Tutorial identification hyperelastic constants

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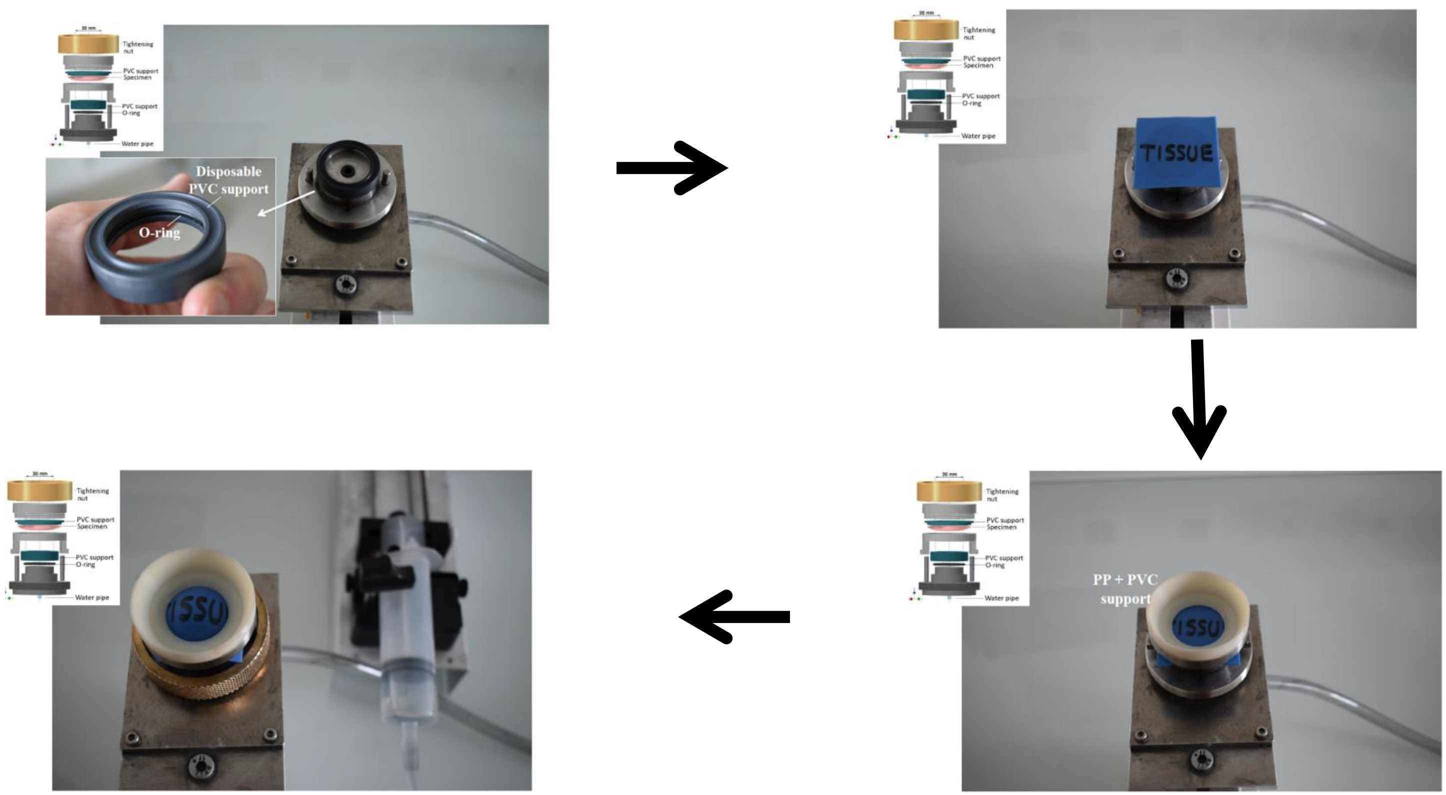
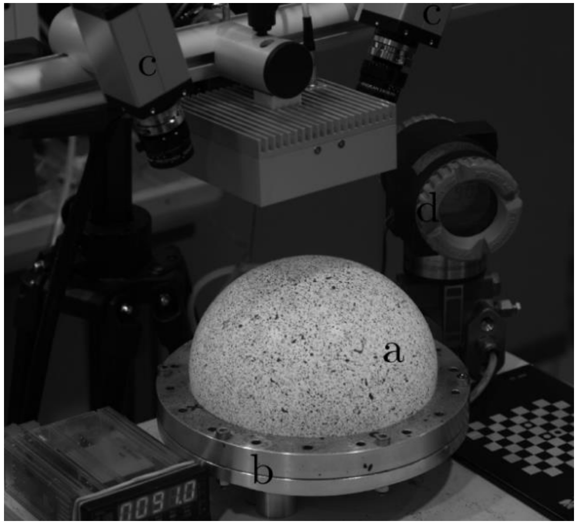
Introduction

