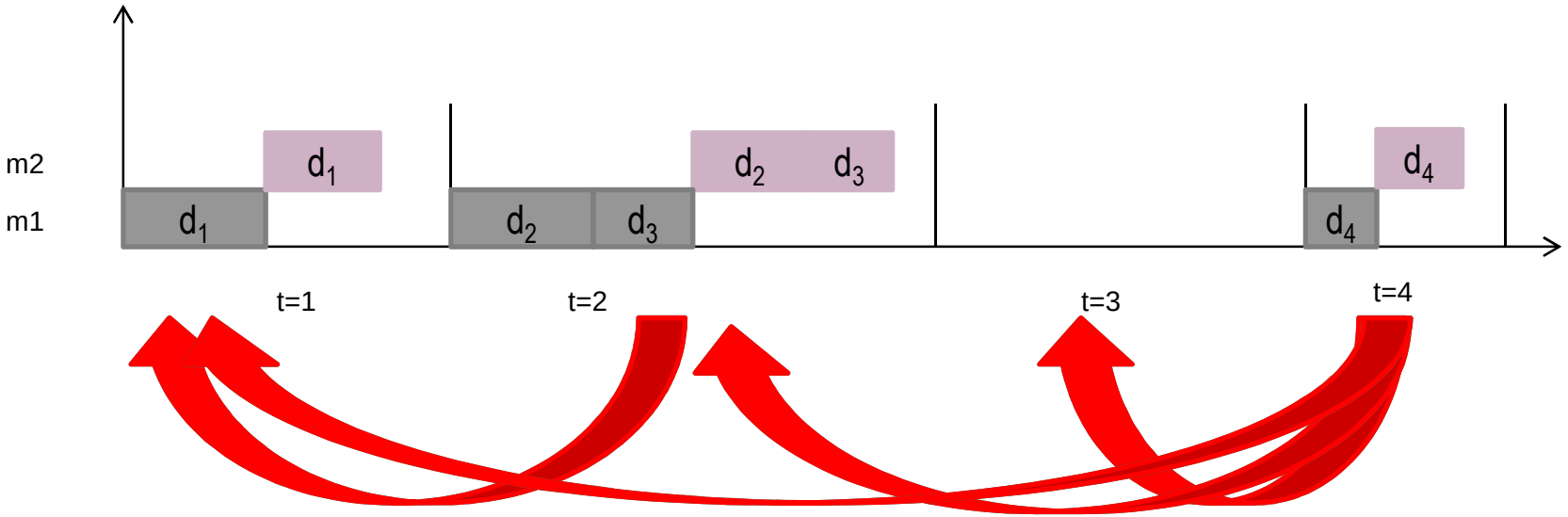

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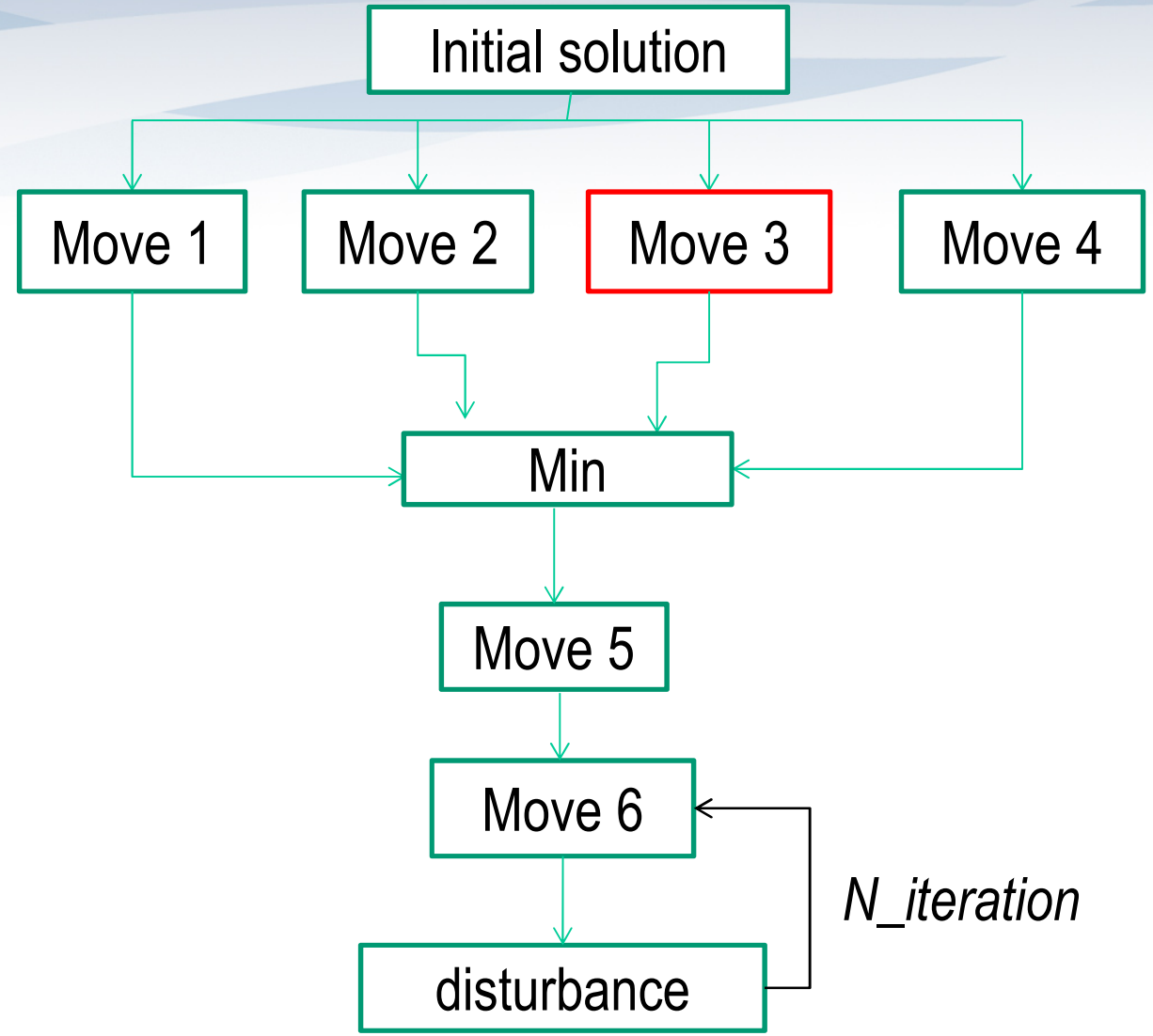
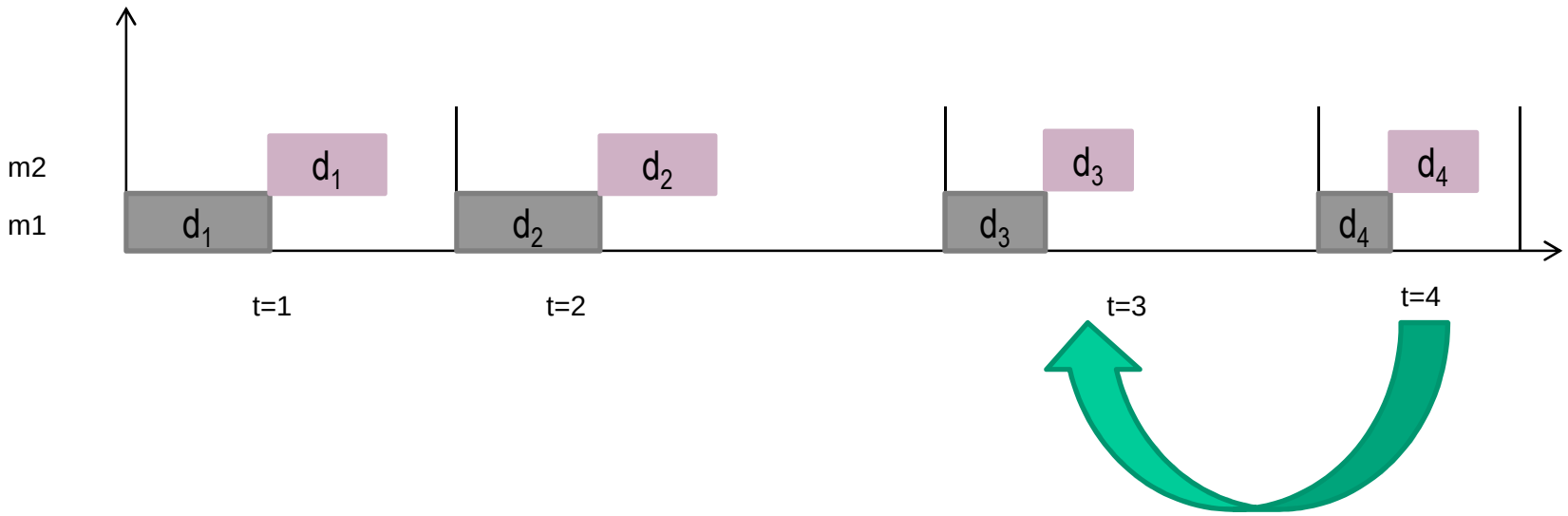


