Incorporating some sustainability issues in lot-sizing models

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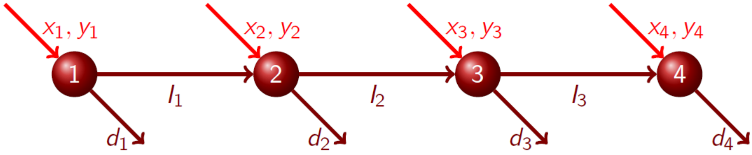
Outline

We fill focus on sustainability issues in operational decisions.

1. Lot-sizing with remanufacturing: joint and separate set-up cost
   - Natural MIP formulations
   - Complexity results
   - Reformulations
   - Computational tests
2. Lot-sizing with an emission constraint
   - Complexity results
   - Algorithms
     - Lagrangian heuristic
     - Pseudo-polynomial algorithm for *co-behaving* costs and emissions
     - Fully polynomial time approximation schemes (FPTASes)
   - Computational tests
3. Bi-objective lot-sizing: minimize costs and emissions
   - Finding Pareto points
   - Complexity results
   - Efficient DP Algorithms for special cases

Classic economic lot-sizing problem



Data:

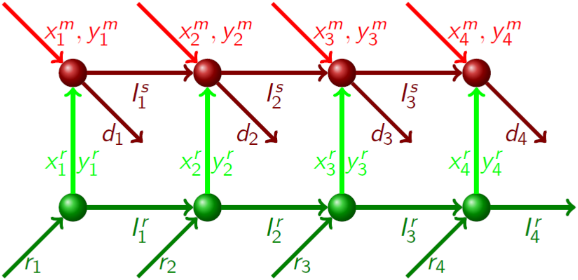
- T : planning horizon indexed by t
- d_t : demand ($D_{t,\tau} = \sum_{s=t}^{\tau} d_s$)
- (K_t, p_t, h_t) : (setup, unit production, unit inventory) costs

Decisions:

- (y_t, x_t, I_t) : (setup, production, inventory) decisions

- Trade-off between (fixed) set-up costs and (linear) holding costs.
- Solvable in $\mathcal{O}(T \log T)$ time
- subproblem in MRP/ERP (usually solved heuristically)

Part I: Lot-sizing with remanufacturing



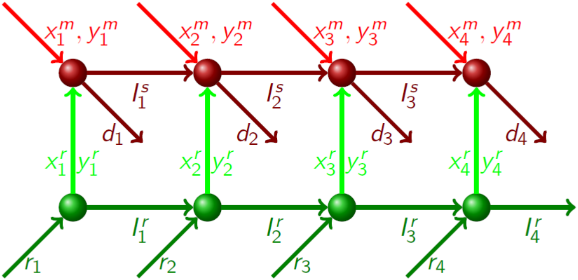
Classic single-item, uncapacitated lot-sizing problem (upper layer) *plus*:

- A known quantity of used products r_t is returned from customers in each period t.
- ‘Returns’ can be remanufactured, so that they are as good as new.
- When to set up the (re-)manufacuring process?

Extra decisions:

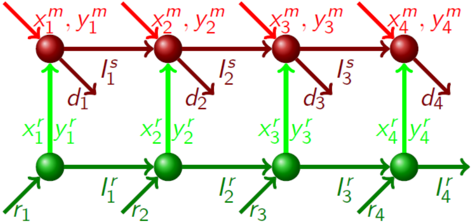
- (y_t^r, x_t^r, I_t^r) : remanufacturing (setup, production, inventory) decisions

Lot-sizing with remanufacturing



- 'Returns' can be remanufactured, so that they are as good as new.
  - no choice: printer cartridges, single-use cameras
  - service contract: copiers
- A known quantity of used products is returned from customers in each period.
  - remanufacturing included in more and more MRP/ERP systems
- The manufacturing and remanufacturing process have either *joint* (ELSRj) or *separate* (ELSRs) set-up costs (Teunter et al., 2006)
  - manufacturing and remanufacturing on the same or different production lines

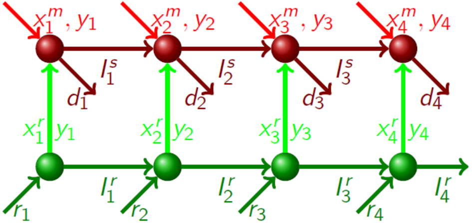
Natural formulation: separate set-ups



$$\min \sum_{t=1}^{T} \left(K_t^m y_t^m + p_t^m x_t^m + h_t^s I_t^s + K_t^r y_t^r + p_t^r x_t^r + h_t^r I_t^r \right) \tag{1}$$

$$
\begin{array}{lrcll}
\text{s.t.} & I_t^s & = & I_{t-1}^s + x_t^m + x_t^r - d_t & t = 1, \ldots, T \quad (2) \\
 & I_t^r & = & I_{t-1}^r - x_t^r + r_t & t = 1, \ldots, T \quad (3) \\
 & x_t^m & \leq & D_{t,T}\, y_t^m & t = 1, \ldots, T \quad (4) \\
 & x_t^r & \leq & D_{t,T}\, y_t^r & t = 1, \ldots, T \quad (5) \\
 & x_t^m, x_t^r, I_t^s, I_t^r \geq 0 & & y_t^m, y_t^r \in \{0, 1\} & t = 1, \ldots, T \quad (6) \\
 & I_0^s = I_0^r & = & 0 & \quad (7)
\end{array}
$$