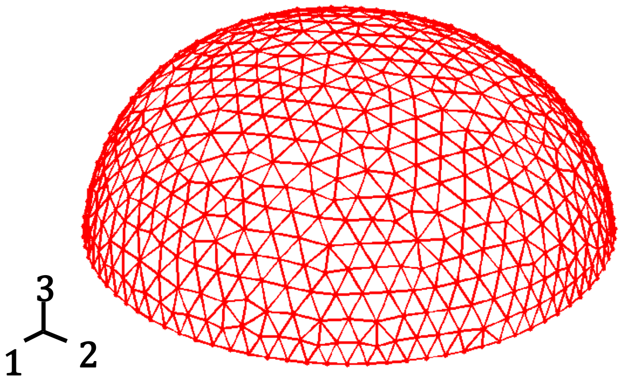
- The objective of this tutorial is to identify the parameters of a hyperelastic constitutive law of an tissue sample which is tested in bulge inflation as in the experiment shown below



Data of the problem

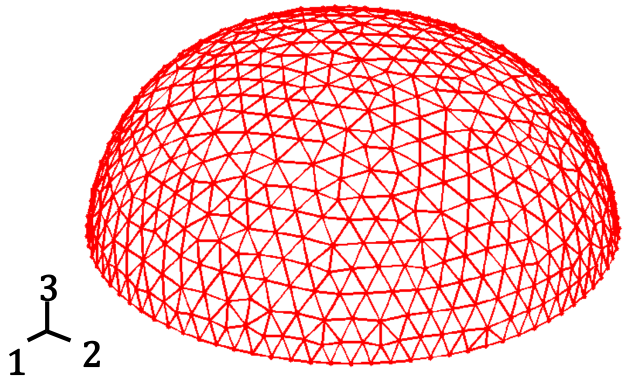
- The displacement field is measured during the stereo image correlation swelling test.





Data of the problem

- A cloud of 644 points is defined in the field of view in initial position which is defined as the reference configuration. The coordinates in the reference configuration are (x_0, y_0, z_0). For each point, the digital image stereo-correlation method can reconstruct the coordinates (x, y z) for 43 loading steps. Each loading step corresponds to a pressure P applied in the system.
- The problem data are available in Matlab format in the data_TP.mat file. You will find there (x_0, y_0, z_0), (x, y z) and P.



Mechanical response

- Plot the curve of the variation of P as a function of the maximum value of z.
- The inverse problem consists in finding a constitutive law for the membrane which would reproduce the same behavior curve. One solution would be to develop a finite element model of the bulge inflation test and then to update the parameters of the model so as to minimize the difference between the measured displacements and the displacements predicted by the model.
- We will develop an alternative approach using the VFM.

Differentiation in the local basis

- We define a triangular mesh from the point cloud (x_0, y_0, z_0), each point being a node.
- The connectivity matrix of this mesh is given in the form of a Matlab matrix called TRI.
- We will reconstruct at the center of gravity of each triangle the deformation gradient and the Green Lagrange strains from the available data. They are provided in DEF_GRADIENT.mat and EPSILON.mat if necessary.
- The code is provided (calcul_F.m) as well as the results if necessary in Matlab matrices.
- Run calcul_F.m.