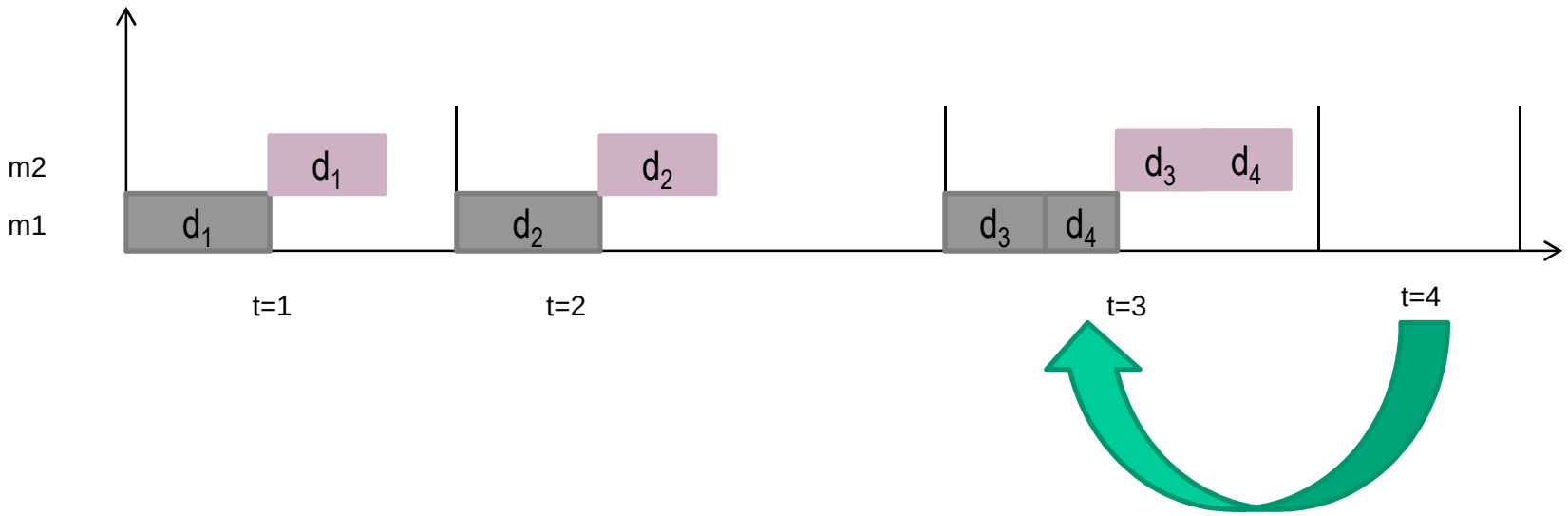
Heuristic H1

Move 3



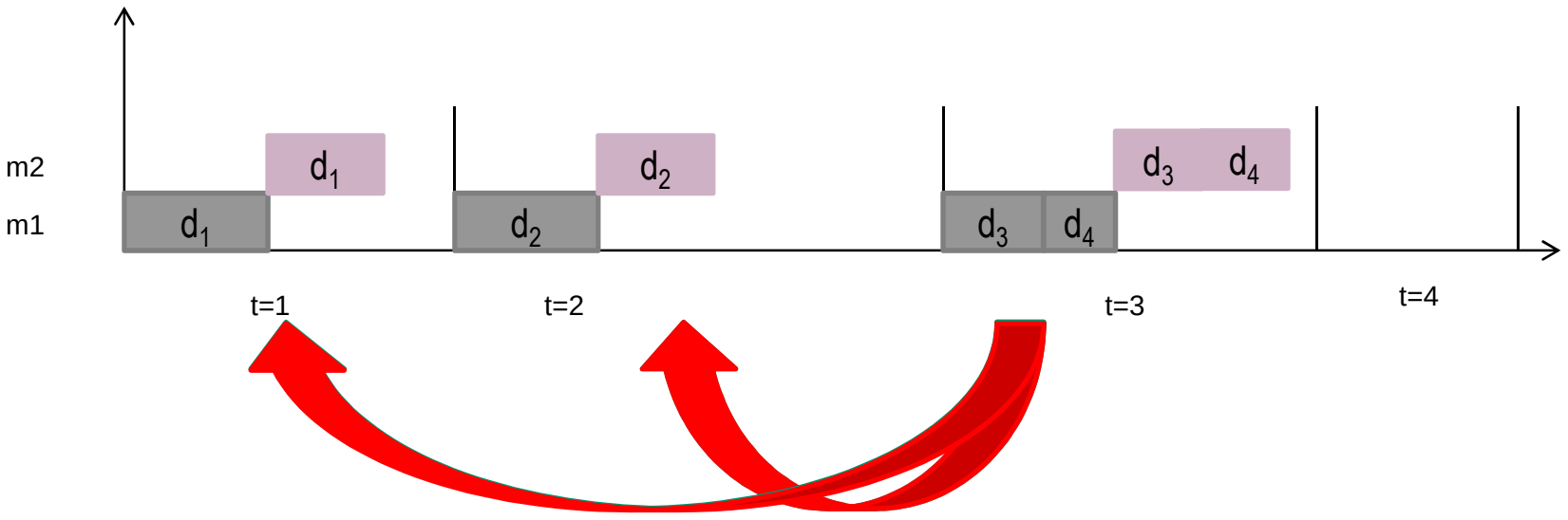
Heuristic H1

Move 3



Heuristic H1

Move 3



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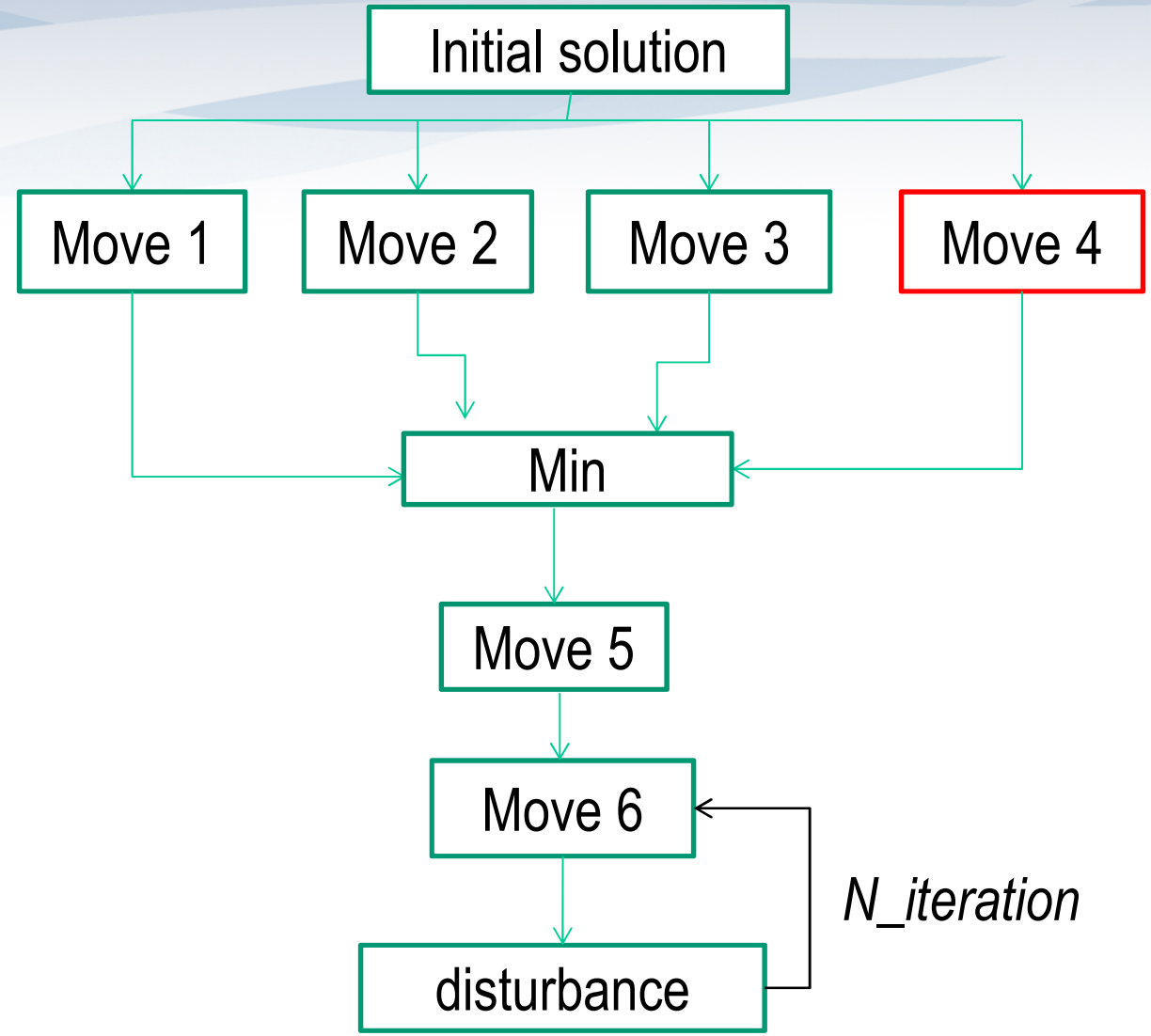
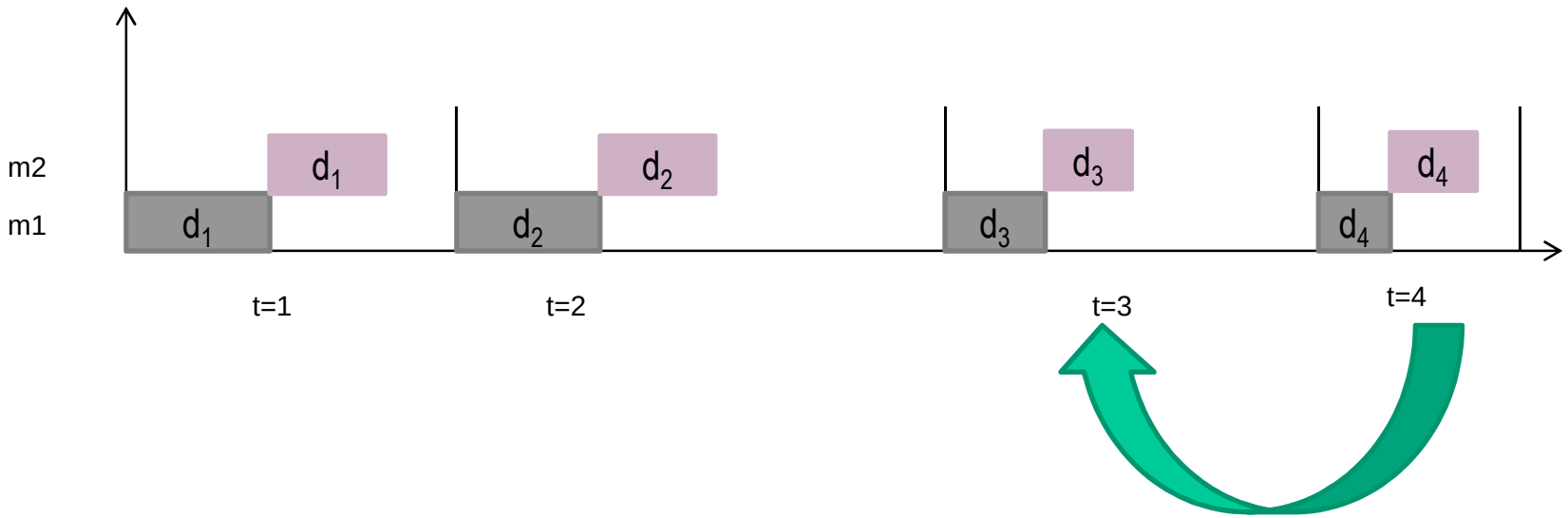