Natural formulation: joint set-ups

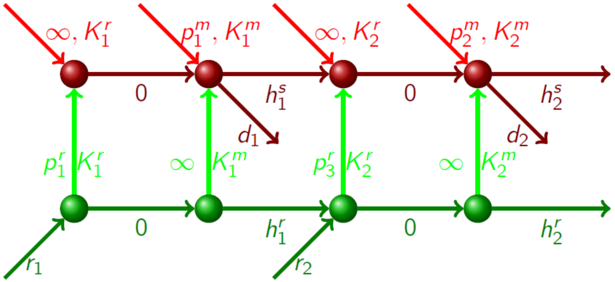


$$\min \sum_{t=1}^{T} \left(K_t y_t + p_t^m x_t^m + h_t^s I_t^s + p_t^r x_t^r + h_t^r I_t^r \right) \quad (8)$$

$$\begin{array}{rrcll}
\text{s.t.} & I_t^s & = & I_{t-1}^s + x_t^m + x_t^r - d_t & t = 1, \ldots, T \quad (9) \\
& I_t^r & = & I_{t-1}^r - x_t^r + r_t & t = 1, \ldots, T \quad (10) \\
& x_t^m + x_t^r & \leq & D_{t,T}\, y_t & t = 1, \ldots, T \quad (11) \\
& x_t^m, x_t^r, I_t^s, I_t^r \geq 0 & & y_t \in \{0, 1\} & t = 1, \ldots, T \quad (12) \\
& I_0^s = I_0^r & = & 0 & \quad (13)
\end{array}$$

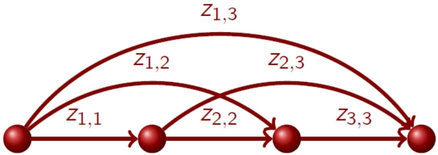
Complexity results

- ELSRs is $\mathcal{NP}$-hard for constant cost parameters.
  - reduction from PARTITION (see Van den Heuvel, 2006)
- ELSRj is solvable in $\mathcal{O}(T^4)$ for constant cost parameters (Teunter et al., 2006).
- ELSRj is $\mathcal{NP}$-hard in general.
  - ELSRs is a special case of ELSRj



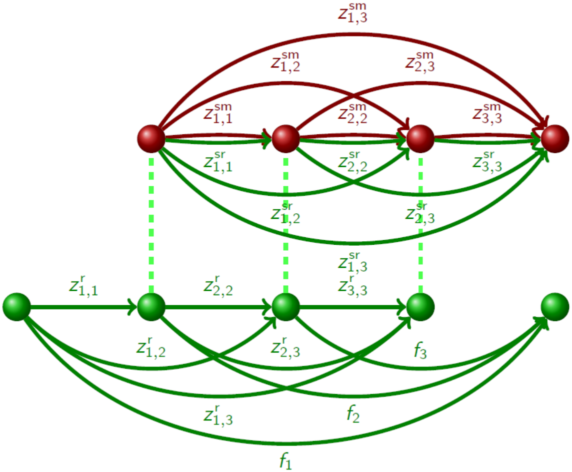
Shortest path reformulation: single-item problem

- Based on Eppen & Martin's (1987) shortest path reformulation of the capacitated problem (without remanufacturing).
- $z_{i,j}$ is fraction of demand in *each* of the periods i until j that is satisfied by production in period i.
- Send a unit flow through the network.
- LP relaxation gives integer optimal solution.



Shortest path reformulation: separate set-ups

- Extension to remanufacturing
  - Also gives the possibility of final inventories (through f_1, f_2, f_3).



Shortest path reformulation: separate set-ups

$$\min \left(\sum_{t=1}^{T} \left(K_t^m y_t^m + K_t^r y_t^r + c_t^f f_t \right) + \sum_{i=1}^{T} \sum_{j=i}^{T} \left(C_{i,j}^{\mathsf{sm}} z_{i,j}^{\mathsf{sm}} + C_{i,j}^{\mathsf{sr}} z_{i,j}^{\mathsf{sr}} + C_{i,j}^{r} z_{i,j}^{\mathsf{r}} \right) \right) \quad (14)$$

s.t.

$$1 = \sum_{j=1}^{T} \left(z_{1,j}^{\mathsf{sm}} + z_{1,j}^{\mathsf{sr}} \right) \quad (15)$$

$$\sum_{i=1}^{t-1} \left(z_{i,t-1}^{\mathsf{sm}} + z_{i,t-1}^{\mathsf{sr}} \right) = \sum_{j=t}^{T} \left(z_{t,j}^{\mathsf{sm}} + z_{t,j}^{\mathsf{sr}} \right) \qquad t = 2, \ldots, T \quad (16)$$

$$\sum_{j=t}^{T} z_{t,j}^{\mathsf{sm}} \leq y_t^m \qquad t = 1, \ldots, T \quad (17)$$

$$\sum_{j=t}^{T} z_{t,j}^{\mathsf{sr}} \leq y_t^r \qquad t = 1, \ldots, T \quad (18)$$

$$1 = \sum_{j=1}^{T} z_{1,j}^{\mathsf{r}} + f_1 \quad (19)$$

$$\sum_{i=1}^{t-1} z_{i,t-1}^{\mathsf{r}} = \sum_{j=t}^{T} z_{t,j}^{\mathsf{r}} + f_t \qquad t = 2, \ldots, T \quad (20)$$

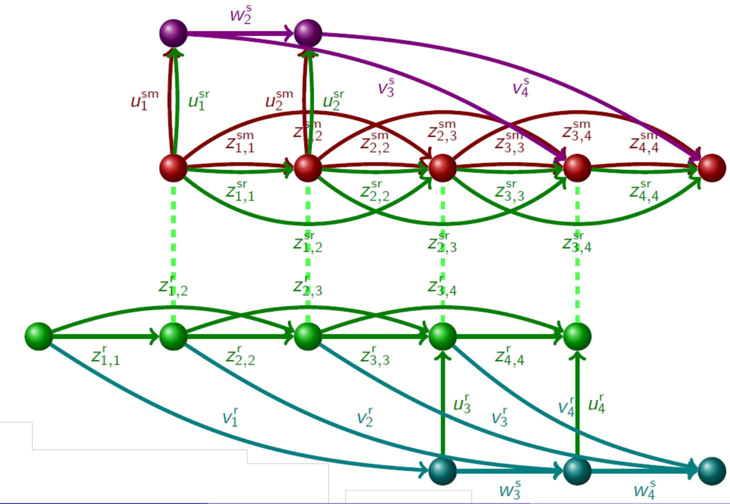
$$\sum_{i=1}^{t} z_{i,t}^{\mathsf{r}} \leq y_t^r \qquad t = 1, \ldots, T \quad (21)$$

$$\sum_{i=1}^{t} R_{i,t}\, z_{i,t}^{\mathsf{r}} = \sum_{j=t}^{T} D_{t,j}\, z_{t,j}^{\mathsf{sr}} \qquad t = 1, \ldots, T \quad (22)$$

$$z_{i,j}^{\mathsf{sm}}, z_{i,j}^{\mathsf{sr}}, z_{i,j}^{\mathsf{r}} \geq 0 \qquad 1 \leq i \leq j \leq T \quad (23)$$

Partial shortest path reformulation: separate set-ups

- To reduce the number of variables, use partial shortest path reformulation.
  - See Van Vyve & Wolsey (2006) and Stadtler (1997) for classic lot-sizing problem.