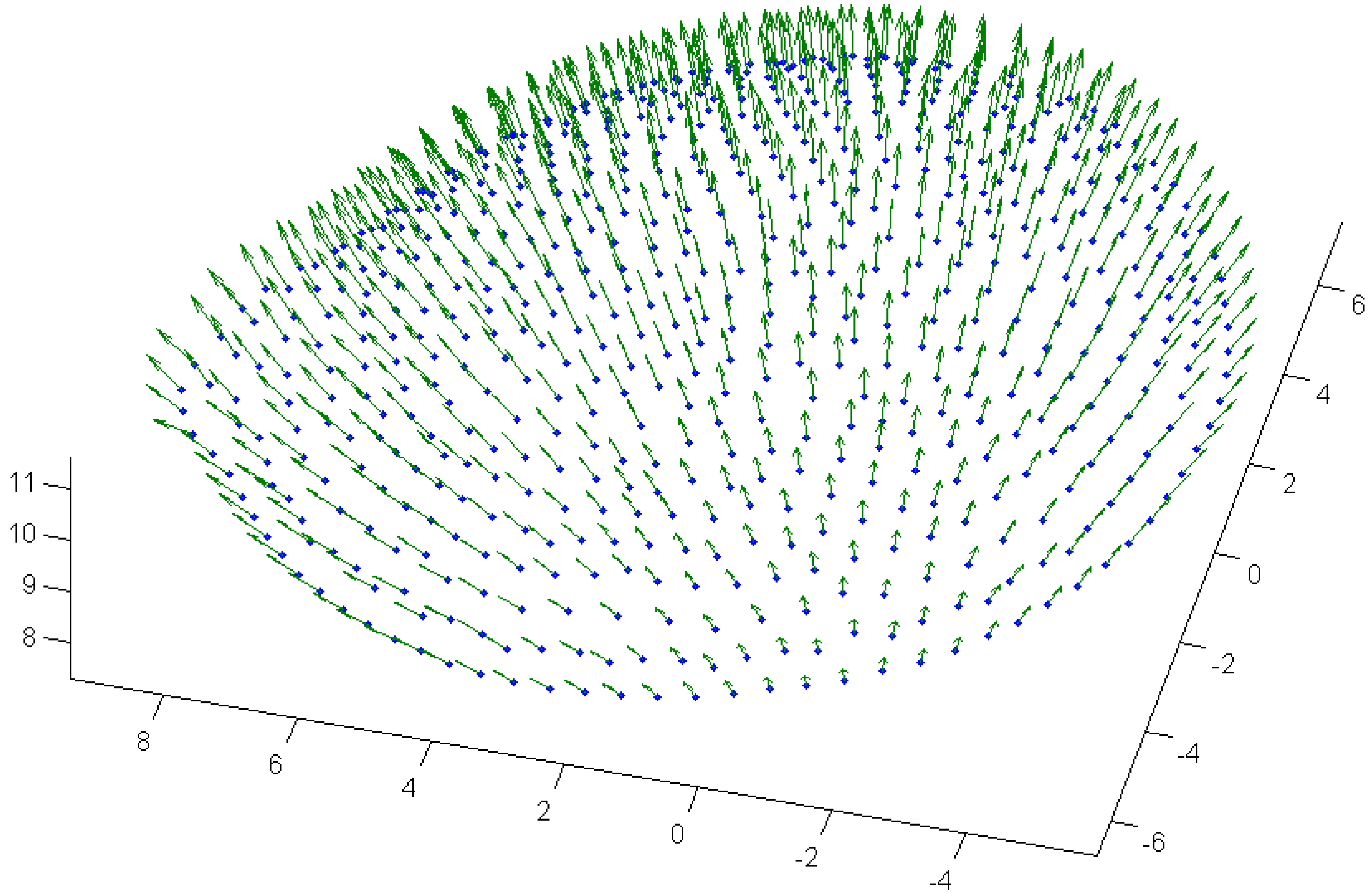
Important variables created by calcul_F.m

- xGdeb, yGdeb, zGdeb: coordinates of the centers of gravity of the triangles in the initial state
- xGfin, yGfin, zGfin: coordinates of the centers of gravity of the triangles in the final state
- vect3: vector normal to the surface in the initial state
- vect3b: vector normal to the surface in the final state
- F_tensor: contains F11, F22, F12 and F22 (components of the deformation gradient) for each triangle and each pressure value
- F_tensor: contains E11, E22, E12 (components of the Green Lagrange strains) for each triangle and each pressure value