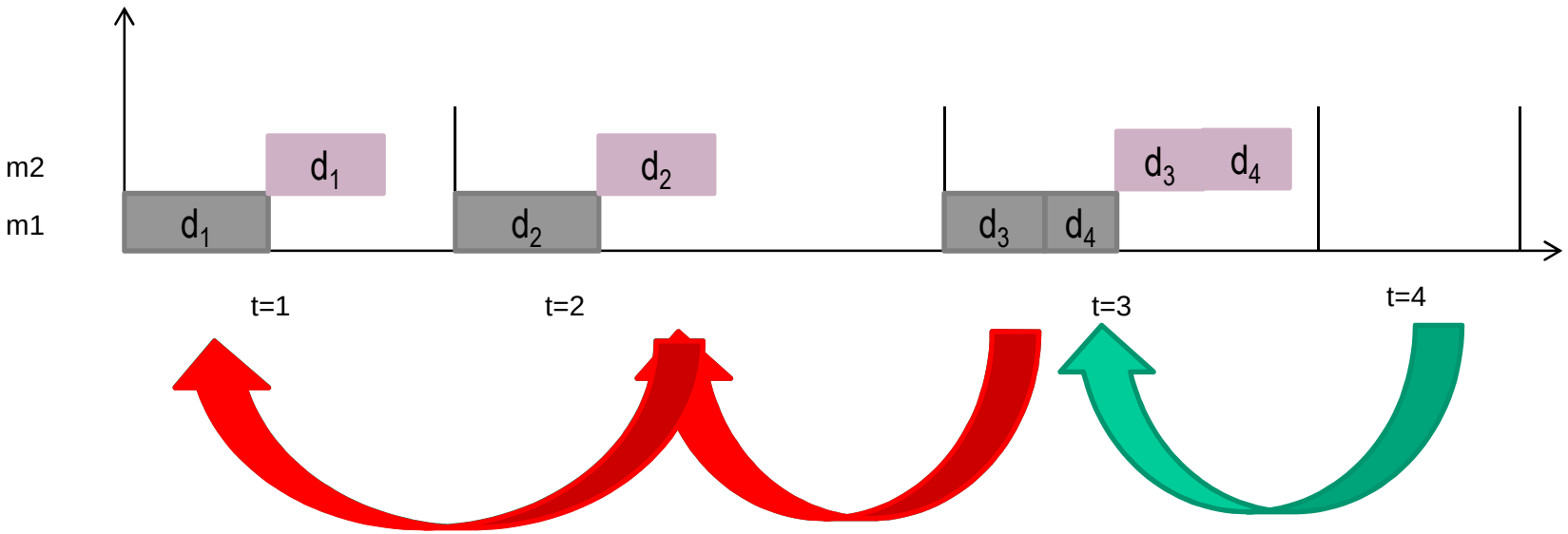
Heuristic H1

Move 4



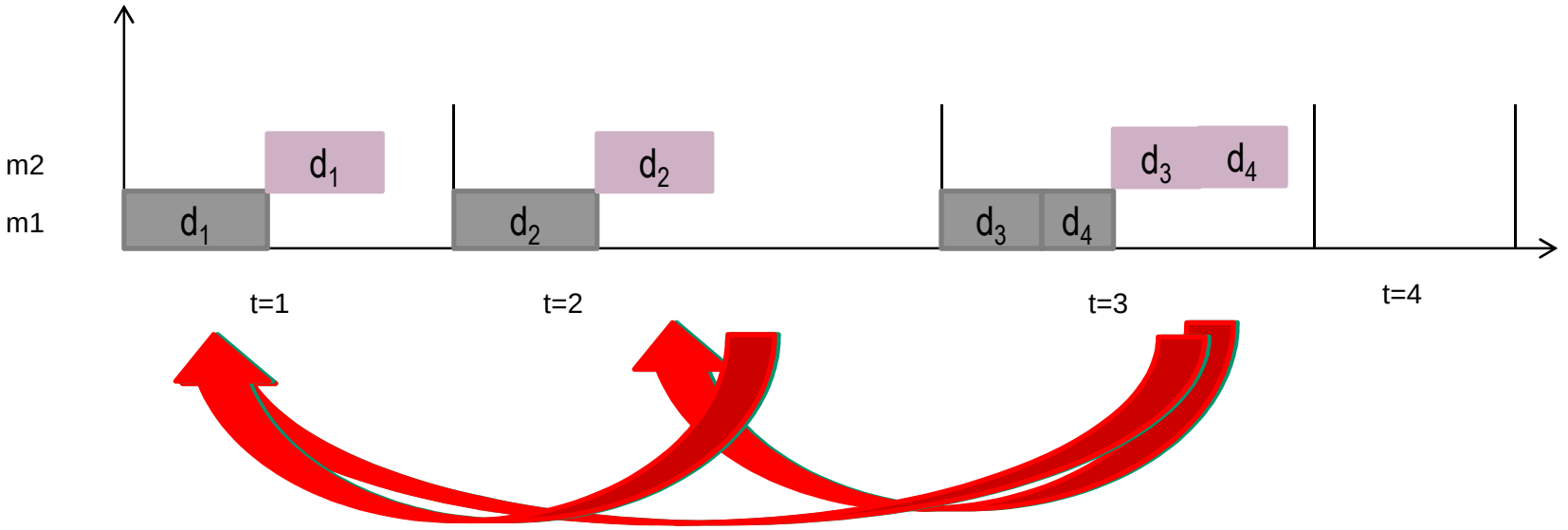
Heuristic H1

Move 4

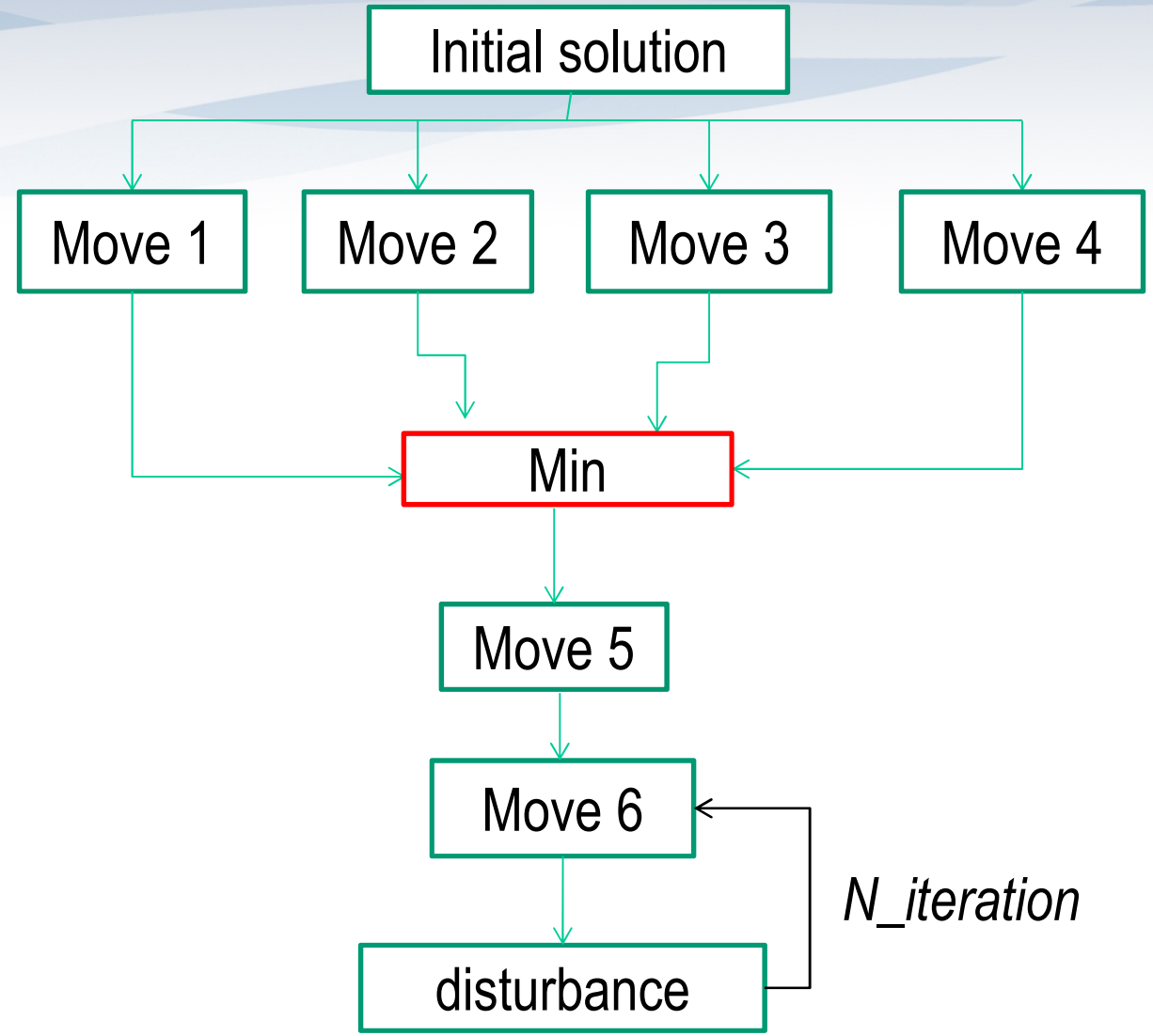


