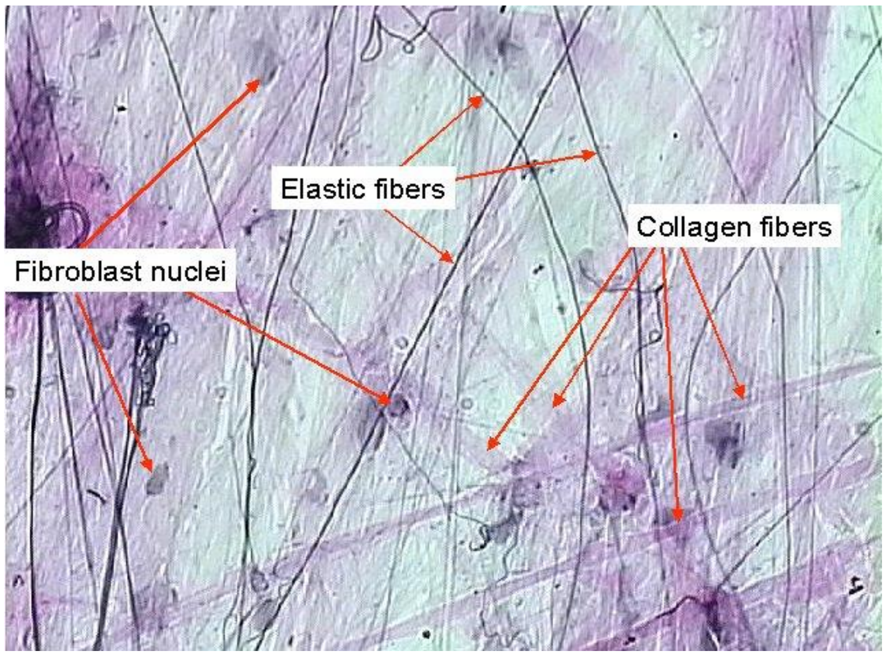
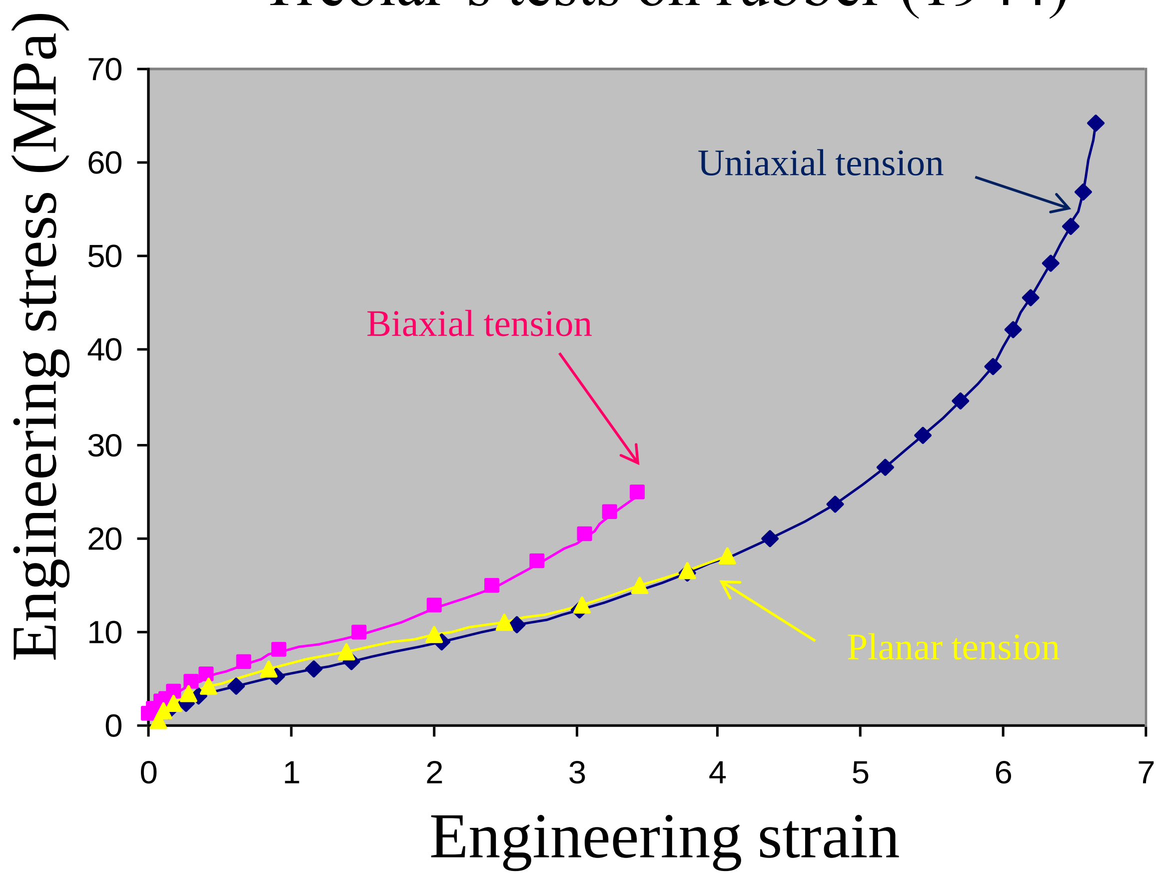
# Mechanics of soft biological tissues

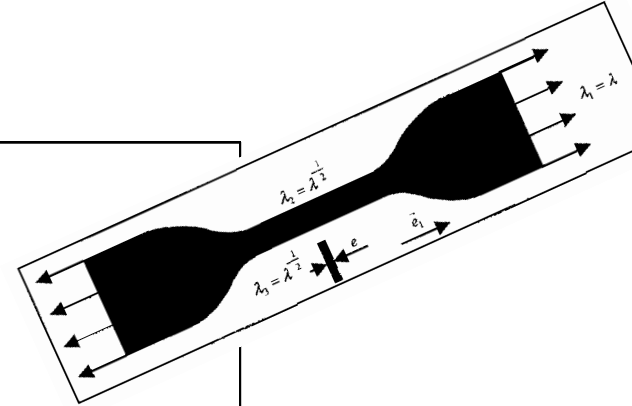

Prof Stéphane AVRIL

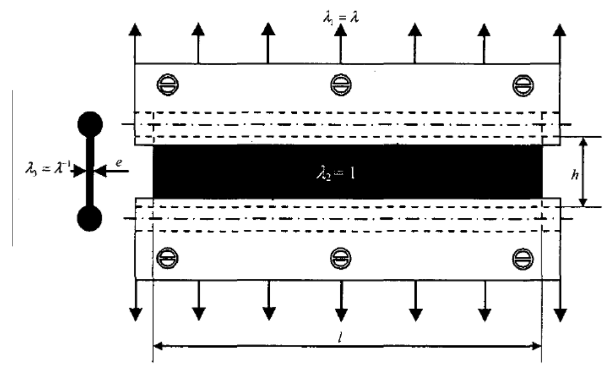
# Soft biological tissues: many challenges for continuum mechanics

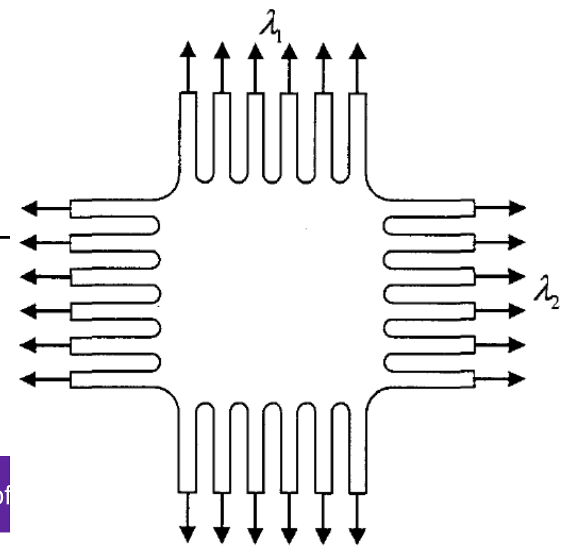



# Treolar’s tests on rubber (1944)







# Identification approach

We choose a constitutive model and we determine the parameters so as to minimize the following cost function:

$$E = \sum_{i=1}^{n} \left( \frac{F_i^{mes} - F_i^{th}}{\overline{F}_i^{mes}} \right)^2$$