(l,S,WW) valid inequalities

- The (l,S,WW) inequalities are strong valid inequalities for the single item, uncapacitated lot-sizing problem. (Pochet and Wolsey, 1994)
- We adapted them for both the returns and serviceables layer of lot-sizing with remanufacturing.
- For separate set-ups:

$$
\begin{aligned}
I^s_{i-1} + \sum_{t=i}^{j} D_{t,j}\,(y^m_t + y^r_t) &\geq D_{i,j} \qquad 2 \leq i \leq j \leq T && (24)\\
I^r_j + \sum_{t=i}^{j} R_{i,t}\; y^r_t &\geq R_{i,j} \qquad 1 \leq i \leq j \leq T && (25)
\end{aligned}
$$

- For joint set-ups:

$$
\begin{aligned}
I^s_{i-1} + \sum_{t=i}^{j} D_{t,j}\; y_t &\geq D_{i,j} \qquad 2 \leq i \leq j \leq T && (26)\\
I^r_j + \sum_{t=i}^{j} R_{i,t}\; y_t &\geq R_{i,j} \qquad 1 \leq i \leq j \leq T && (27)
\end{aligned}
$$

- We added these to the Natural formulation.

Computational tests: data

Parameter settings:

parameter	T	d_t	r_t
value	$\{25, 50, 75\}$	$N(100, 50)$	$\{N(10, 5), N(50, 25), N(90, 45)\}$

parameter	$K_t^m = K_t^r = K$	$h_t^s = h_t^r = h^r$	$p_t^m = p_t^r = p^r$
value	$\{125, 250, 500, 1000\}$	1	0

- 10 replications for each setting
- so 360 problem instances generated (both for ELSRs and ELSRj)
- solved with CPLEX 10.1 and 1 hour time limit

Setting k^s and k^r in the partial shortest path formulation:

- Use approximations of time between orders in EOQ setting (Van der Laan and Teunter, 2006):

$$TBO^s = \sqrt{\frac{2\bar{K}^s}{\bar{h}^s(\bar{d} - \bar{r})}} \text{ and } TBO^r = \sqrt{\frac{2\bar{K}^r}{\bar{h}^r \bar{r}}}.$$

- We choose $k^s = \lceil 2 \cdot TBO^s \rceil$ and $k^r = \lceil 2 \cdot TBO^r \rceil$ (formulation PSP2).

Computational tests: results separate, 50 periods

set-up costs		average returns 10				50				90			
		Natural	SP	PSP2	(l, S, WW)	Natural	SP	PSP2	(l, S, WW)	Natural	SP	PSP2	(l, S, WW)
125	avg. sol. time MIP (s)	12	**0.4**	0.7	6	606	426	**252**	296	152	325	194	**106**
	avg. LP gap (%)	91	**1.7**	**1.7**	21	89	**7.0**	**7.0**	20	45	**7.3**	**7.3**	11
250	solved to optimality	3	**10**	**10**	**10**	3	**10**	**10**	7	8	**10**	**10**	9
	avg. MIP gap (%)	1.7	**0**	**0**	**0**	1.7	**0**	**0**	0.5	0.4	**0**	**0**	0.1
	avg. sol. time MIP (s)	3272	**0.5**	1.1	369	2859	**843**	989	2000	1122	811	**805**	1006
	avg. LP gap (%)	89	**1.0**	**1.0**	18	88	**6.3**	**6.3**	17	56	**7.8**	**7.8**	11
500	solved to optimality	0	**10**	**10**	**10**	3	**10**	**10**	**10**	6	**9**	**9**	**9**
	avg. MIP gap (%)	5.1	**0**	**0**	**0**	1.4	**0**	**0**	**0**	0.99	0.21	**0.18**	0.20
	avg. sol. time MIP (s)	3600	**0.5**	1.3	335	3144	**53**	91	729	1576	**390**	450	607
	avg. LP gap (%)	86	**1.1**	**1.1**	18	86	**4.7**	**4.7**	14	64	**7.7**	**7.7**	11
1000	solved to optimality	0	**10**	**10**	**10**	1	**10**	**10**	**10**	9	**10**	9	9
	avg. MIP gap (%)	3.6	**0**	**0**	**0**	3.4	**0**	**0**	**0**	0.48	**0**	0.25	0.19
	avg. sol. time MIP (s)	3600	**0.4**	1.0	401	3582	**25**	45	283	724	**424**	592	517
	avg. LP gap (%)	83	**0.67**	**0.67**	14	83	**3.8**	**3.8**	12	69	**6.2**	**6.2**	9.3