Heuristic H1

Move 4



Heuristic H1



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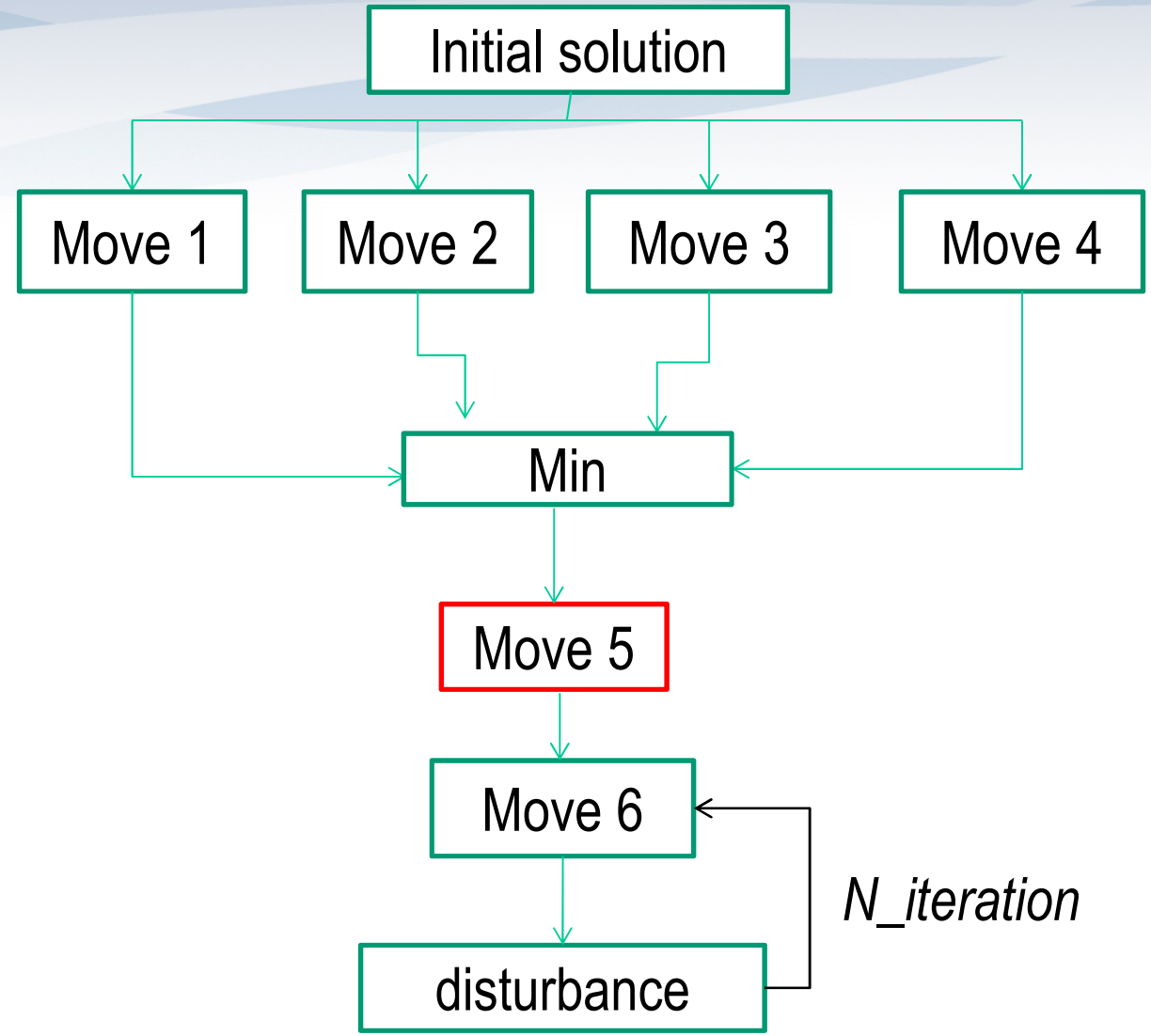
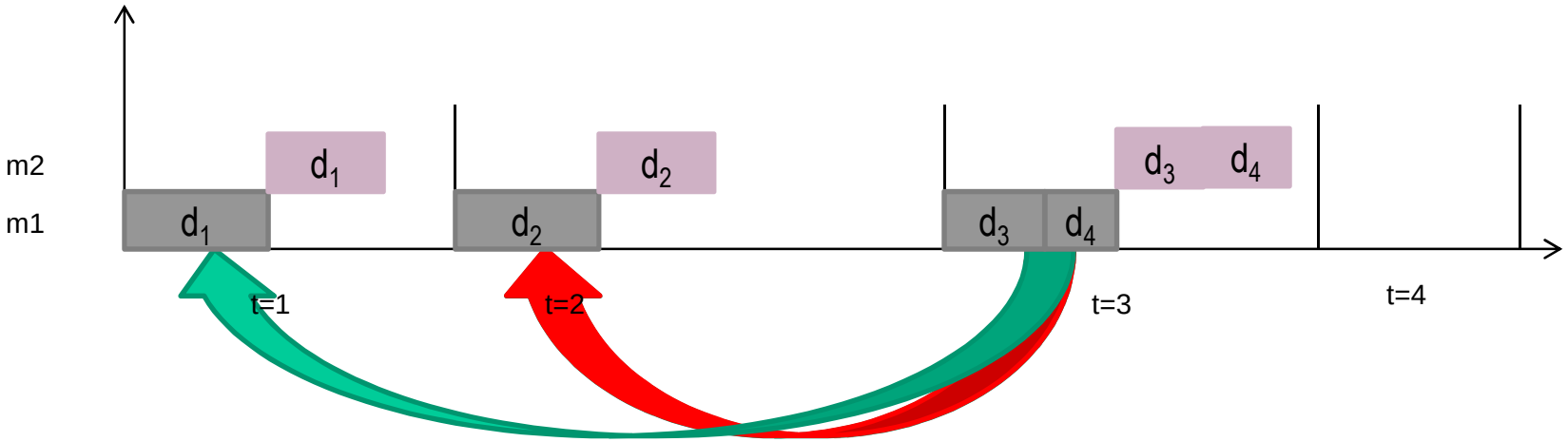