$F_i^{mes}$ are the experimentally measured forces

$\overline{F}_i^{mes}$ is their average

$F_i^{th}$ are the model predictions of the forces

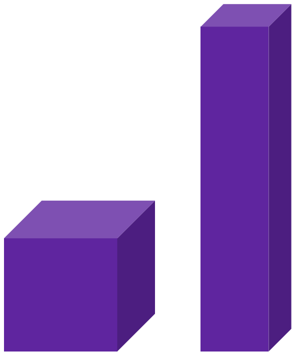

avril@emse.fr

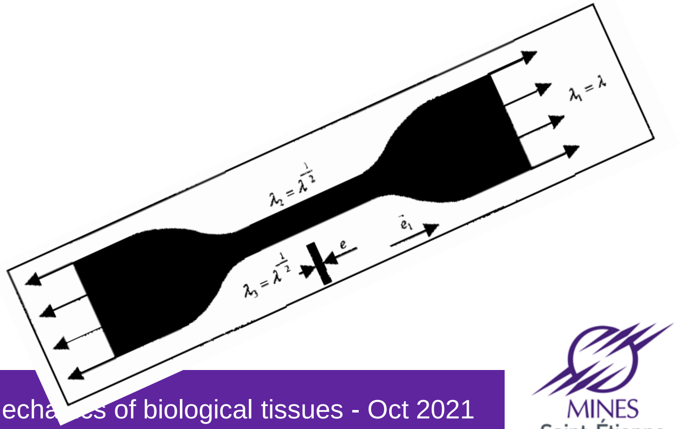
# Identification procedure

Uniaxial tension

Equibiaxial tension

Planar tension



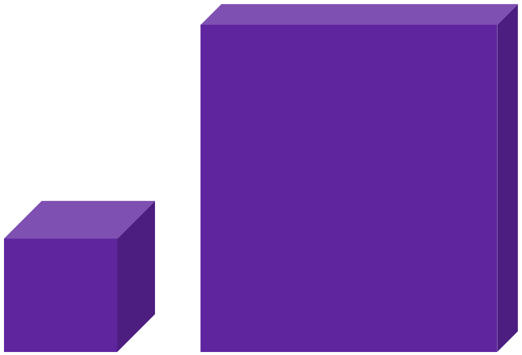


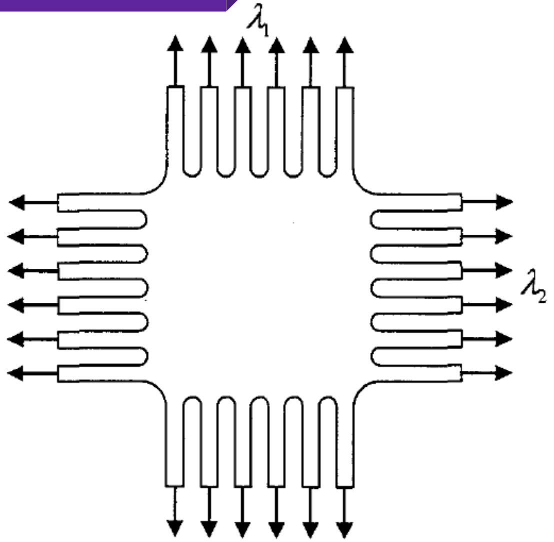
# Identification procedure

Uniaxial tension

Equibiaxial tension

Planar tension



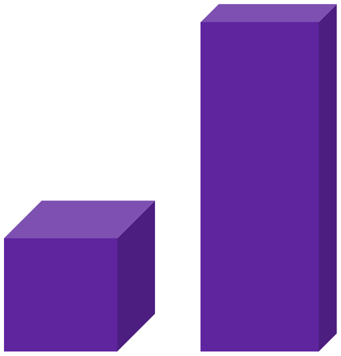


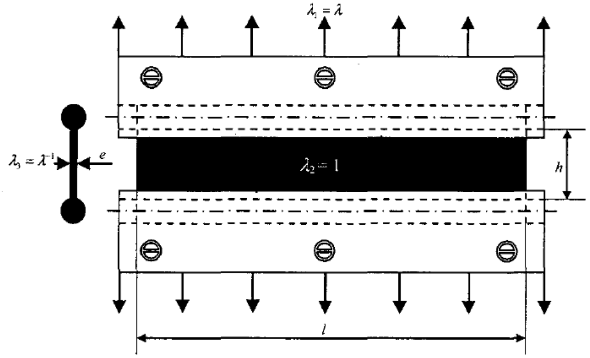
# Identification procedure

Uniaxial tension

Equibiaxial tension

Planar tension





# Uniaxial tension

$$\underline{\underline{F}} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda^{-\frac{1}{2}} & 0 \\ 0 & 0 & \lambda^{-\frac{1}{2}} \end{bmatrix}$$

$$\underline{\underline{\sigma}} = \begin{bmatrix} \frac{\lambda F}{S_0} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$I_1 = \lambda^2 + 2\lambda^{-1} \quad I_2 = \lambda^{-2} + 2\lambda$$

$$\underline{\underline{\sigma}} = \left[\left(2\frac{\partial\Psi}{\partial I_1} + 2I_1\frac{\partial\Psi}{\partial I_2}\right)\underline{\underline{F}}.{}^t\underline{\underline{F}} - 2\frac{\partial\Psi}{\partial I_2}\underline{\underline{F}}.{}^t\underline{\underline{F}}.\underline{\underline{F}}.{}^t\underline{\underline{F}}\right] + c\underline{\underline{I}}$$


avril@emse.fr

# Uniaxial tension

$$\frac{\lambda F}{S_0} = \left[\left(2\frac{\partial\Psi}{\partial I_1} + 2\left(\lambda^2 + \frac{2}{\lambda}\right)\frac{\partial\Psi}{\partial I_2}\right)\left(\lambda^2 - \lambda^{-1}\right) - 2\frac{\partial\Psi}{\partial I_2}\left(\lambda^4 - \lambda^{-2}\right)\right]$$

$$\boxed{\frac{F}{S_0} = 2\left(1 - \lambda^{-3}\right)\left(\lambda\frac{\partial\Psi}{\partial I_1} + \frac{\partial\Psi}{\partial I_2}\right)}$$

Remark:

$$I_1 = \lambda^2 + 2\lambda^{-1} \qquad I_2 = \lambda^{-2} + 2\lambda$$

$$\frac{dI_1}{d\lambda} = 2\lambda\left(1 - \lambda^{-3}\right) \qquad \frac{dI_2}{d\lambda} = 2\left(1 - \lambda^{-3}\right)$$

$$\boxed{\frac{F}{S_0} = \frac{\partial\Psi}{\partial\lambda}}$$

# Equibiaxial tension

$$\underline{\underline{F}} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^{-2} \end{bmatrix}$$

$$I_1 = 2\lambda^2 + \lambda^{-4} \qquad I_2 = 2\lambda^{-2} + \lambda^4$$

$$\underline{\underline{\sigma}} = \begin{bmatrix} \dfrac{\lambda F}{S_0} & 0 & 0 \\ 0 & \dfrac{\lambda F}{S_0} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underline{\underline{\sigma}} = \left[ \left( 2\frac{\partial \Psi}{\partial \mathrm{I}_1} + 2\mathrm{I}_1 \frac{\partial \Psi}{\partial \mathrm{I}_2} \right) \underline{\underline{\mathrm{F}}}.^t \underline{\underline{\mathrm{F}}} - 2\frac{\partial \Psi}{\partial \mathrm{I}_2} \underline{\underline{\mathrm{F}}}.^t \underline{\underline{\mathrm{F}}}.\underline{\underline{\mathrm{F}}}.^t \underline{\underline{\mathrm{F}}} \right] + c\underline{\underline{\mathrm{I}}}$$

# Equibiaxial tension

$$\frac{\lambda F}{S_0} = \left[ \left( 2\frac{\partial \Psi}{\partial I_1} + 2\left(2\lambda^2 + \lambda^{-4}\right)\frac{\partial \Psi}{\partial I_2} \right)\left(\lambda^2 - \lambda^{-4}\right) - 2\frac{\partial \Psi}{\partial I_2}\left(\lambda^4 - \lambda^{-8}\right) \right]$$

$$\boxed{\frac{F}{S_0} = 2\left(\lambda - \lambda^{-5}\right)\left( \frac{\partial \Psi}{\partial I_1} + \lambda^2 \frac{\partial \Psi}{\partial I_2} \right)}$$

Remark:

$$I_1 = 2\lambda^2 + \lambda^{-4} \qquad I_2 = 2\lambda^{-2} + \lambda^4$$

$$\frac{dI_1}{d\lambda} = 4\left(\lambda - \lambda^{-5}\right) \qquad \frac{dI_2}{d\lambda} = 4\lambda^2\left(\lambda - \lambda^{-5}\right)$$

$$\boxed{\frac{F}{S_0} = \frac{1}{2}\frac{\partial \Psi}{\partial \lambda}}$$

# Planar tension

$$\underline{\underline{F}} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda^{-1} \end{bmatrix}$$

$$\underline{\underline{\sigma}} = \begin{bmatrix} \frac{\lambda F}{S_0} & 0 & 0 \\ 0 & ? & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$I_1 = \lambda^2 + \lambda^{-2} + 1 \quad I_2 = \lambda^2 + \lambda^{-2} + 1$$

$$\underline{\underline{\sigma}} = \left[ \left( 2\frac{\partial \Psi}{\partial I_1} + 2 I_1 \frac{\partial \Psi}{\partial I_2} \right) \underline{\underline{F}}.{}^t\underline{\underline{F}} - 2\frac{\partial \Psi}{\partial I_2} \underline{\underline{F}}.{}^t\underline{\underline{F}}.\underline{\underline{F}}.{}^t\underline{\underline{F}} \right] + c\underline{\underline{I}}$$



MINES Saint-Étienne

# Planar tension

$$\frac{\lambda F}{S_0} = \left[\left(2\frac{\partial \Psi}{\partial I_1} + 2\left(\lambda^2 + \lambda^{-2} + 1\right)\frac{\partial \Psi}{\partial I_2}\right)\left(\lambda^2 - \lambda^{-2}\right) - 2\frac{\partial \Psi}{\partial I_2}\left(\lambda^4 - \lambda^{-4}\right)\right]$$

$$\boxed{\frac{F}{S_0} = 2\left(\lambda - \lambda^{-3}\right)\left(\frac{\partial \Psi}{\partial I_1} + \frac{\partial \Psi}{\partial I_2}\right)}$$