Normal vectors

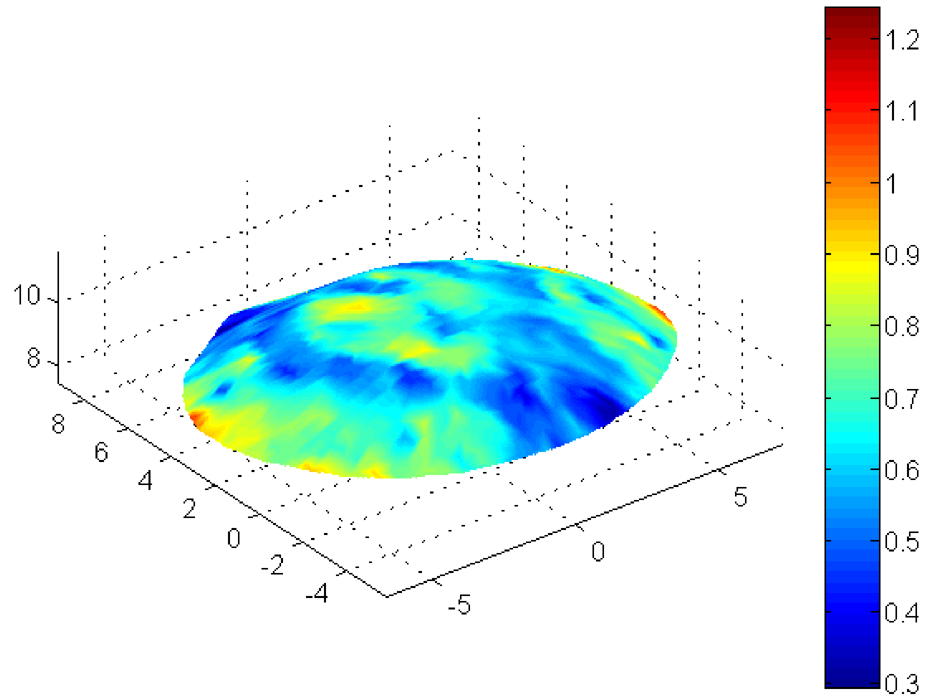
- Why did we calculate the normal vectors at the surface?
- From the norms, one can define a local coordinate system of the membrane and then calculate the deformation gradient and the Green Lagrange strains.
- How are normal vectors calculated?
- We compute the normal vectors at the surface at each node from the data. This is done by fitting the surface locally by a quadric.
- Matlab code is provided: initial_normals.m and final_normals.m

Normal vectors



Strain fields

- Represent graphically the obtained fields (E11, E22, E12,… at different pressure values) in color maps using the function provided (Myplot.m). What do we see?



Stress fields

- Assume that the behavior of the tissue has a NeoHookean hyperelastic behavior and that its hyperelastic constant is 1 MPa.
- Find the expression of the stress as a function of the strain for an incompressible NeoHookean material.

$$\underline{\underline{\sigma}} = 2C_{10}\,\underline{\underline{F}}.{}^{t}\underline{\underline{F}} + c\underline{\underline{I}}$$

- Reconstruct the Cauchy stress field in the local coordinate system.
- To check if the stresses obtained are at equilibrium, we will write the principle of virtual powers at each loading step.

Field of virtual velocities

- The virtual velocity field which is proposed here is a “swelling” field.
- Each point in the tissue sample is moved b virtually in the normal direction.
- Use calcul_def_virt.m to find the field of virtual strain velocities at each stage of loading.
- These virtual strain rates are reconstructed in the deformed local coordinate system (Eulerian expression).
- They are provided in EPSILON_VIRT.mat if necessary.