Computational tests: results joint, 50 periods

set-up costs		average returns 10			50			90		
		Natural	SP	(l, S, WW)	Natural	SP	(l, S, WW)	Natural	SP	(l, S, WW)
125	avg. sol. time MIP (s)	**0.0**	**0.0**	0.2	**0.0**	0.1	0.3	**1.4**	4.9	3.9
	integer solutions LP	0	**9**	1	0	0	0	0	0	0
	avg. LP gap (%)	89	**0.009**	0.91	85	**1.0**	1.4	42	**3.3**	3.5
250	avg. sol. time MIP (s)	0.8	**0.0**	0.2	**0.1**	**0.1**	0.4	**2.1**	4.8	3.9
	integer solutions LP	0	**10**	4	0	**1**	0	0	0	0
	avg. LP gap (%)	87	**0**	0.70	85	**0.48**	0.97	53	**3.5**	4.0
500	avg. sol. time MIP (s)	35	**0.0**	0.2	1.9	**0.0**	0.3	3.4	**3.2**	3.7
	integer solutions LP	0	**10**	6	0	**6**	3	0	0	0
	avg. LP gap (%)	84	**0**	0.68	83	**0.11**	0.47	61	**3.1**	3.7
1000	avg. sol. time MIP (s)	168	**0.0**	0.2	16.8	**0.0**	0.2	6.2	**0.9**	2.4
	integer solutions LP	0	**10**	6	0	**7**	5	0	0	0
	avg. LP gap (%)	80	**0**	0.45	81	**0.009**	0.16	66	**2.0**	2.4

Part II: Lot-sizing with an emission constraint

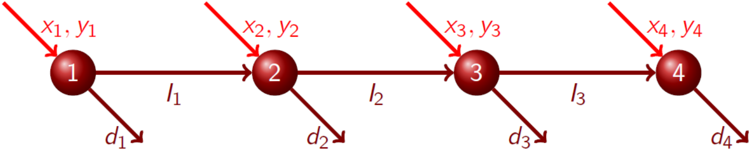
Quantitative models for carbon footprint in SCM:

- Emissions and the design of the supply chain
  e.g. (Cachon, 2011)
- Emissions and the choice of transportation mode
  e.g. (Hoen et al., 2014)
- Emissions and the level of data aggregation
  e.g. (Velázquez-Martínez et al., 2014)
- Emissions and the management of inventory
  e.g. (Hua et al., 2011)
- Emissions and operational decisions
  e.g. (Benjaafar et al., 2013)

Lot-sizing with an emission constraint

- Focus on both costs and environmental implications of the production process
  - Limit emissions of pollutants, such as CO_2.
  - Legal restrictions
  - Reducing carbon footprint in pursuit of a ‘greener’ image
- Not only *financial costs*, but also *emission* levels associated with production, keeping inventory and setting up the production process
- Difference with capacitated lot-sizing (with set-up times):
  - Constraint for each period vs. one global constraint
  - Keeping inventory may also emit pollutants.

Lot-sizing with an emission constraint: definition



$$\min \quad \sum_{t=1}^{T} (p_t(x_t) + h_t(I_t)) \tag{28}$$

$$\begin{aligned}
\text{s.t.} \quad I_t &= I_{t-1} + x_t - d_t \quad t = 1, \dots, T & (29)\\
I_0 &= 0 & (30)\\
x_t, I_t &\geq 0 \quad t = 1, \dots, T & (31)\\
\sum_{t=1}^{T} \left(\hat{p}_t(x_t) + \hat{h}_t(I_t)\right) &\leq \hat{C} & (32)
\end{aligned}$$

- All functions are assumed concave, nondecreasing and nonnegative.
  - Includes the case with fixed set-up costs (and emissions) and linear production and holding costs (and emissions).

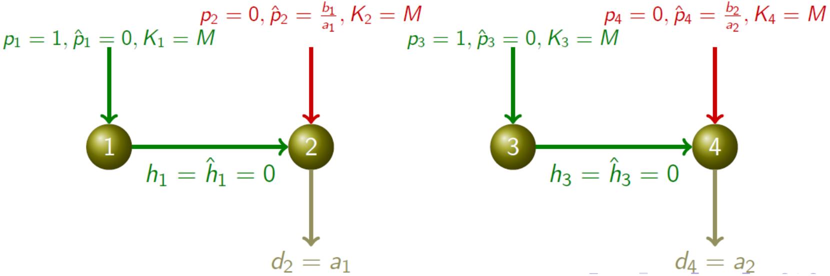
Some related literature

- Benjaafar et al. (2013):
  - Four emission policies: (i) taxes, (ii) caps, (iii) cap-and-trade mechanisms and (iv) offsets
  - It illustrates the impact of lot-sizing decisions on emissions
- Absi et al. (2013):
  - Unit emission cap of the combination of production modes
  - Four models: (i) periodic, (ii) cumulative, (iii) global and (iv) rolling
  - Periodic polynomial, the rest $\mathcal{NP}$-complete

Complexity result

- Lot-sizing with an emission constraint is $\mathcal{NP}$-hard, even if only production emits pollutants and these production emissions are linear.
  - Reduction from KNAPSACK

- Consequence: lot-sizing *with two production modes* in each period is $\mathcal{NP}$-hard, even if only production emits pollutants (linearly) and either all (financial) costs or all emissions are time-invariant.



Lagrangian heuristic

$$\begin{aligned}
\min \quad & \sum_{t=1}^{T} \left(p_t(x_t) + h_t(I_t)\right) + \lambda \sum_{t=1}^{T} \left(\hat{p}_t(x_t) + \hat{h}_t(I_t) - \hat{C}\right) \\
= \quad & \sum_{t=1}^{T} \left(p_t(x_t) + \lambda\hat{p}_t(x_t) + h_t(I_t) + \lambda\hat{h}_t(I_t)\right) - \lambda\hat{C}
\end{aligned}$$

$$\text{s.t.} \quad \begin{array}{rcll}
I_t & = & I_{t-1} + x_t - d_t & t = 1, \ldots, T \\
x_t, I_t & \geq & 0 & t = 1, \ldots, T \\
I_0 & = & 0 & \\
\lambda & \geq & 0 &
\end{array}$$

- Dualise the emission constraint.
- For a given λ:
  - $p_t + \lambda\hat{p}_t$ and $h_t + \lambda\hat{h}_t$ are concave functions
  - $\lambda\hat{C}$ is a constant
  - we have a classic (uncapacitated) lot-sizing problem
  - solvable in $\mathcal{O}(T^2)$

Lagrangian heuristic

- For a given λ we have a classic (uncapacitated) lot-sizing problem.