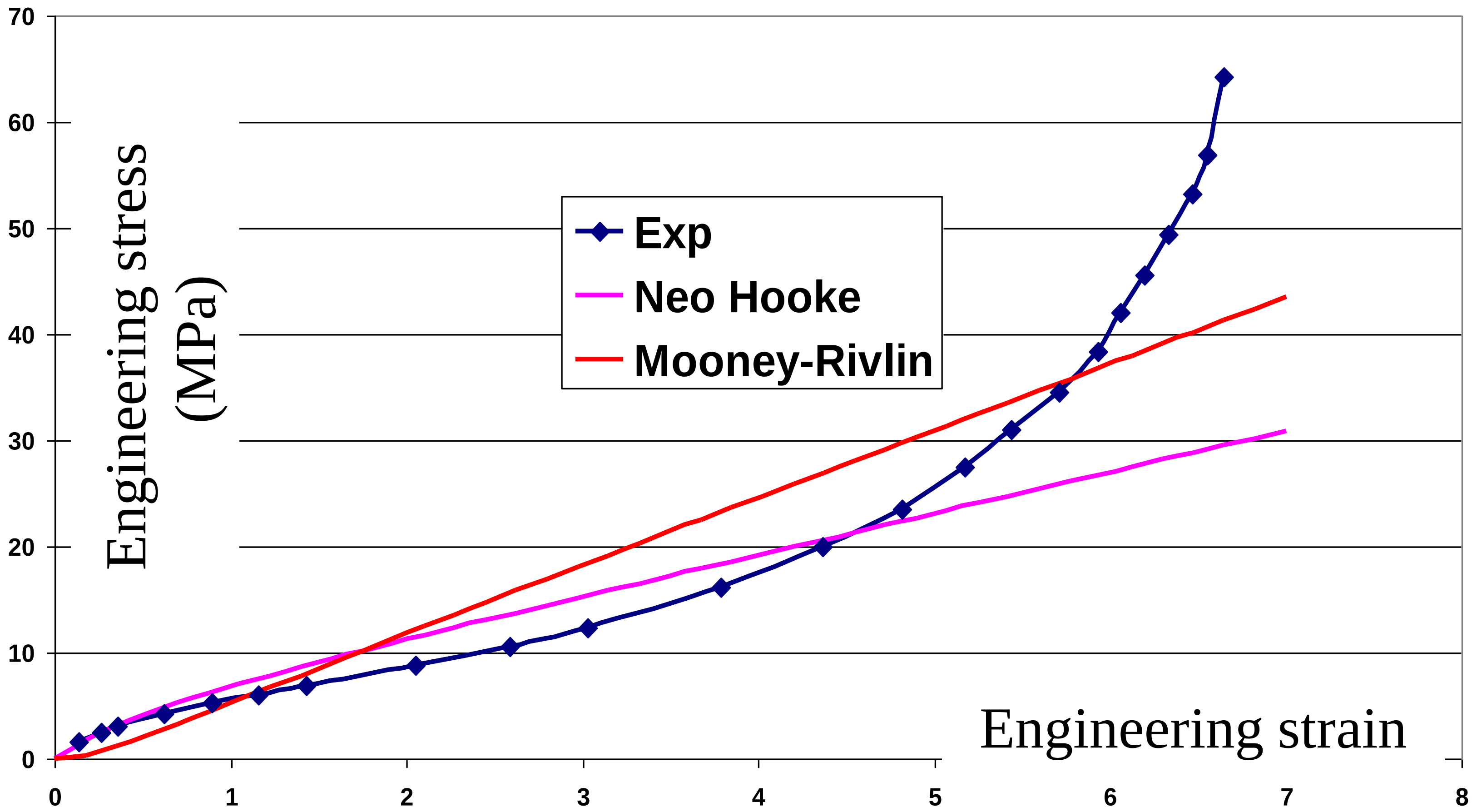
Remark:

$$I_1 = \lambda^2 + \lambda^{-2} + 1 \qquad I_2 = \lambda^2 + \lambda^{-2} + 1$$

$$\frac{dI_1}{d\lambda} = 2\left(\lambda - \lambda^{-3}\right) \qquad \frac{dI_2}{d\lambda} = 2\left(\lambda - \lambda^{-3}\right)$$

$$\boxed{\frac{F}{S_0} = \frac{\partial \Psi}{\partial \lambda}}$$

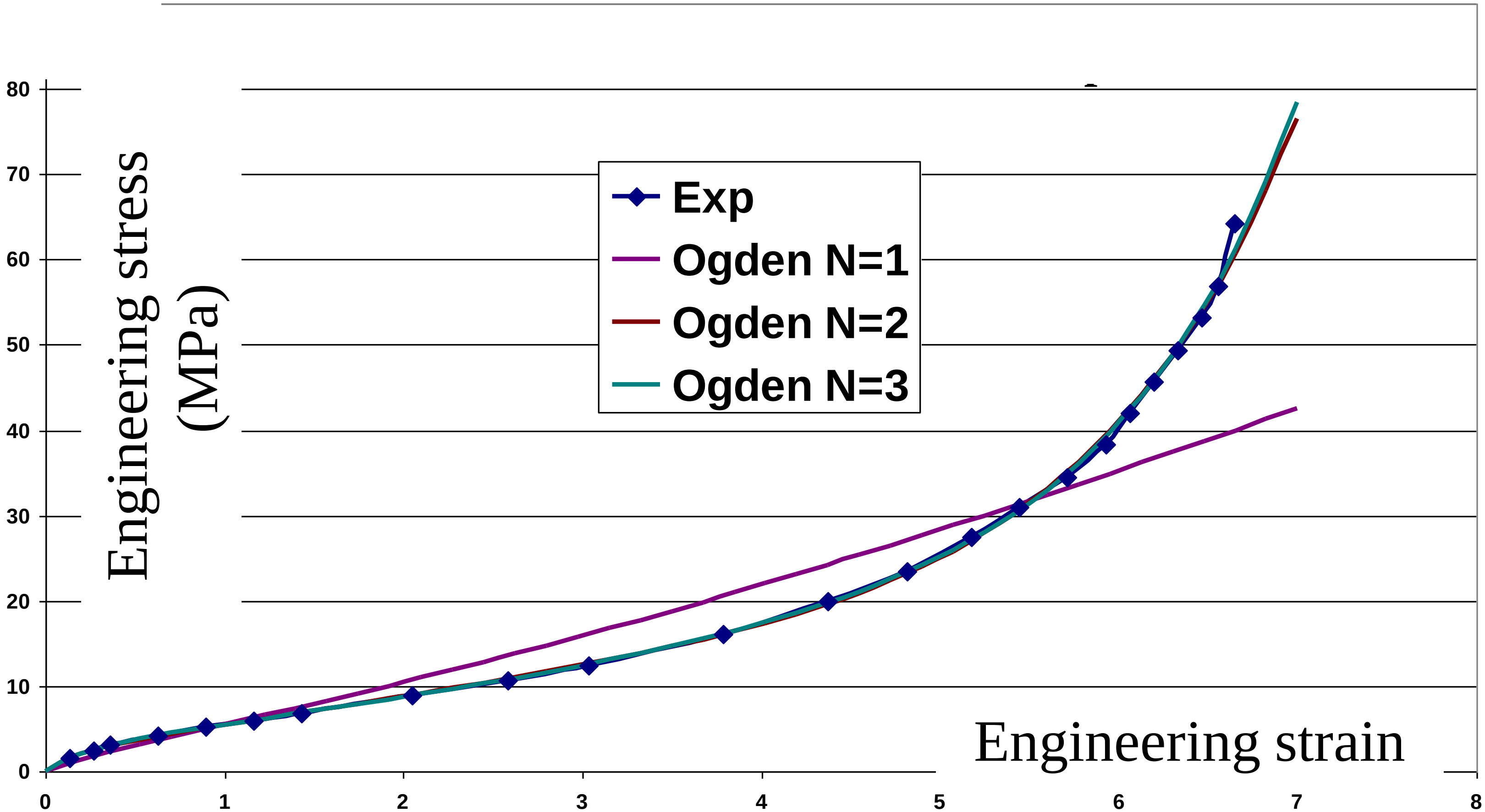
# Identification of parameters from the data of the uniaxial tensile test only






avril@emse.fr

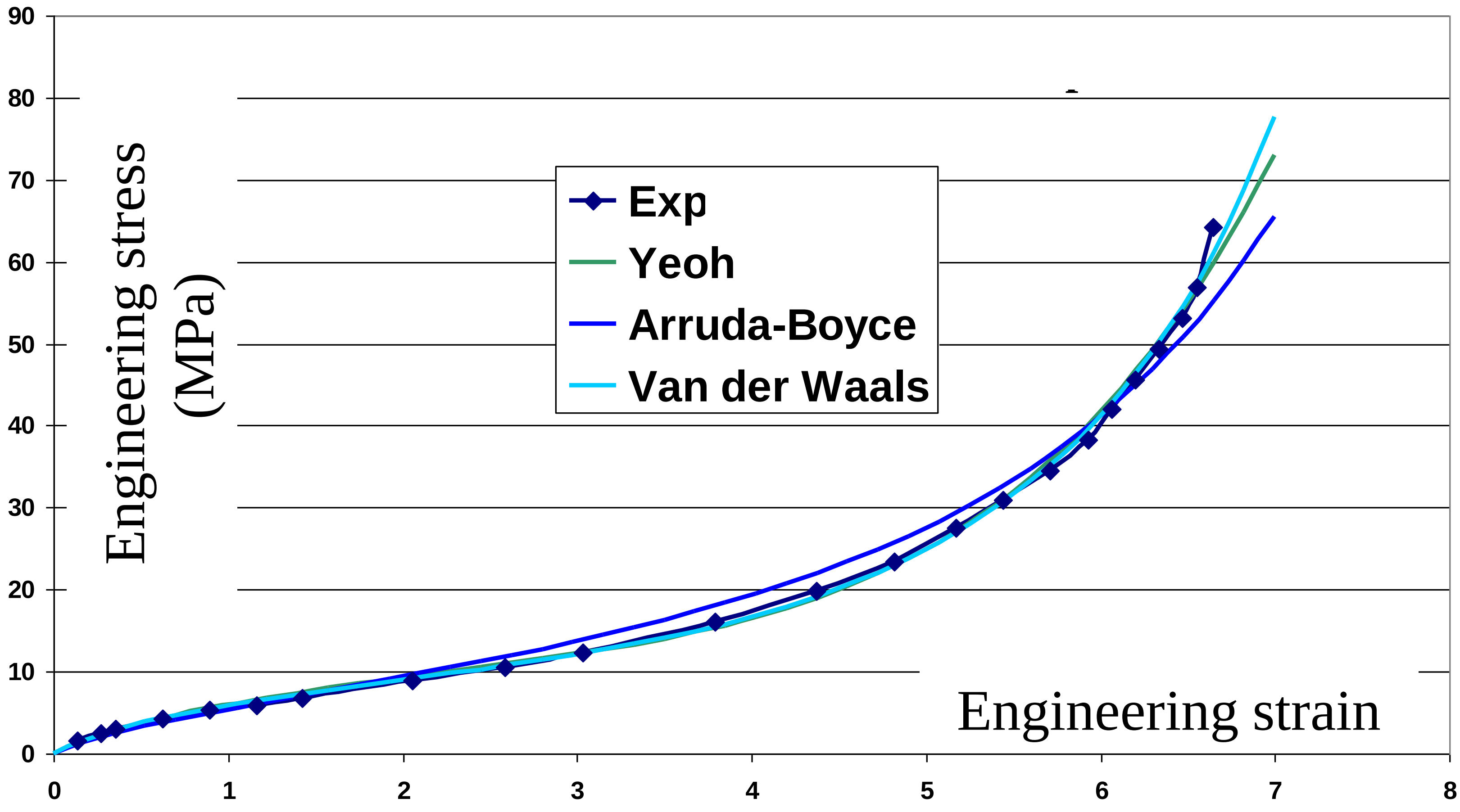
# Identification of parameters from the data of the uniaxial tensile test only