Principle of virtual power

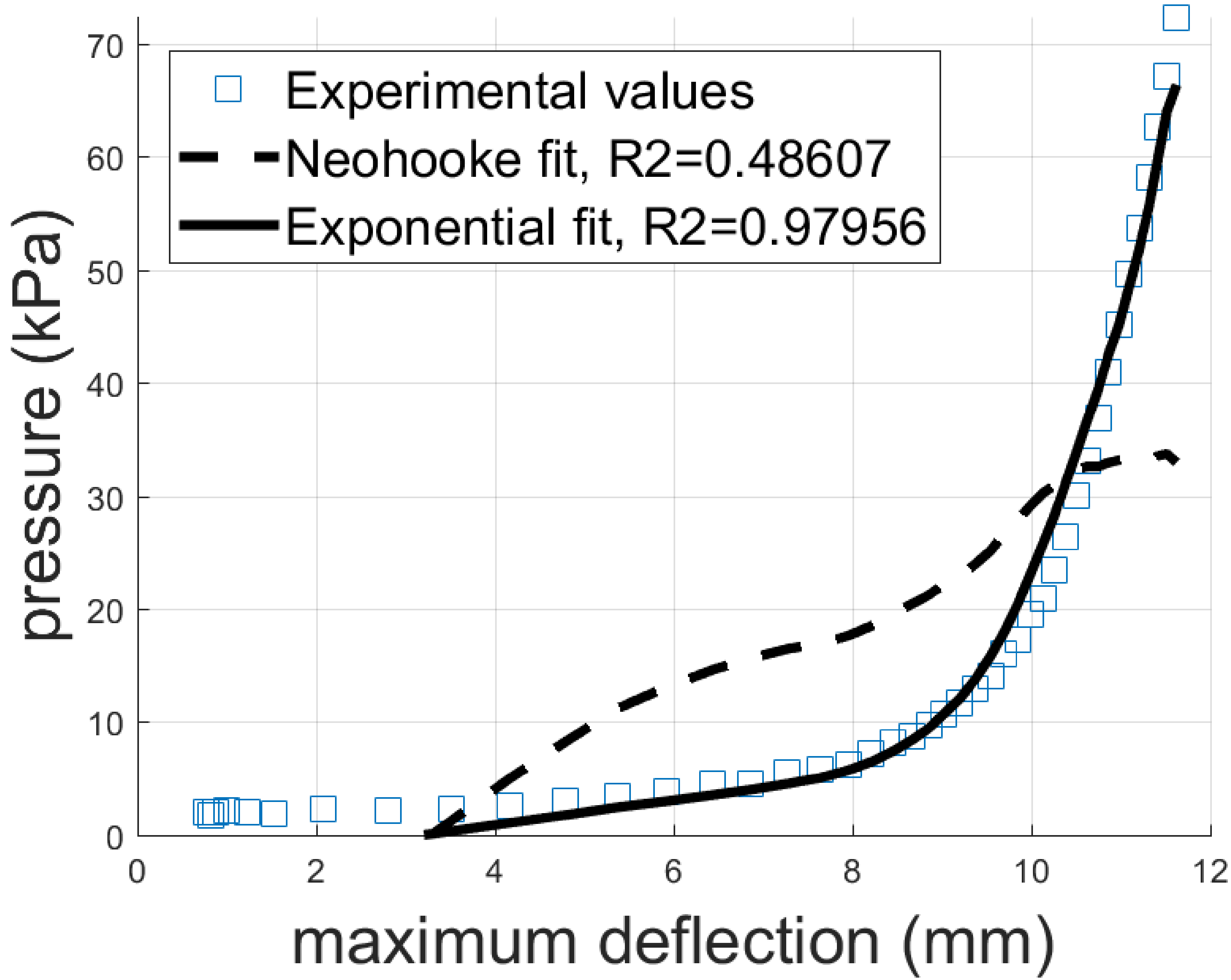
- Reconstruct the internal virtual power vector over the entire measurement area.
- Reconstruct the power of the external forces by showing that it reduces to the virtual power of the forces of pressure.
- Plot the variations of these virtual powers (interior and exterior) according to the different stages of loading.

Identification of a hyperelastic constant

- Set up a least squares method to identify the hyperelastic constant of the NeoHookean model.
- This is equivalent to fit the external virtual power curve by the internal virtual power curve, which depends on the constant to be determined.
- After identifying the constant, calculate the coefficient of determination R2.

Identification of other models

- In order to improve R2, perform the identification with other more suitable hyperelastic behavior laws (for example, Ogden, Holzapfel).
- Set up least squares regression for one of these models.



Experimental values
Neohooke fit, R2=0.48607
Exponential fit, R2=0.97956
pressure (kPa)
maximum deflection (mm)
0
10
20
30
40
50
60
70
0
2
4
6
8
10
12

Spatial variations of hyperelastic constants

- Show that the identification method that you have just implemented can be applied to any area of the sample to reconstruct the spatial distribution of hyperelastic constant.
- Under which assumptions is the developed method valid?
- Under what conditions is this assumption likely to be called into question?
- What to do if this assumption is not valid.