$$
\begin{aligned}
z(\lambda) = \min \quad & \sum_{t=1}^{T} \Big(p_t(x_t) + \lambda \hat{p}_t(x_t) + h_t(I_t) + \lambda \hat{h}_t(I_t) \Big) \\
\text{s.t.} \quad & I_t = I_{t-1} + x_t - d_t \quad t = 1, \ldots, T \\
& x_t, I_t \geq 0 \quad t = 1, \ldots, T \\
& I_0 = 0
\end{aligned}
$$

- A lower bound and corresponding λ^* is found by solving

$$\max_{\lambda \geq 0} \{ z(\lambda) \}$$

- There is an algorithm (see Megiddo (1979), Gusfield (1983), Wagelmans (1990)) that:
  - finds an interval such that λ^* is one of the endpoints;
  - runs in $\mathcal{O}\left(T^4\right)$ time.
  - For λ^*, we are indifferent between two solutions, of which one is feasible and the other infeasible.
  - Thus, we get both a feasible solution and a lower bound.

Structural properties

We can show that an optimal solution satisfies the following properties:

- Let the production periods (sources) be given. If, for each period, the cheapest source to satisfy demand is also the cleanest:
  - single sourcing property holds
    (a period's demand is all procured from one production period)
  - Costs and emissions that satisfy this property are called *co-behaving*.
  - This includes the case with *Wagner-Whitin* (non-speculative) costs and emissions.
  - Also includes the case in which emissions are time-invariant and holding emissions are zero
    (or costs are time-invariant and holding costs are zero).

- In general:
  - Single sourcing in all but (at most) one period.

Pseudo-polynomial algorithm (co-behaving costs & emissions)

- Assume integer parameters.
- Minimise emissions under budget constraint.
- Because single sourcing property holds, we can extend Wagner and Whitin's algorithm.
- $f(t, B)$ gives the minimum emissions for periods t until T, given budget B.
- With $f(T+1, B) = 0$ with have for $1 \leq t \leq T$ the recursion:

$$f(t, B) = \min_{s>t:\ B \geq c(t,s)} \{e(t, s) + f(s+1, B - c(t, s))\} \tag{33}$$

$$\tag{34}$$

where
$$c(t, s) := p_t\left(D_{t,s}\right) + \sum_{\tau=t}^{s-1} h_\tau\left(D_{\tau,s}\right) \tag{35}$$

$$e(t, s) := \hat{p}_t\left(D_{t,s}\right) + \sum_{\tau=t}^{s-1} \hat{h}_\tau\left(D_{\tau,s}\right) \tag{36}$$

- $f(1, B)$ gives the minimum emissions given budget B.
- Try budget $B = 1, 2, 3, \ldots$ until minimum emissions $\leq$ emission cap.

Pseudo-polynomial algorithm (co-behaving costs & emissions)

- Recursion:

$$
\begin{aligned}
f(T+1,B) &= 0, \\
f(t,B) &= \min_{s>t:\ B\geq c(t,s)} \{e(t,s)+f(s+1,B-c(t,s))\} \ , \ t\leq T,
\end{aligned}
$$

where

$$
\begin{aligned}
c(t,s) &:= p_t\left(D_{t,s}\right)+\sum_{\tau=t}^{s-1} h_\tau\left(D_{\tau,s}\right) \\
e(t,s) &:= \hat{p}_t\left(D_{t,s}\right)+\sum_{\tau=t}^{s-1} \hat{h}_\tau\left(D_{\tau,s}\right)
\end{aligned}
$$

- $f(1,B)$ gives the minimum emissions given budget B.
- Try budget $B=1,2,3,\ldots$ until minimum emissions $\leq$ emission cap.
- Optimal production quantities can be found with a simple backtracking procedure.
- Required memory is $\mathcal{O}(T\,opt)$.
- Running time is $\mathcal{O}\left(T^2 opt\right)$ with *opt* the optimal objective value.

FPTAS for co-behaving costs & emissions

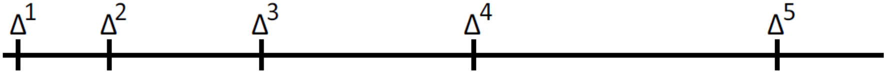
- We turn the pseudo-polynomial algorithm into a FPTAS by reducing the number of states.

$$f(t,B) = \min_{s>t:\ B\geq c(t,s)} \{e(t,s) + f(s+1, \text{round}(B - c(t,s)))\}$$

- Round down the budget to the nearest value of

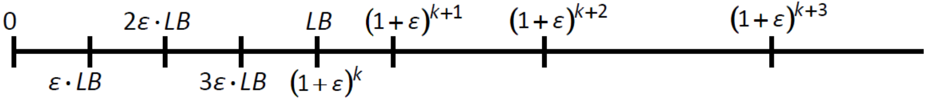
$$\Delta^k := \left(1 + \frac{\varepsilon}{(e-1)(T+1)}\right)^k$$

- Try budget $B = \Delta^1, \Delta^2, \Delta^3, \ldots$ until minimum emissions $\leq$ emission cap.
- This leads to an FPTAS with $\mathcal{O}\left(\frac{T^3 \ln(opt)}{\varepsilon}\right)$ running time and $\mathcal{O}\left(\frac{T^2 \ln(opt)}{\varepsilon}\right)$ memory consumption



Using the heuristic to speed up the FPTAS

- We can use the heuristic's LB to avoid small intervals at the start.
- Using this idea leads to improved running times:
  - $\mathcal{O}\left(\frac{T^3 \ln\left(\frac{opt}{LB}\right)}{\varepsilon}\right)$ for the FPTAS for WW costs and emissions;
  - $\mathcal{O}\left(\frac{T^4 \ln^2\left(\frac{opt}{LB}\right)}{\varepsilon^2}\right)$ for the general FPTAS.