avril@emse.fr

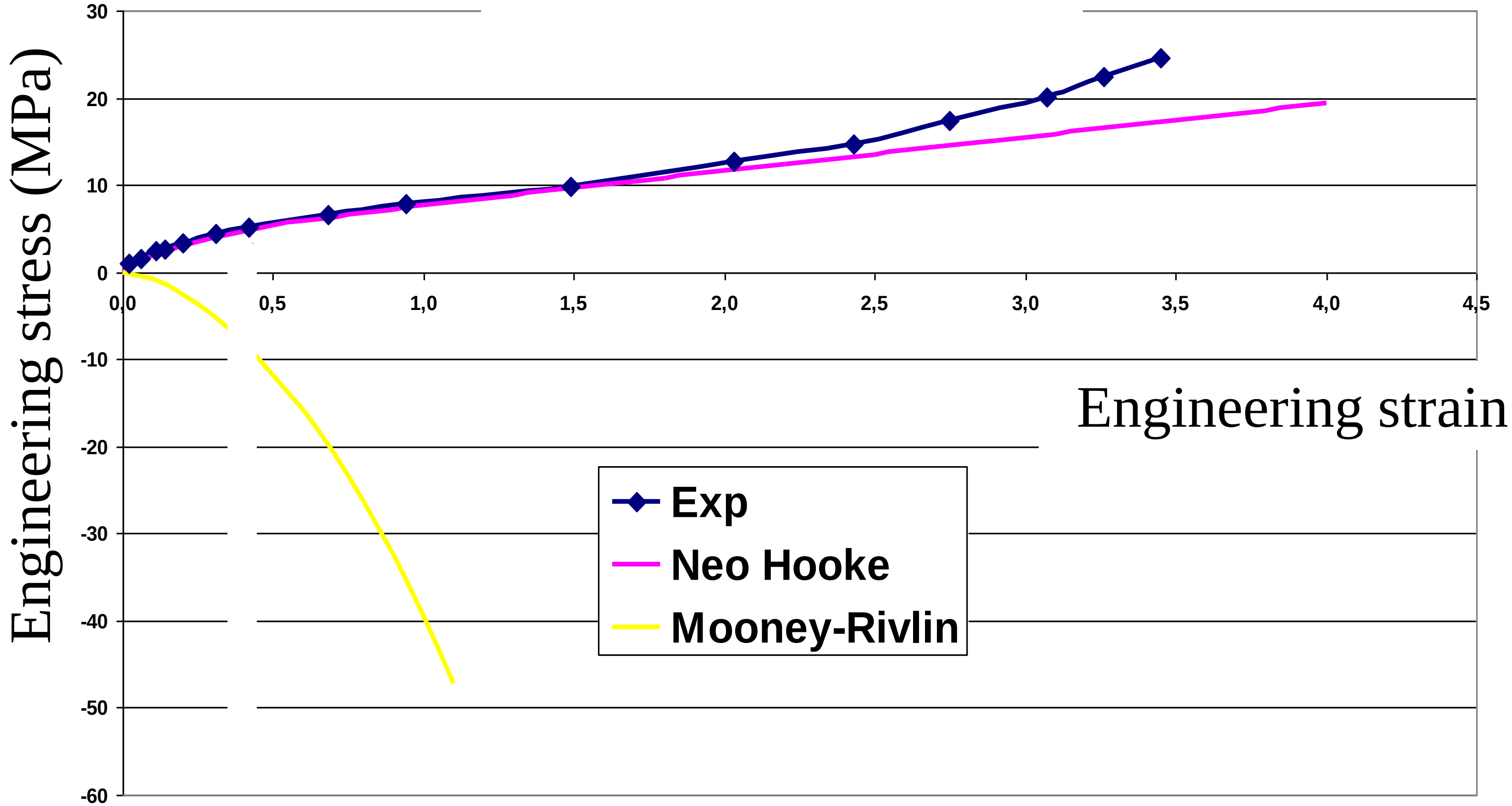
# Identification of parameters from the data of the uniaxial tensile test only





avril@emse.fr

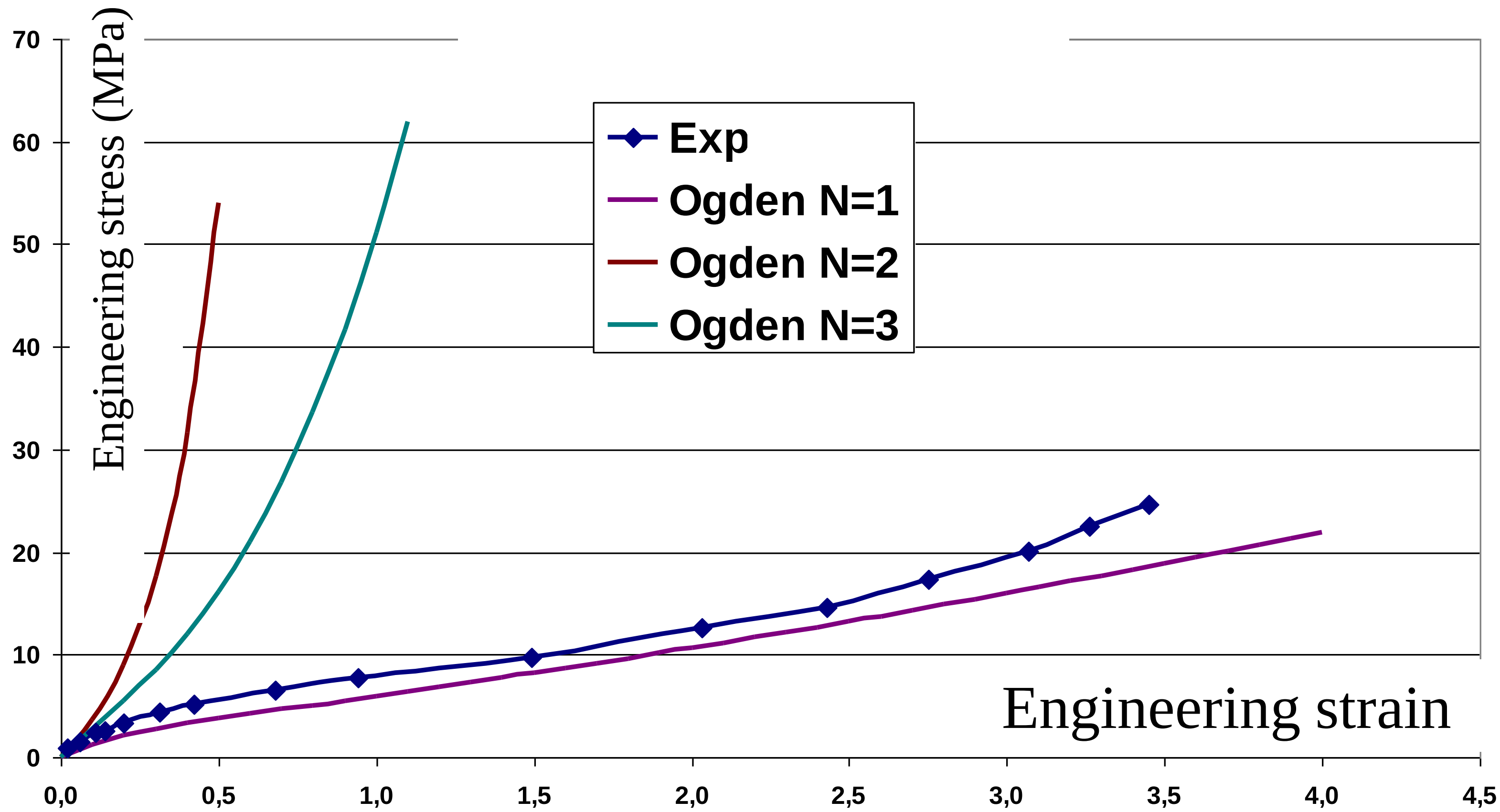
# Identification of parameters from the data of the uniaxial tensile test only
# Prediction of the planar tension data





avril@emse.fr

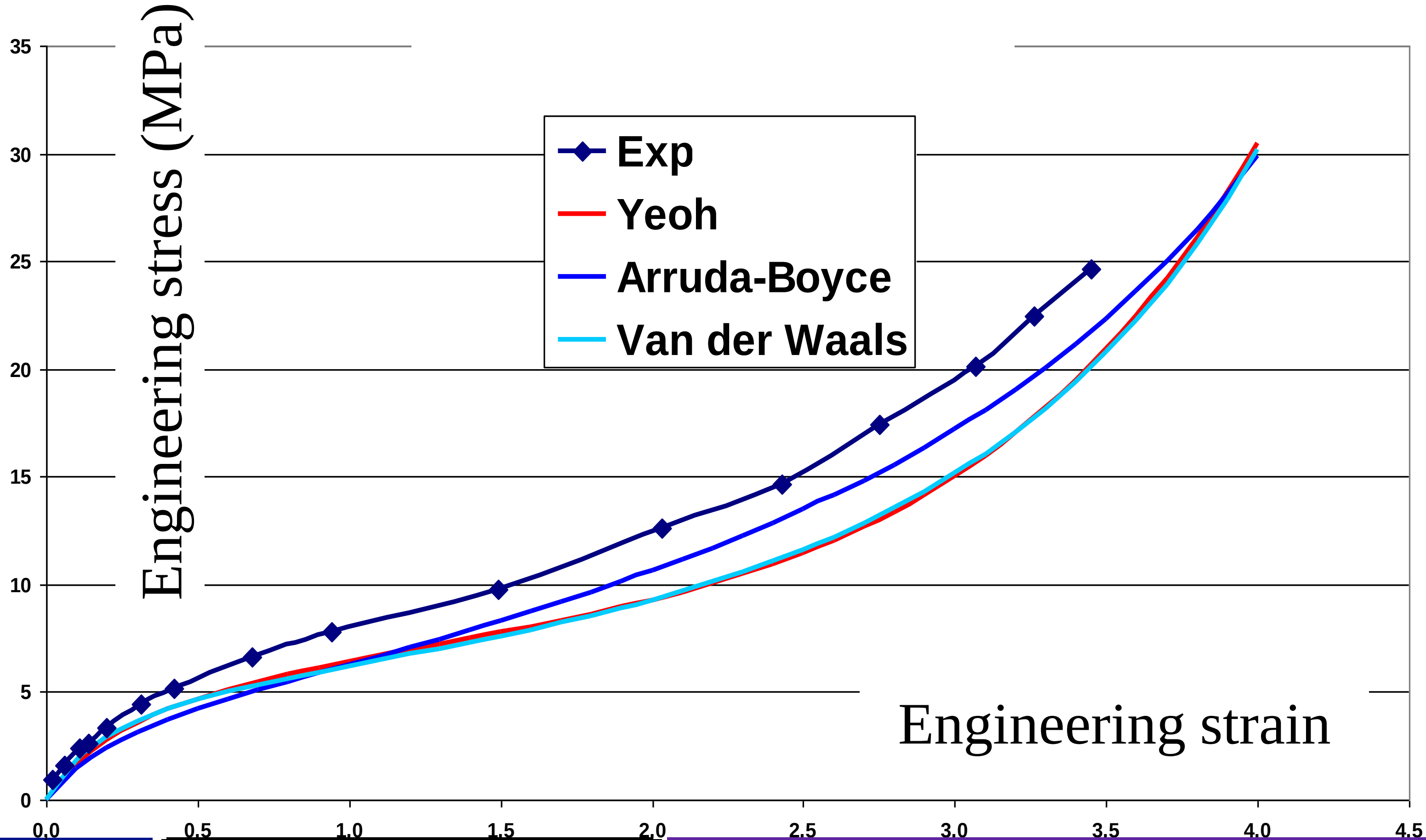
# Identification of parameters from the data of the uniaxial tensile test only
# Prediction of the planar tension data