Heuristic H1

Move 5

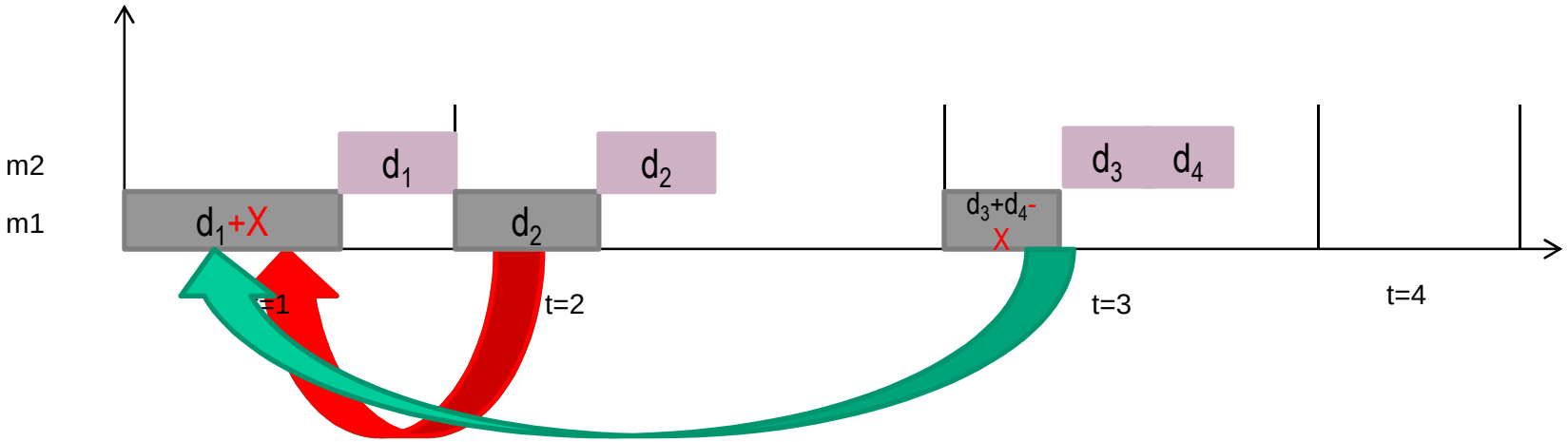
- **Without creating overlap**



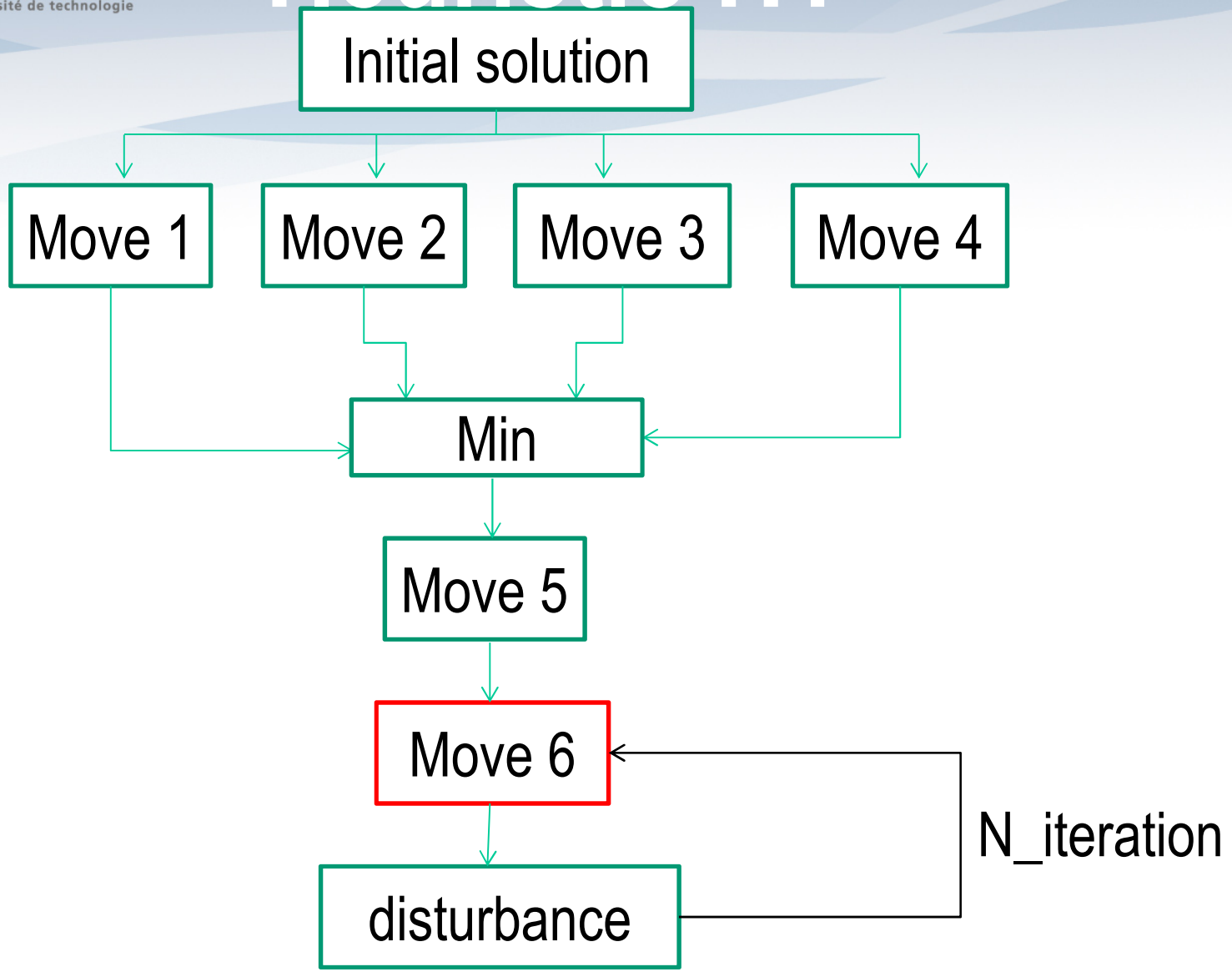
Heuristic H1

Move 5

- **Without creating overlap**



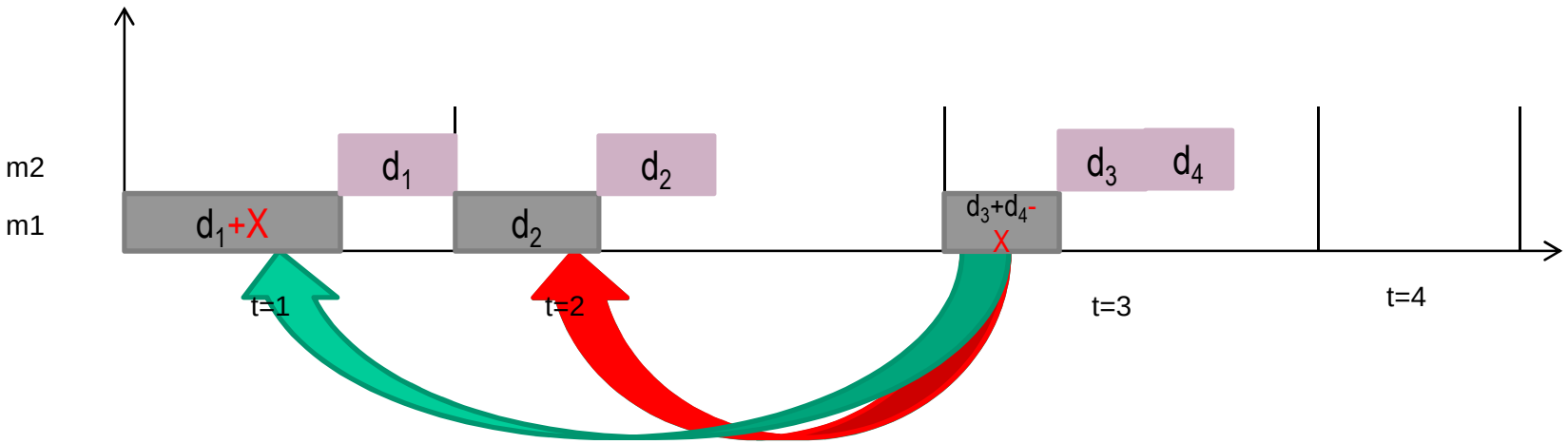
Heuristic H1



Heuristic H1

Move 6

- **Possibility to creat overlap**



Heuristic H1

Move 6

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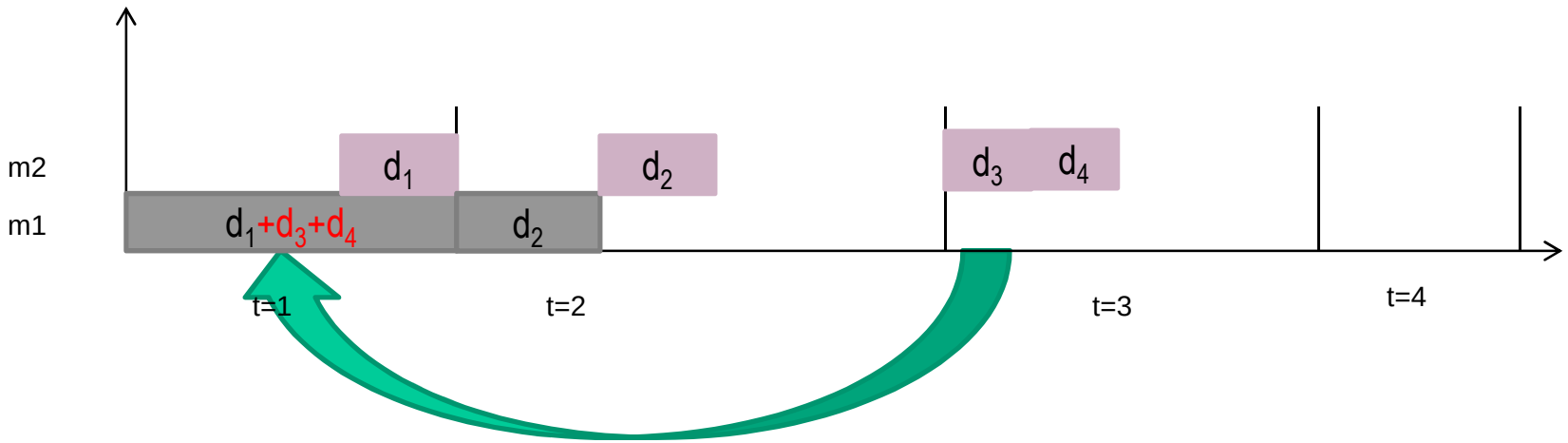