Computational tests: data

- We generated 1800 problem instances to test all algorithms.
  - CPLEX 10.1 for comparison
  - 'natural' formulation and shortest path reformulation
- All costs and emissions were fixed-plus-linear.
- Three 'degrees of co-behaviour':
  - co-behaving costs and emissions
  - $\frac{1}{2}T$ pairs (t, s) violate the co-behaviour property
  - all periods form pairs, such that the instance corresponds to a problem with $\frac{1}{2}T$ periods with two production modes: 'cheap & dirty' and 'expensive & clean'
- $T = 25, 50, 100$
- $\varepsilon = 0.10, 0.05, 0.01$

Computational tests: results (algorithms for co-behaving inst.)

	T	25	50	100
Megiddo	Avg. sol. time (s)	0.001	0.013	0.16
	Avg. post. gap (%)	1.5	0.85	0.41
	Avg. true gap (%)	0.47	0.41	0.26
FPTAS-CB-LB(0.1)	Avg. sol. time (s)	0.002	0.019	0.20
	Avg. post. gap (%)	0.81	0.44	0.17
	Avg. true gap (%)	0.021	0.024	0.015
FPTAS-CB-LB(0.05)	Avg. sol. time (s)	0.003	0.024	0.24
	Avg. post. gap (%)	0.55	0.34	0.16
	Avg. true gap (%)	0.0022	0.0067	0.0060
FPTAS-CB-LB(0.01)	Avg. sol. time (s)	0.009	0.067	0.59
	Avg.post. gap (%)	0.15	0.12	0.075
	Avg.true gap (%)	0.00044	0.00016	0.00014
FPTAS-CB(0.1)	Avg. sol. time (s)	0.008	0.052	0.35
	Avg. post. gap (%)	3.4	3.4	3.5
	Avg. true gap (%)	0.010	0.020	0.017
FPTAS-CB(0.05)	Avg. sol. time (s)	0.018	0.11	0.77
	Avg. post. gap (%)	1.7	1.7	1.7
	Avg. true gap (%)	0.0021	0.0054	0.0042
FPTAS-CB(0.01)	Avg. sol. time (s)	0.093	0.67	5.2
	Avg. post. gap (%)	0.33	0.34	0.34
	Avg. true gap (%)	0.000088	0.00015	0.00015
PP-CB	Avg. sol. time (s)	0.24	1.8	22
CPLEX 10.1 Nat.	Avg. sol. time (s)	0.045	0.44	–
CPLEX 10.1 SP	Avg. sol. time (s)	0.030	0.069	0.22

Computational tests: results (general algorithms)

	T	25	25	26	50	50	50	100	100	100
	data set	co-bhv.	gen.	2 modes	co-bhv.	gen.	2 modes	co-bhv.	gen.	2 modes
Megiddo	Avg. sol. time (s)	0.001	0.001	0.001	0.013	0.013	0.014	0.16	0.16	0.16
	Avg. post. gap (%)	1.5	2.8	12	0.85	1.3	6.2	0.41	0.61	2.8
	Avg. true gap (%)	0.47	1.2	6.1	0.41	0.74	3.8	0.26	0.41	2.1
FPTAS-gen-LB(0.1)	Avg. sol. time (s)	0.006	0.010	0.028	0.082	0.12	0.26	1.3	1.6	2.8
	Avg. post. gap (%)	1.0	1.6	3.7	0.45	0.62	2.3	0.16	0.21	0.69
	Avg. true gap (%)	0.0053	0.070	0.028	0.0065	0.028	0.042	0.0048	0.017	0.0080
FPTAS-gen-LB(0.05)	Avg.sol.time(s)	0.010	0.019	0.59	0.14	0.22	0.57	2.4	3.0	5.9
	Avg. post. gap (%)	0.92	1.4	2.3	0.44	0.61	1.9	0.16	0.21	0.69
	Avg. true gap (%)	0.00055	0.043	0.028	0.0011	0.025	0.038	0.0014	0.014	0.0080
FPTAS-gen-LB(0.01)	Avg.sol.time(s)	0.042	0.13	0.66	0.63	1.4	6.9	11	17	58
	Avg. post. gap (%)	0.41	0.46	0.54	0.31	0.38	0.53	0.15	0.20	0.46
	Avg. true gap (%)	0.000014	0.012	0.013	0.000080	0.014	0.0048	0.0000087	0.011	0.0076
FPTAS-gen(0.1)	Avg. sol. time (s)	0.051	0.26	0.75	0.68	2.4	8.5	11	25	68
	Avg. post. gap (%)	6.6	6.6	6.4	6.6	6.6	6.4	6.6	6.6	6.4
	Avg. true gap (%)	0.0042	0.038	0.021	0.0048	0.023	0.0063	0.0046	0.016	0.011
FPTAS-gen(0.05)	Avg. sol. time (s)	0.10	0.90	2.9	1.4	8.5	32	22	72	234
	Avg. post. gap (%)	3.3	3.3	3.2	3.3	3.3	3.2	3.3	3.3	3.2
	Avg.true gap (%)	0.00048	0.018	0.0047	0.00084	0.018	0.015	0.0017	0.012	0.0069
FPTAS-gen(0.01)	Avg. sol. time (s)	0.56	23	69	7.3	191	765	113	1280	5163
	Avg. post. gap (%)	0.67	0.66	0.65	0.67	0.66	0.64	0.67	0.66	0.64
	Avg. true gap (%)	0.000027	0.0057	0.00073	0.000064	0.0068	0.0026	0.000049	0.0044	0.0040
CPLEX 10.1 Nat.	Avg. sol. time (s)	0.045	0.041	0.035	0.44	0.38	0.12	–	–	–
CPLEX 10.1 SP	Avg. sol. time (s)	0.030	0.031	0.053	0.069	0.072	0.14	0.22	0.26	0.55

Part III: Bi-Objective Lot-Sizing Problems

Data:

- T : planning horizon indexed by t
- d_t : demand ($d_{t,\tau} = \sum_{s=t}^{\tau} d_s$)
- (f_t, c_t, h_t) : (setup, unit production, unit inventory) costs
- $(\hat{f}_t, \hat{c}_t, \hat{h}_t)$: (setup, unit production, unit inventory) emissions
- ℓ : length of the emission block