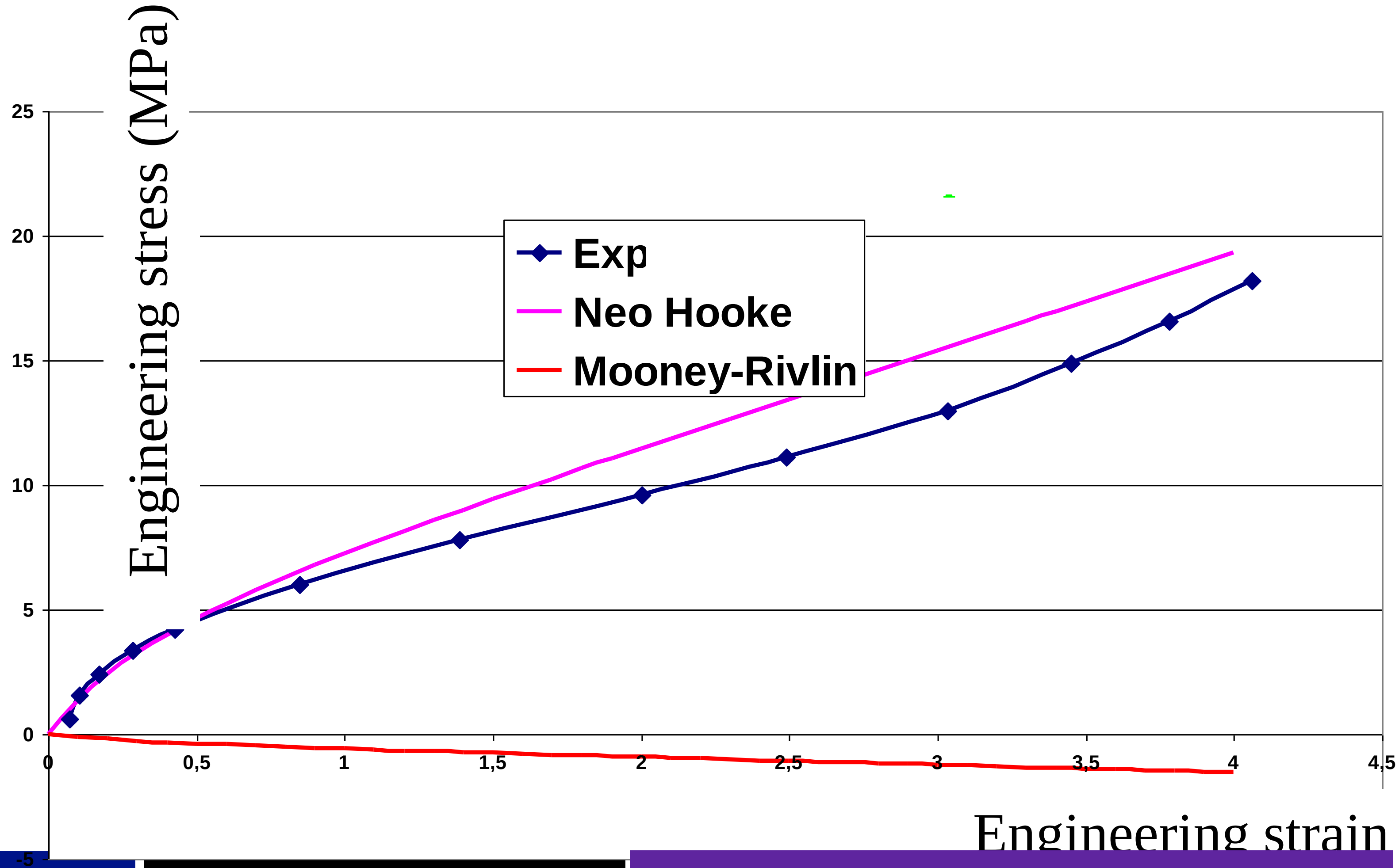


avril@emse.fr

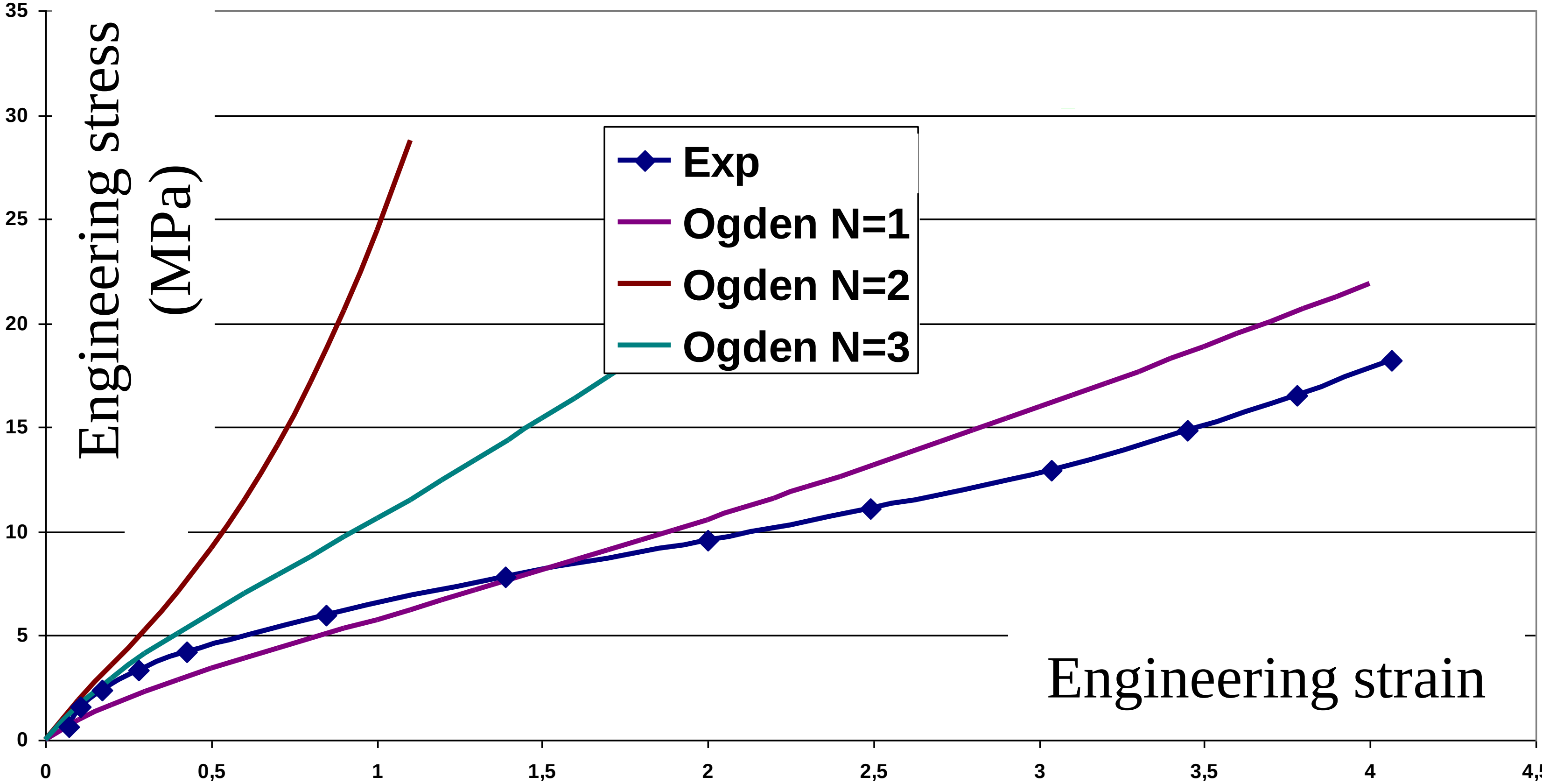
# Identification of parameters from the data of the uniaxial tensile test only
# Prediction of the planar tension data