Heuristic H1

Move 6

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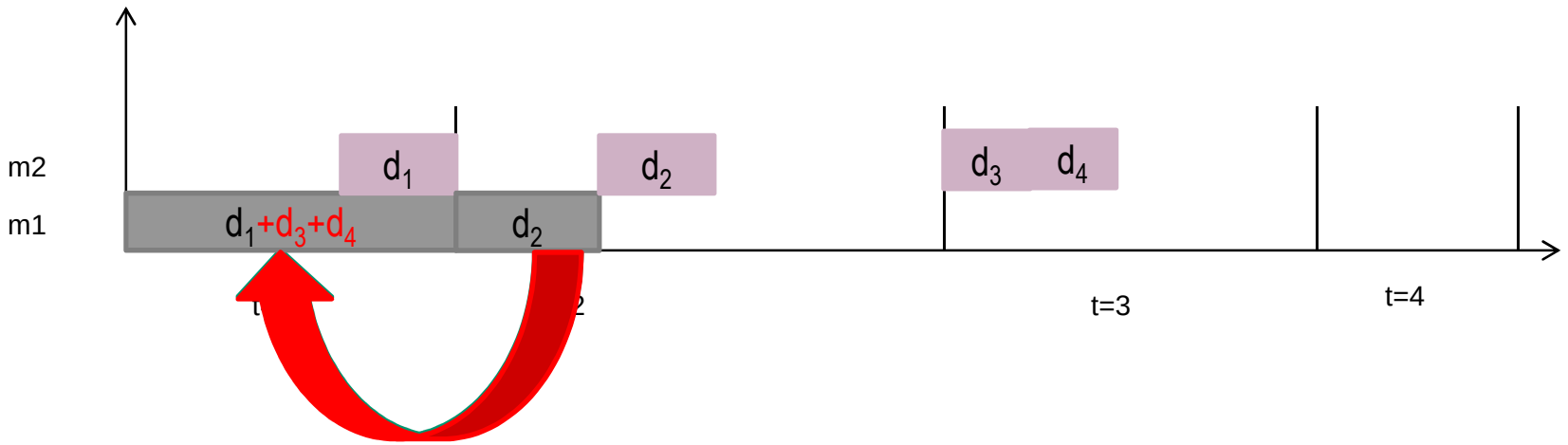
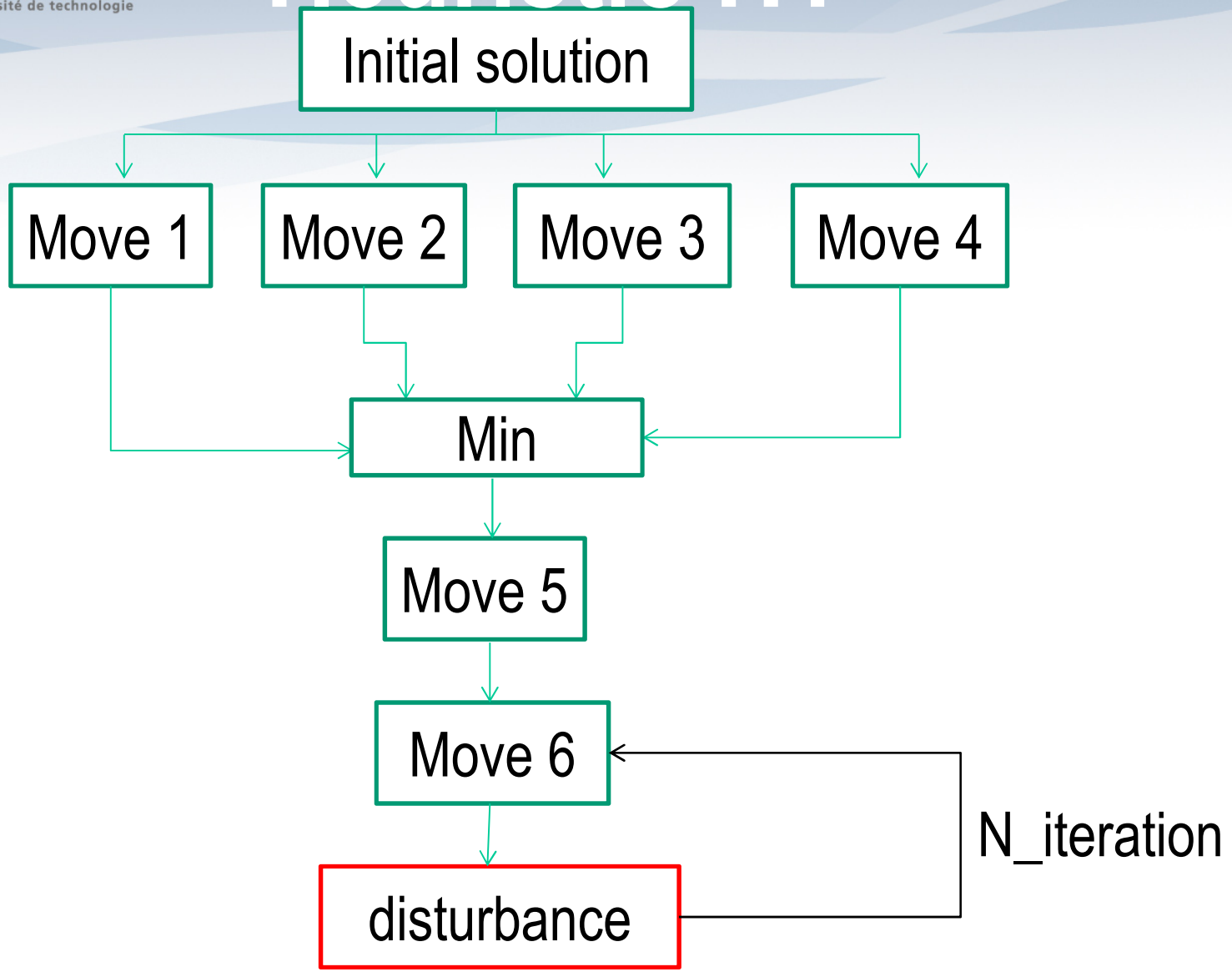
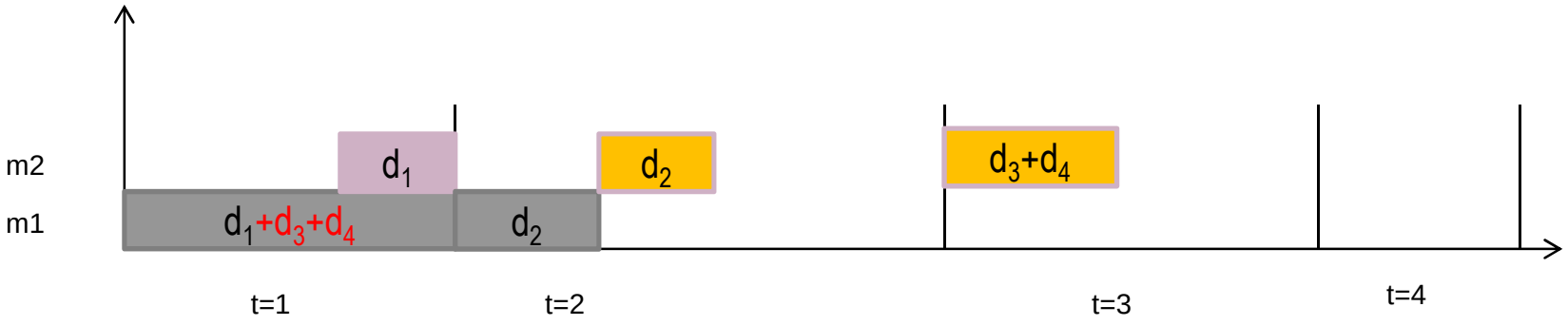


Heuristic H1

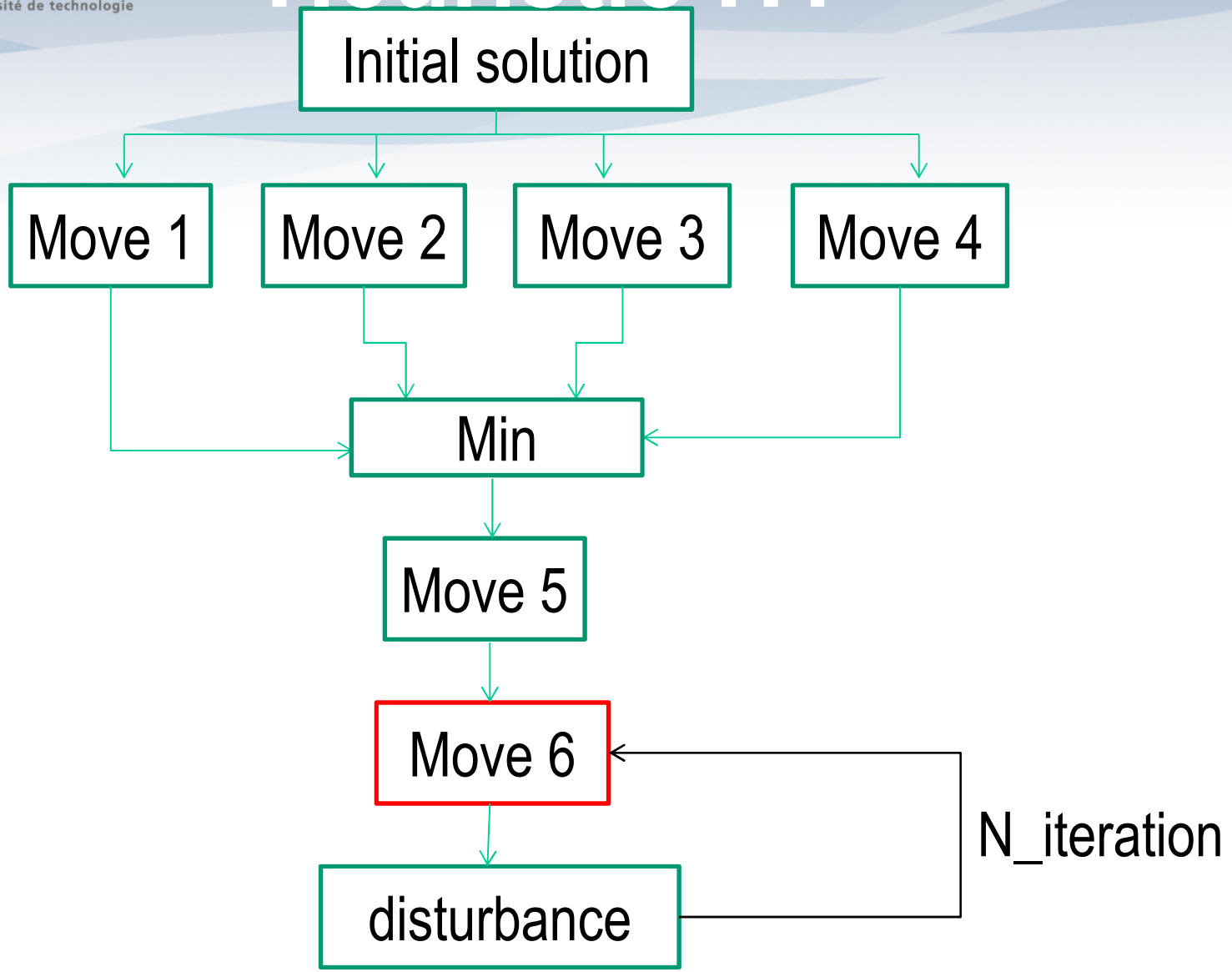


Heuristic H1

disturbance



Heuristic H1



Parameters

- **Let :**
  - $N \in \{7..10\}$
  - $T \in \{10..16\}$
  - The parameters follow uniform distribution