Decisions:

- (y_t, x_t, I_t) : (setup, production, inventory) decisions

Different levels of aggregation for emissions

Partitioning the planning horizon:

- We partition the planning horizon into blocks of length ℓ
- We minimize the max emission over the blocks (T/ℓ blocks in total)
- Different levels of aggregation depending on ℓ

Special cases:

- Whole-horizon emissions ($\ell = T$)
- Period emissions ($\ell = 1$)

The Bi-Objective Economic Lot-Sizing Problem

$$\text{minimize} \left(\overbrace{\sum_{t=1}^{T}[f_t y_t + c_t x_t + h_t I_t]}^{\text{Costs: } g^{\mathrm{C}}()}, \overbrace{\max_{i=1,\ldots,T/\ell} \sum_{t=(i-1)\ell+1}^{i\ell} [\hat{f}_t y_t + \hat{c}_t x_t + \hat{h}_t I_t]}^{\text{Block emissions: } g^{\mathrm{E}}()} \right)$$

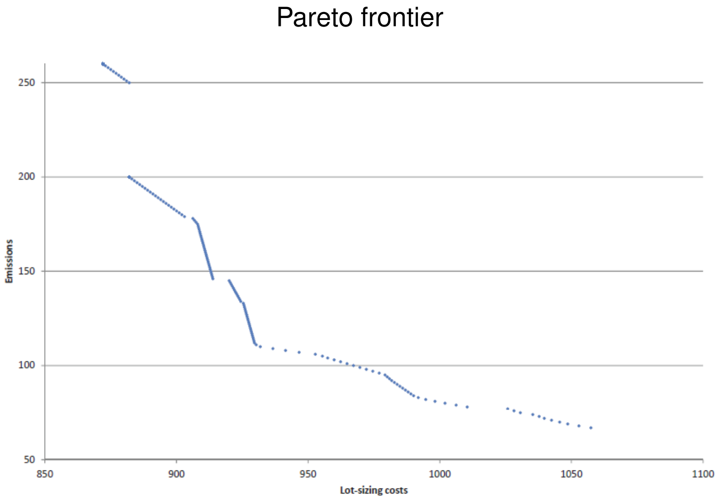
subject to (BOLS$^{(\ell)}$)

$$\begin{aligned} x_t + I_{t-1} &= d_t + I_t & t &= 1, \ldots, T \\ x_t &\leq d_{1,T} y_t & t &= 1, \ldots, T \\ I_0 &= 0 & & \\ y_t &\in \{0, 1\} & t &= 1, \ldots, T \\ x_t, I_t &\geq 0 & t &= 1, \ldots, T \end{aligned}$$

The Pareto frontier

Example instance with $T = 15$:

demands	cost parameters	emission parameters
(10,10,4,2,1,40,10,10,10,10,10,3,4,7,3)	(25,5,1)	(0, 5, 1)



Pareto efficient outcomes

Pareto efficient outcomes are found by, for each $\hat{b} \in \mathbb{R}_+$, solving

$$\text{minimize} \sum_{t=1}^{T} [f_t y_t + c_t x_t + h_t I_t]$$

subject to $(\mathcal{P}^{(\ell)}(\hat{b}))$

$$\begin{aligned}
x_t + I_{t-1} &= d_t + I_t && t = 1, \ldots, T \\
x_t &\leq d_{1,T} y_t && t = 1, \ldots, T \\
I_0 &= 0 \\
y_t &\in \{0, 1\} && t = 1, \ldots, T \\
x_t, I_t &\geq 0 && t = 1, \ldots, T \\
\sum_{t=(i-1)\ell+1}^{i\ell} [\hat{f}_t y_t + \hat{c}_t x_t + \hat{h}_t I_t] &\leq \hat{b} && i = 1, \ldots, T/\ell
\end{aligned}$$

ε-dominating sets

Difficulty of multi-objective problems:

- Describing the Pareto frontier is challenging in Multi-Objective Combinatorial Optimization Ulungu and Teghem (1994); Ehrgott and Gandibleux (2000, 2004)
- Focus on ε-dominating set in the outcome space, see e.g. Blanquero and Carrizosa (2002)

Definition: Z^* is an ε*-dominating set* for

$$\min_{z \in Z} \quad (g^{\mathrm{C}}(z), g^{\mathrm{E}}(z))$$

if, for each $z \in Z$, there exists a $z^* \in Z^*$ such that

$$\begin{aligned} g^{\mathrm{C}}(z^*) &\leq g^{\mathrm{C}}(z) + \varepsilon \text{ and} \\ g^{\mathrm{E}}(z^*) &\leq g^{\mathrm{E}}(z) + \varepsilon \end{aligned}$$

Finding an ε-dominating set

Let $\hat{b}^{\min}$ ($\hat{b}^{\max}$) be the minimum (maximum) value of $\hat{b}$

Simple algorithm for finding an ε-dominating set

Step 0. Calculate $\hat{b}^{\min}$ and $\hat{b}^{\max}$
Let $\{\hat{b}^i\}$ be a grid of $[\hat{b}^{\min}, \hat{b}^{\max}]$ of width ε
Set $Z^* = \emptyset$

Step 1. $\forall i$, solve ($\mathcal{P}^{(\ell)}(\hat{b}^i)$) and add its optimal solution to Z^*

Goal:

- finding efficient algorithms for ($\mathcal{P}^{(\ell)}(\hat{b})$)
- we will focus on special cases with time-invariant cost parameters

Time-invariant whole-horizon case

$(\mathcal{P}^{(T)}(\hat{b}))$ is equivalent to solving lot-sizing problems with a fixed number of setups n, $\forall n = 1, \ldots, T$,

$$\text{minimize } \sum_{t=1}^{T} I_t$$

subject to

$$\begin{aligned}
x_t + I_{t-1} &= d_t + I_t & t = 1, \ldots, T \\
x_t &\leq d_{1,T} y_t & t = 1, \ldots, T \\
I_0 &= 0 & \\
y_t &\in \{0, 1\} & t = 1, \ldots, T \\
x_t, I_t &\geq 0 & t = 1, \ldots, T \\
\sum_{t=1}^{T} y_t &= n &
\end{aligned}$$