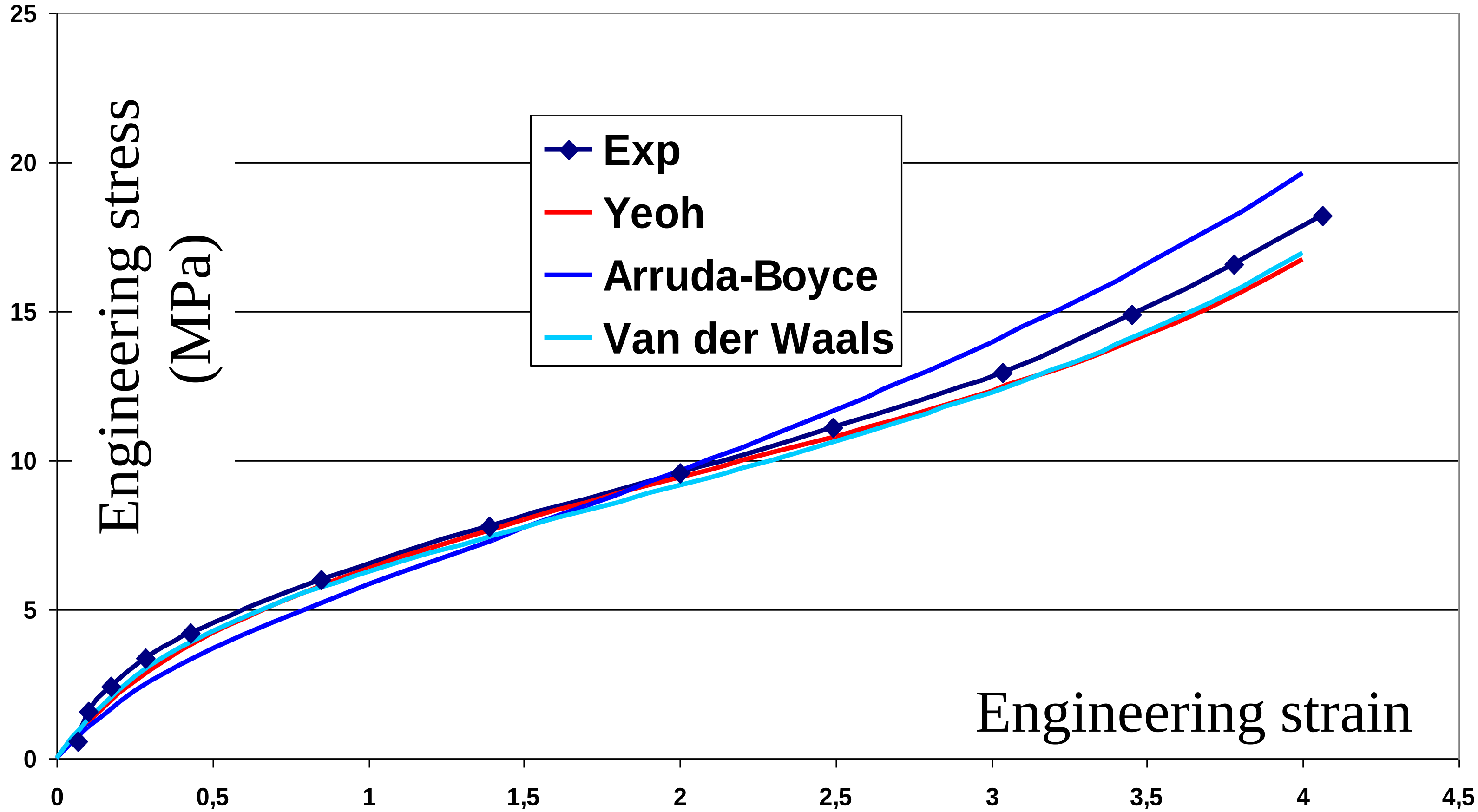
# Identification of parameters from the data of the uniaxial tensile test only
# Prediction of the biaxial tension data



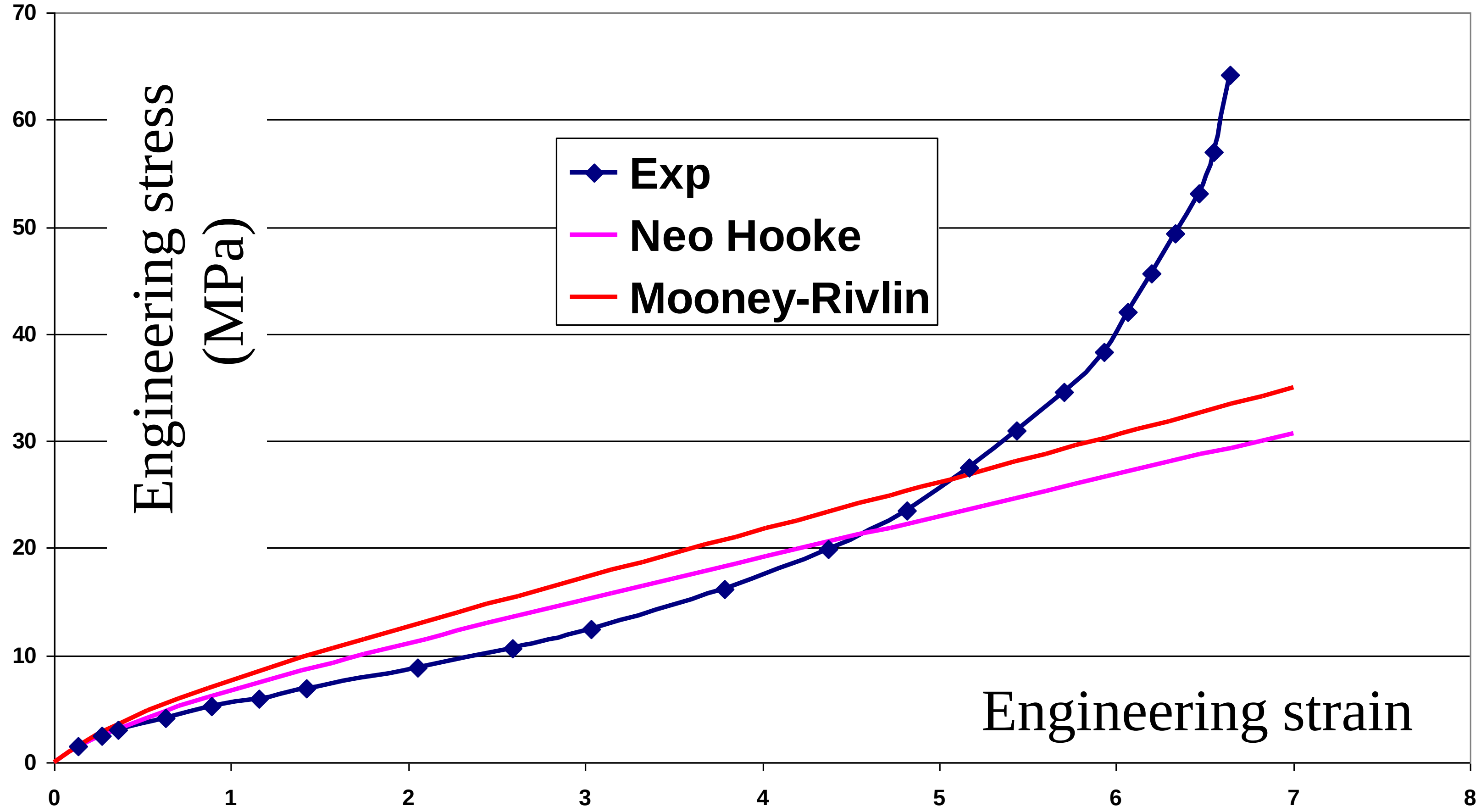
# Identification of parameters from the data of the uniaxial tensile test only
# Prediction of the biaixal tension data



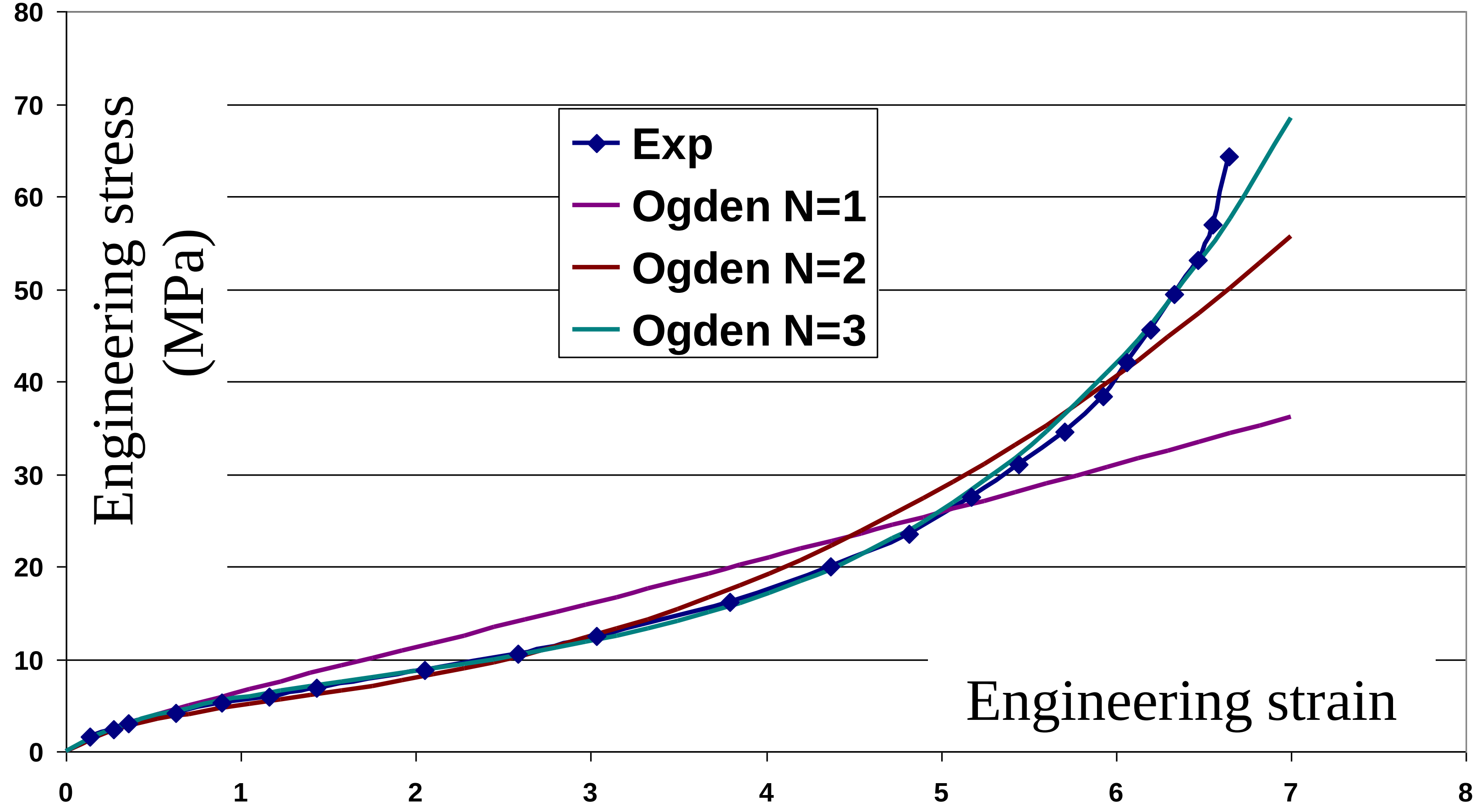
# Identification of parameters from the data of the uniaxial tensile test only
# Prediction of the planar tension data