$p_m \approx U(5,10)$ $\quad\quad$ $\phi_m \approx U(5,10)$

$h \approx U(1,6)$ $\quad\quad$ $W_{m,t} \approx U(10,30)$

$d_t \approx U(0,cap_t)$

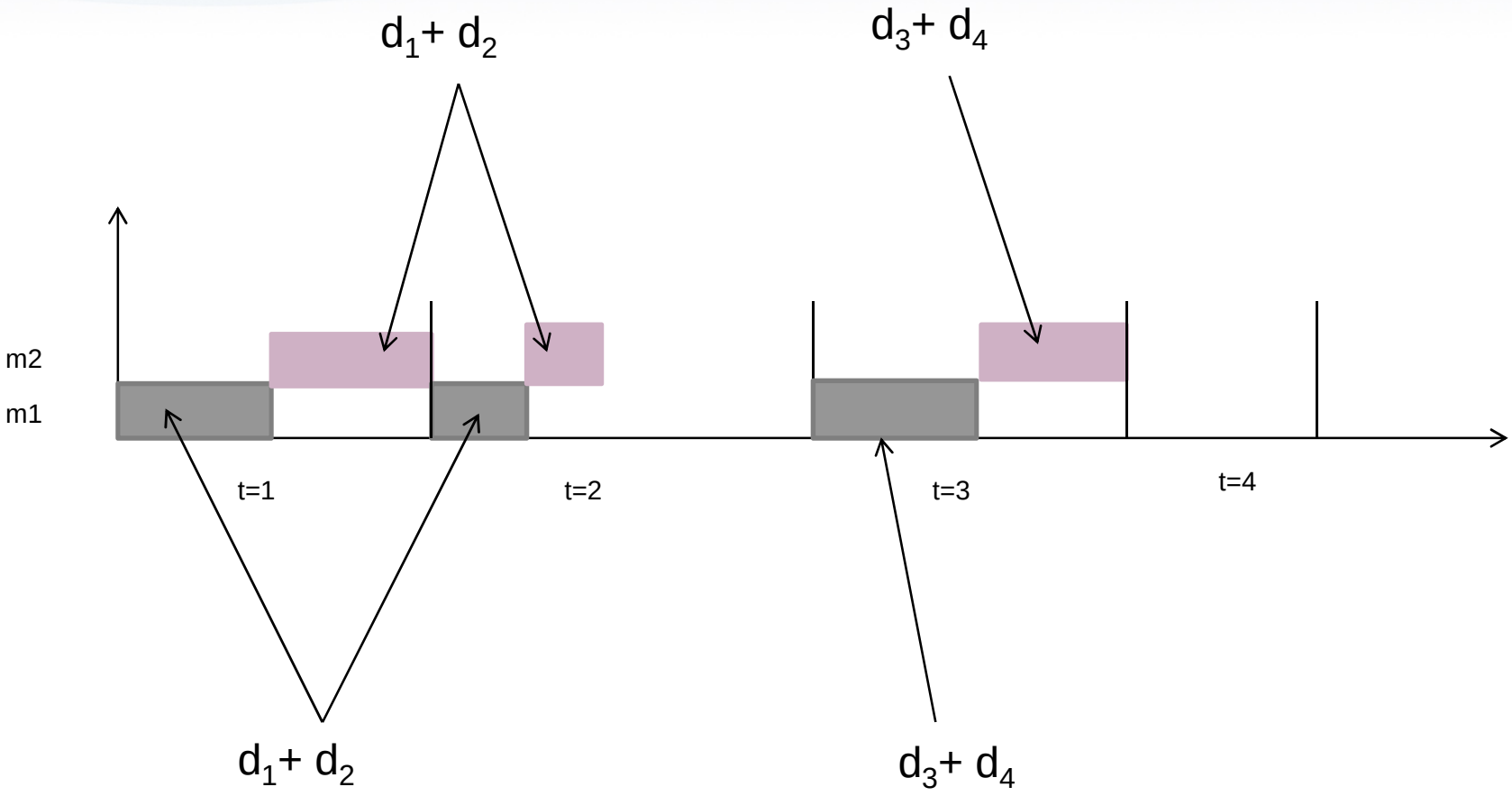
Time of day	Co_t ($/KWh)	Θ_t ($/KW)
7pm-1pm	0,08	1
1pm-7pm	0,16	19,8

Time of use pricing profile (Wang and Li, 2013)

Instances : machines, period_N°	Z_CPLEX	CPU_CPLEX (s)	Z_Heuristic	CPU_Heuristic (s)	Gap (%)
7,12_1	794,5	204	810,5	3	2,01
7,12_2	1064	17	1076	4	1,12
7,12_3	650,22	15	685,22	3	5,38
7,16_1	1302,74	2117	1416,74	3	8,75
7,16_2	1539,11	21	1769,03	3	14,93
7,16_3	1237,2	6342	1245,2	4	0,64
8,16_1	*1663,77*	*14400*	1699,77	4	*2,16*
8,16_2	*977,69*	*14400*	1105,69	3	*13,09*
8,16_3	1535,25	2806	1732,01	3	12,81
9,10_1	849,70	9803	904,7	3	6,47
9,10_2	1015,93	36	1015,93	4	0
9,10_3	806,98	22	806,98	4	0
10,12_1	*1219,84*	*14400*	1275,84	3	*4,16*
10,12_2	1052,27	2495	1052,27	4	0
10,12_3	802,2	34	863,2	3	7,6