Polynomial time algorithm (van Hoesel and Wagelmans, 2000)

Whole-horizon case: complexity overview

The same algorithm works for

ℓ	costs	emissions			running time
T	$f_t = f$, $c_t = c$	$\hat{f}_t = \hat{f}$	$\hat{c}_t = \hat{c}$	$\hat{h}_t = \alpha h_t$	$\mathcal{O}(T^2)$

Complexity overview:

costs		emissions			complexity
f_t	c_t and h_t	$\hat{f}_t$	$\hat{c}_t$	$\hat{h}_t$	
c	non-speculative	c	c	αh_t	polynomially solvable
c	non-speculative	v	0	0	$\mathcal{NP}$-hard
c	non-speculative	0	v	0	$\mathcal{NP}$-hard
c	non-speculative	0	0	c	$\mathcal{NP}$-hard

Our results for the whole-horizon case are tight

Time-invariant period case

Main approach:

- decomposition the problem in subplans $[u, v]$
- compute the cost of a subplan efficiently

Definitions:

- A block/period is called **tight** if its emission constraint is binding
- A block/period is called **extreme** if either tight or no production

Properties:

- The only possible non-tight production period is u
- Consider t $(u < t \leq v)$ with inventory I_t such that:
  - $\bar{x}_t := (\hat{b} - \hat{f} - \hat{h}I_t)/\hat{c} > 0,$
  - $\bar{I}_{t-1} := I_t - \bar{x}_t + d_t > 0.$

  Then t is a tight production period with production quantity $\bar{x}_t$, and incoming inventory equal to $\bar{I}_{t-1}$.

Main result

The optimal cost of all subplans can be calculated in $\mathcal{O}(T^2)$ time by a backward DP algorithm.

Period case: complexity overview

The same algorithm works for

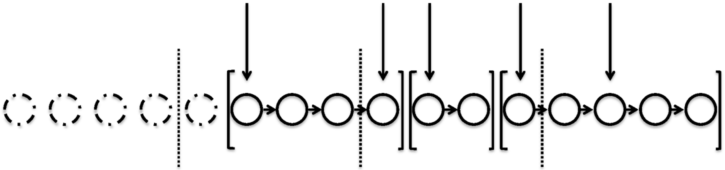
ℓ	costs	emissions			running time
1	$f_t \geq f_{t+1}$, non-speculative	$\hat{f}_t = \hat{f}$	$\hat{c}_t = \hat{c}$	$\hat{h}_t = \hat{h}$	$\mathcal{O}(T^2)$

Complexity overview:

costs		emissions			complexity
f_t	c_t and h_t	$\hat{f}_t$	$\hat{c}_t$	$\hat{h}_t$	
non-increasing	non-speculative	c	c	c	polynomially solvable
c	non-speculative	v	c	0	$\mathcal{NP}$-hard
c	non-speculative	0	v	0	$\mathcal{NP}$-hard
c	non-speculative	0	0	v	open

Our results for the period case are almost tight

Time-invariant block case: production and setup emissions



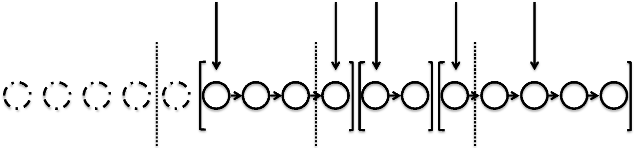
Definitions:

- $b(v)$ ($e(v)$) denotes the first (last) period of the block that contains v
- Two consecutive subplans are called **connected** if they have production in the same block.
- A sequence of connected subplans is called a **maximal sequence of subplans** if it does not contain a smaller sequence.

Proposition:

- Except for the first, all blocks fully contained in a maximal sequence of connected subplans $[t, w]$ are extreme. If the last block is split, then there is no production in $[b(w), w]$

Block case: production and setup emissions



General solution approach:

- Decompose solution into maximal sequences of connected subplans
- Decompose maximal sequences in subplans
- Compute the cost of a subplan efficiently

Main result:

- We can derive a DP algorithm to compute the subplan costs using:
  - $(u, v; w)$, a subplan $[u, v]$ contained in a maximal sequence ending in period w
  - keeping track of the number of production blocks and setups.
- The optimal cost of all subplans can be calculated in $\mathcal{O}(T^7/\ell)$ time by a backward DP algorithm.

Block emissions: complexity overview

Complexity overview in case of block emissions:

costs		emissions			complexity
f_t	c_t and h_t	$\hat{f}_t$	$\hat{c}_t$	$\hat{h}_t$	
v	non-speculative	c	c	0	polynomially solvable
c	non-speculative	v	0	0	$\mathcal{NP}$-hard
c	non-speculative	0	v	0	$\mathcal{NP}$-hard
c	non-speculative	0	0	c	$\mathcal{NP}$-hard

Our results for the block case are tight

Conclusions & further research

1. **Lot-sizing with remanufacturing:**
   - ELSRs and ELSRj are both $\mathcal{NP}$-hard.
   - Computational tests indicate that reformulations SP and PSP perform well

   - Extend shortest path formulations with capacities
   - Use solution of LP relaxation in a rounding heuristic
   - How to incorporate stochastic returns?
2. **Lot-sizing with emission constraint:**
   - is $\mathcal{NP}$-hard
   - A Lagrangean heuristic gives a LB as well as a feasible solution in $\mathcal{O}\left(T^4\right)$.
   - There is an FPTAS which works faster in case of *co-behaviour* property.

   - applying the same technique to create FPTASes for other problems
3. **Bi-objective lot-sizing:**
   - Special classes of problem instances are polynomially solvable
   - We have shown the tightness of our polynomiality results
   - Results can be used to find an ϵ-dominating set

   - Close the complexity gap
   - Overlapping blocks case
   - Algorithms for the $\mathcal{NP}$-complete cases

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