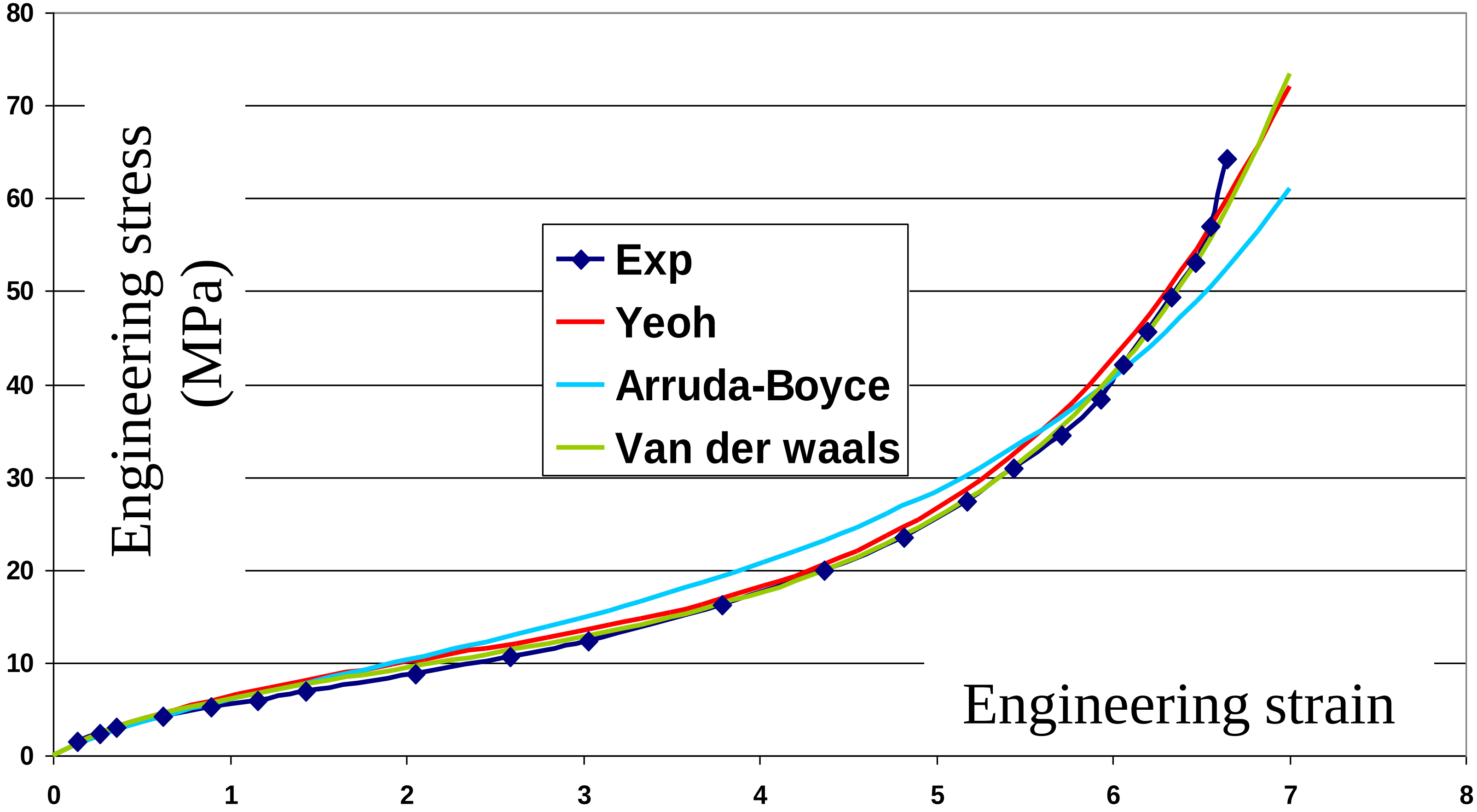
# Identification of parameters from all the data
# Prediction of the uniaxial tensile test



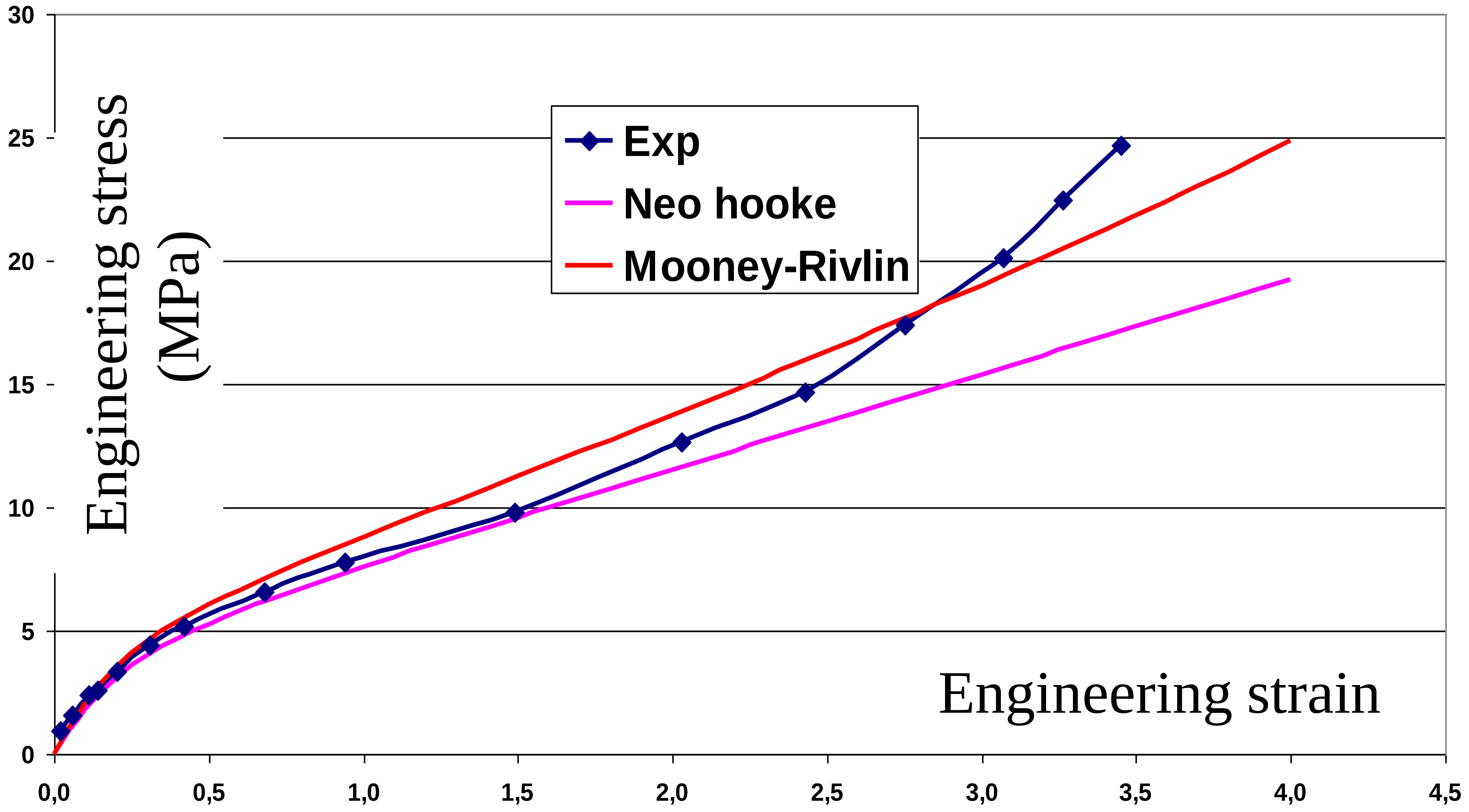
# Identification of parameters from all the data
# Prediction of the uniaxial tensile test