Average gap = 5,27 %

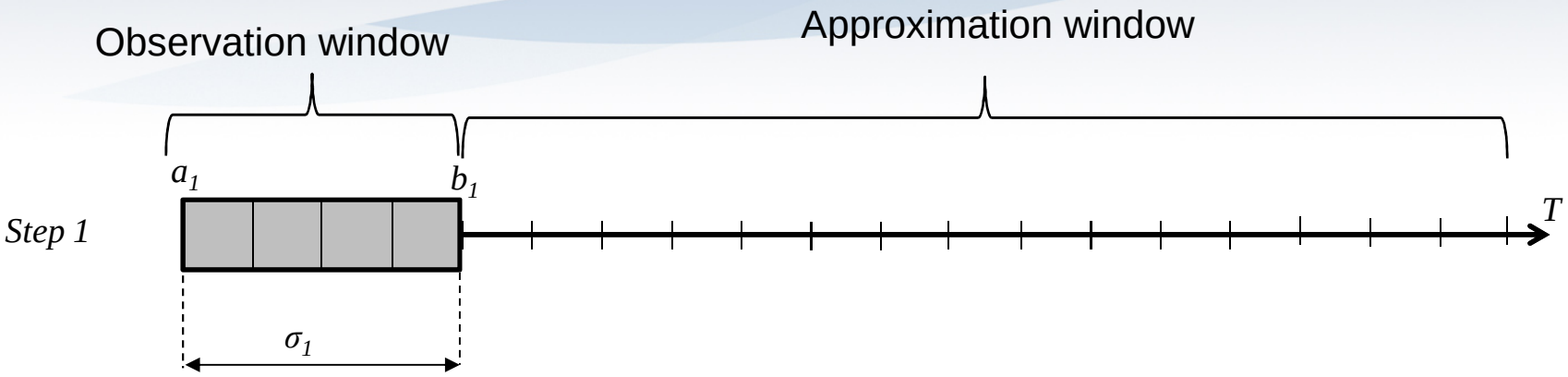
Heuristic H2



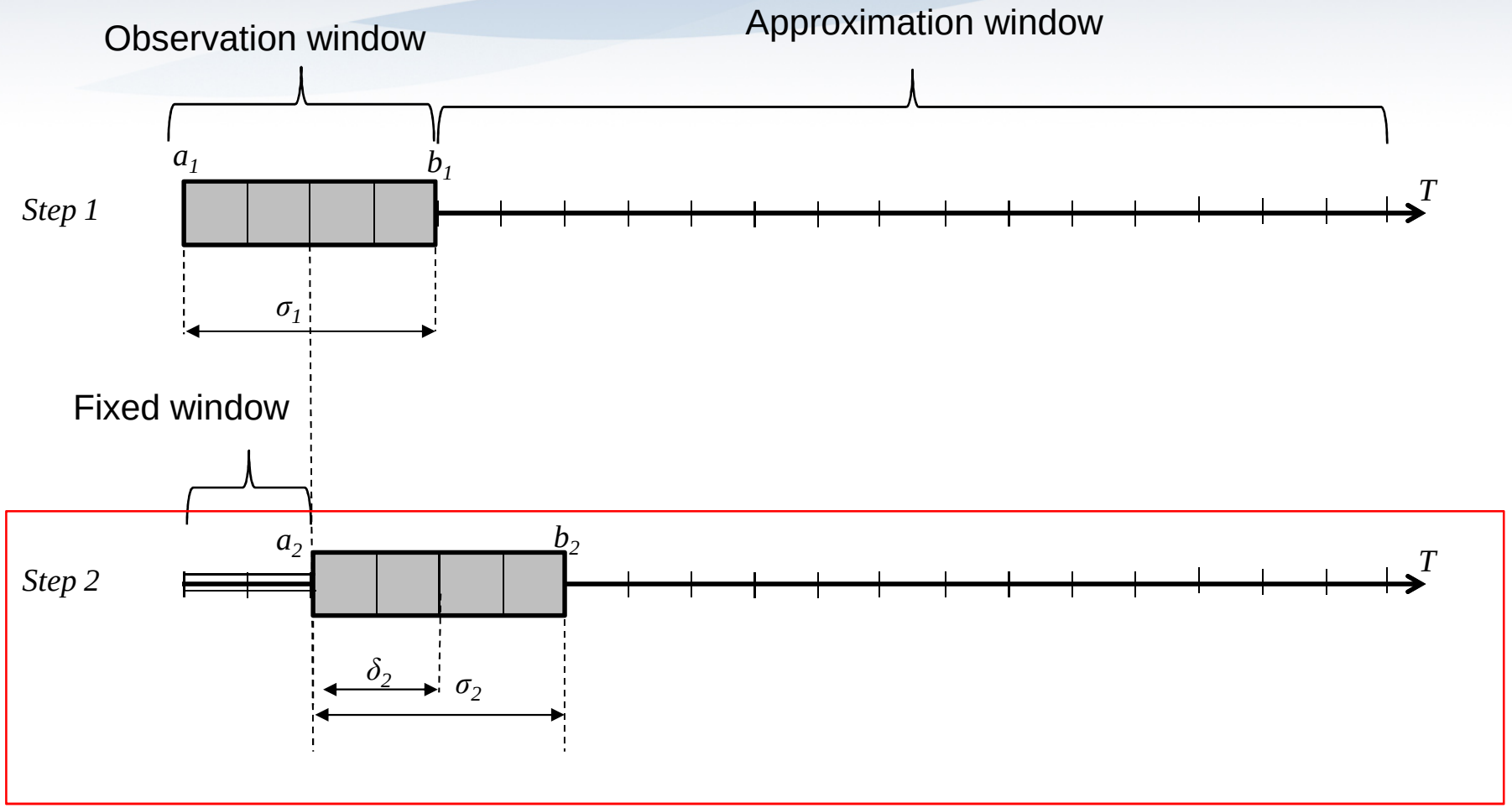
Instances : machines, period_N°	Z_CPLEX	CPU_CPLEX (s)	Z_Heuristic	CPU_Heuristic (s)	Gap (%)
7,12_1	794,5	204	810,5	3	2,01
7,12_2	1064	17	1094	4	2,81
7,12_3	650,22	15	685,22	3	5,38
7,16_1	1302,74	2117	1416,74	3	8,75
7,16_2	1539,11	21	1557,11	3	1,16
7,16_3	1237,2	6342	1237,2	3	0,00
8,16_1	*1663,77*	*14400*	*1675,77*	*4*	*0,72*
8,16_2	*977,69*	*14400*	*996,68*	*3*	*1,32*
8,16_3	1535,25	2806	1732,01	3	12,81
9,10_1	849,70	9803	904,7	4	6,47
9,10_2	1015,93	36	1015,93	3	0,00
9,10_3	806,98	22	806,98	3	0,00
10,12_1	*1219,84*	*14400*	*1275,84*	*4*	*4,16*
10,12_2	1052,27	2495	1052,27	4	0,00
10,12_3	802,2	34	843,2	3	5,11

Average gap = 3,38 %

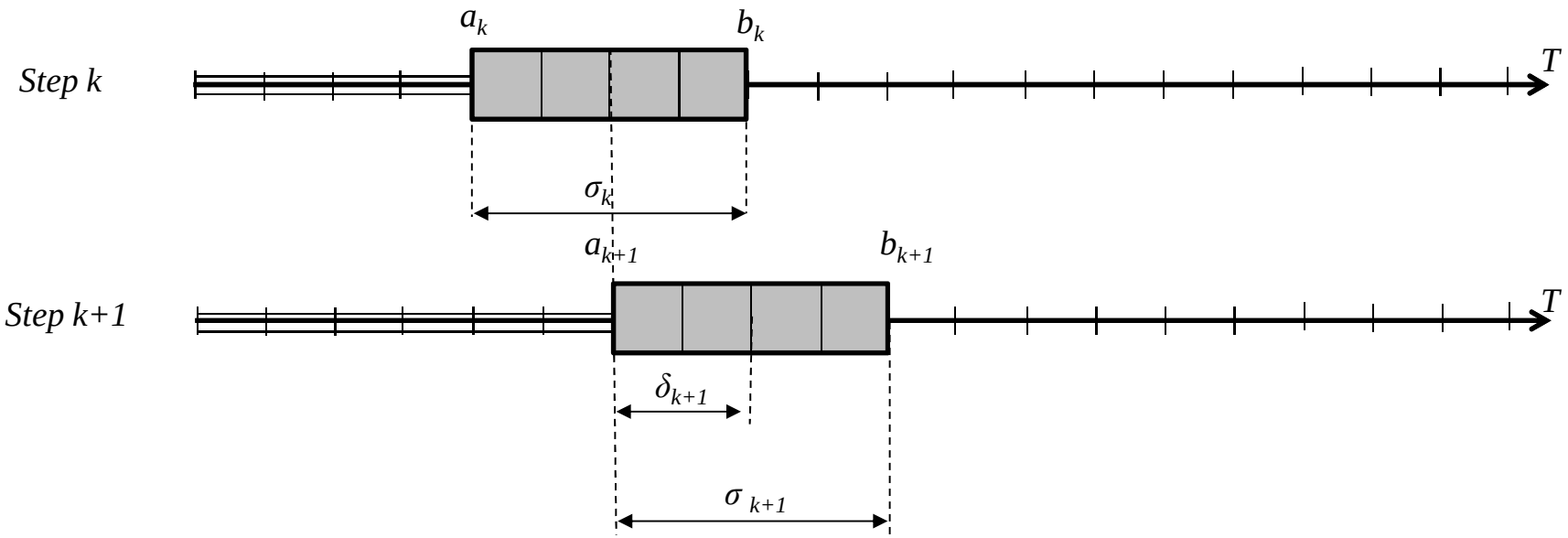
Heuristic Fix & Relax



Heuristic Fix & Relax



Heuristic Fix & Relax



Instances : machines, period_N°	Z_CPLEX	CPU_CPLEX (s)	Z_F & R	CPU_F & R (s)	Gap (%)
7,12_1	794,5	204	807,5	20	1,64
7,12_2	1064	17	1064	14	0,00
7,12_3	650,22	15	650,22	10	0,00
7,16_1	1302,74	2117	1345,73	31	3,30
7,16_2	1539,11	21	1539,11	20	0,00
7,16_3	1237,2	6342	1277,2	24	3,23
8,16_1	*1663,77*	*14400*	*1736,76*	*59*	*4,39*
8,16_2	*977,69*	*14400*	*1020,68*	*58*	*3,76*
8,16_3	1535,25	2806	1535,25	42	0,00
9,10_1	849,70	9803	887,7	47	4,47
9,10_2	1015,93	36	1047,07	53	3,07
9,10_3	806,98	22	806,98	13	0,00
10,12_1	*1219,84*	*14400*	*1286,83*	*194*	*5,06*
10,12_2	1052,27	2495	1094,27	29	3,99
10,12_3	802,2	34	804,2	24	0,24

$\sigma = 6$ $\delta = 4$

Average gap = 2,21 %

Instances : machines, period_N°	Gap_F&R (%)	CPU_F&R (s)	Gap_Heuristic_1 (%)	CPU_Heuristic_1 (s)	Gap_Heuristic_2 (%)	CPU_Heuristic_2 (s)
7,12_1	1,64	20	2,01	3	2,01	3
7,12_2	0,00	14	1,12	4	2,81	4
7,12_3	0,00	10	5,38	3	5,38	3
7,16_1	3,30	31	8,75	3	8,75	3
7,16_2	0,00	20	14,93	3	1,16	3
7,16_3	3,23	24	0,64	4	0,00	3
8,16_1	4,39	59	2,16	4	0,72	4
8,16_2	3,76	58	13,09	3	1,32	3
8,16_3	0,00	42	12,81	3	12,81	3
9,10_1	4,47	47	6,47	3	6,47	4
9,10_2	3,07	53	0	4	0,00	3
9,10_3	0,00	13	0	4	0,00	3
10,12_1	5,06	194	4,16	3	4,16	4
10,12_2	3,99	29	0	4	0,00	4
10,12_3	0,24	24	7,6	3	5,11	3

Av Gap F&R = 2,21 % **Av Gap H1 =5,27 %** **Av Gap H2 = 3,38 %**

Outline

- Introduction
- Bibliographic synthesis
- Proposed model
- Proposed heuristics and results
- Conclusions and perspectives

Conclusions & perspectives

- Develop new lot-sizing problem in flow-shop system with energy consideration
- The proposed heuristics provide promising solutions.

- Next steps :
  - Propose a metaheuristic
  - Reformulate the model to deal with multi products
  - Other alternatives integrating various power resources