# Identification of parameters from all the data
# Prediction of the uniaxial tensile test



# Identification of parameters from all the data
# Prediction of the planar tensile test

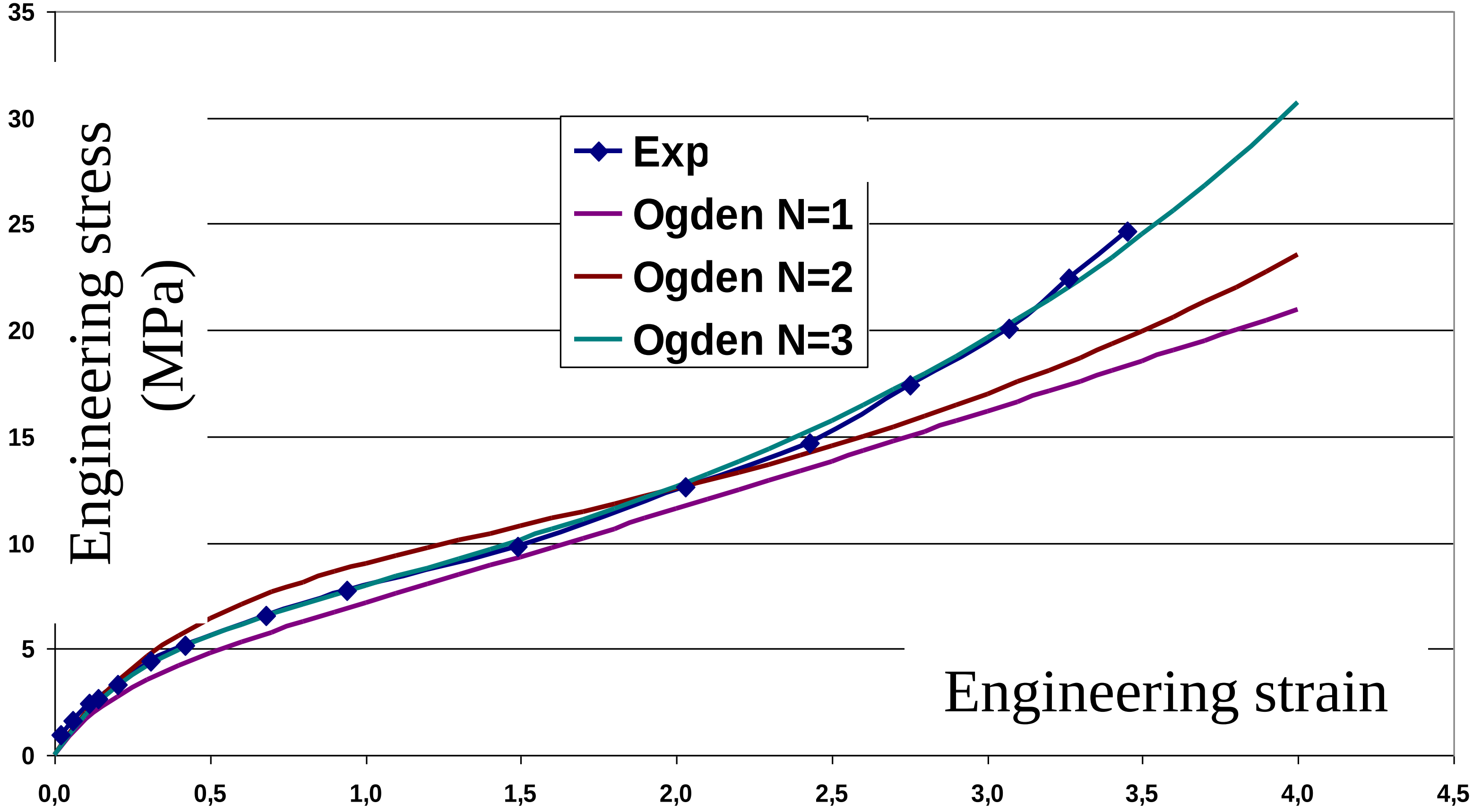




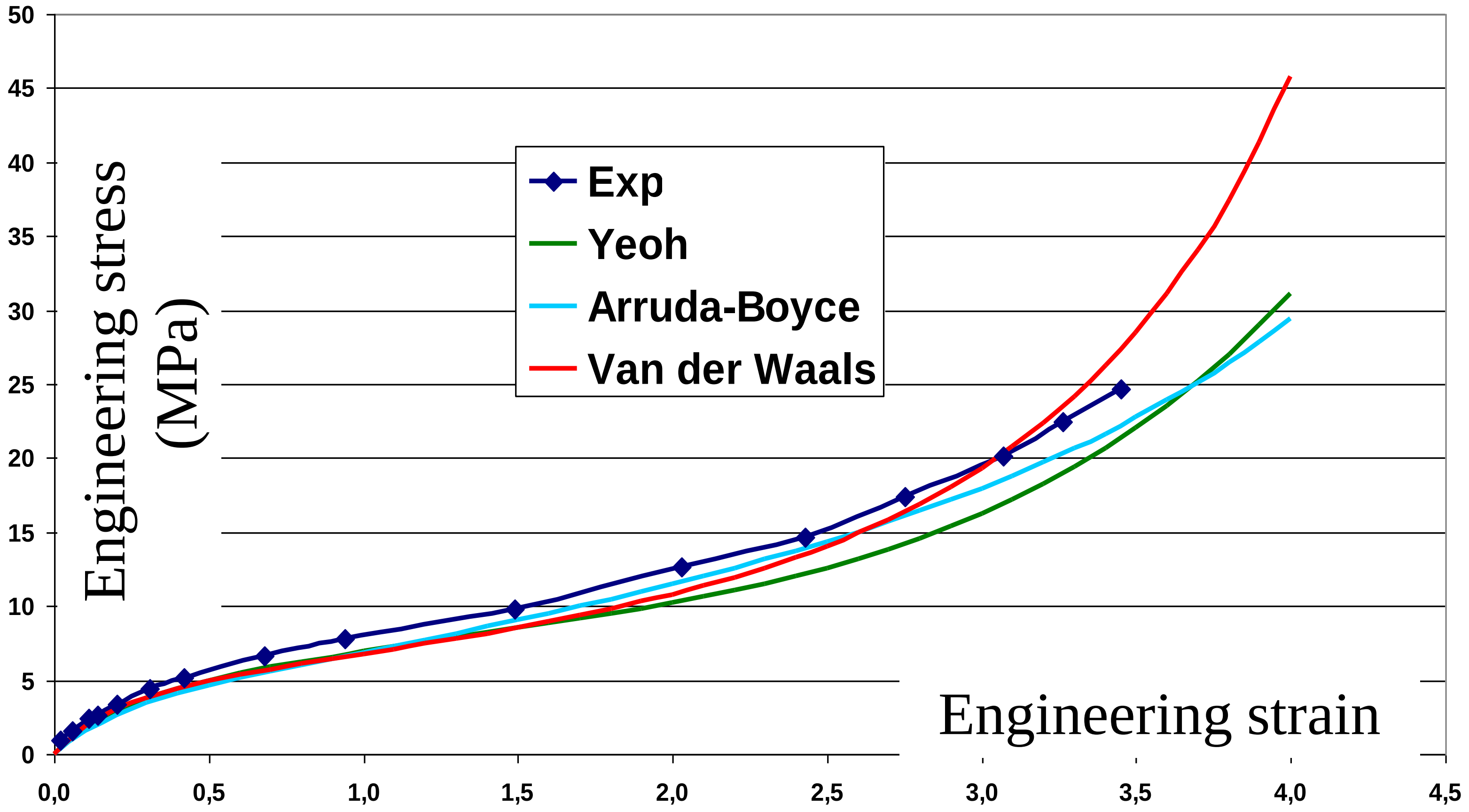
avril@emse.fr

MINES Saint-Étienne

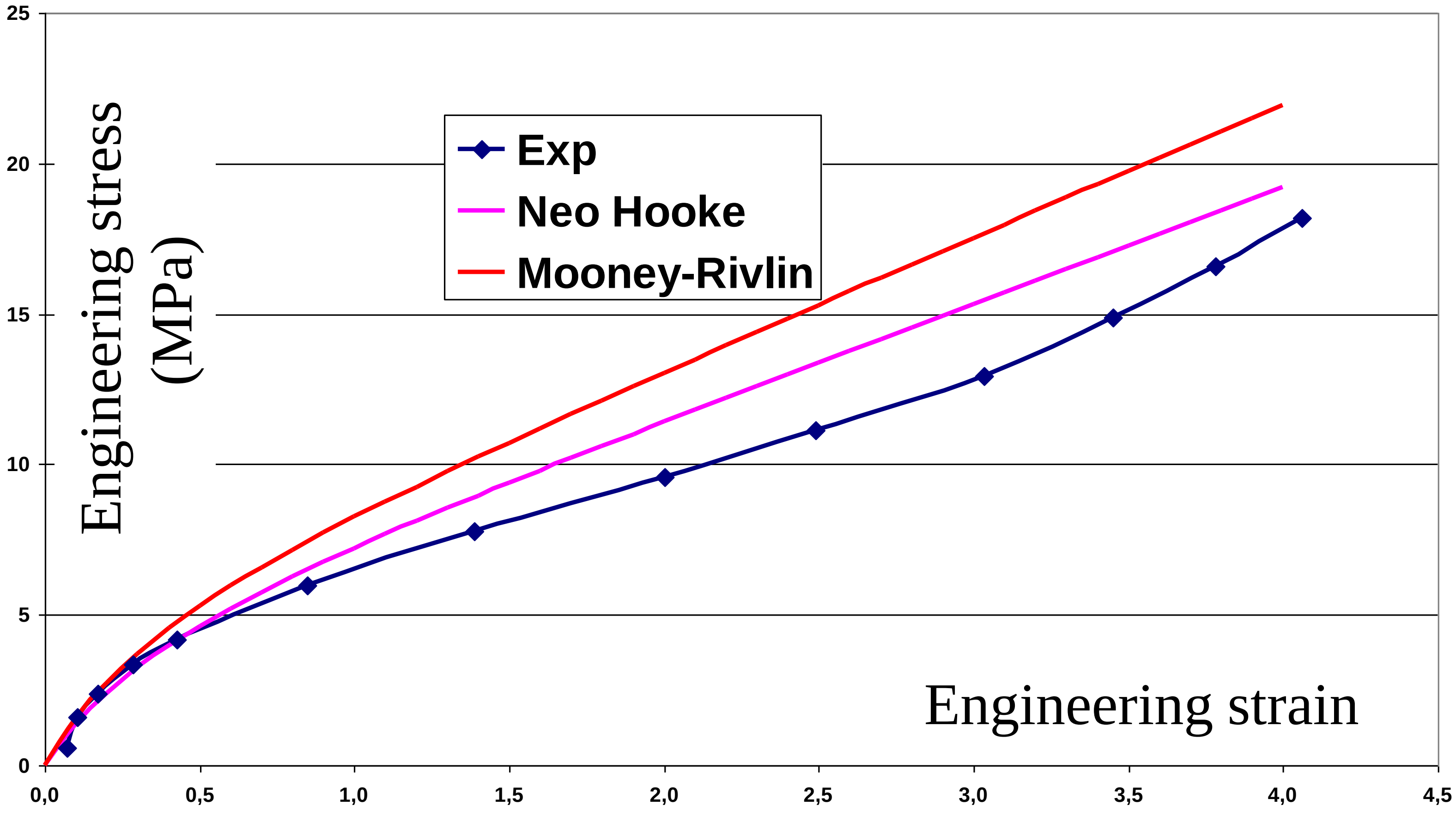
# Identification of parameters from all the data
# Prediction of the planar tensile test



# Identification of parameters from all the data
# Prediction of the uniaxial tensile test



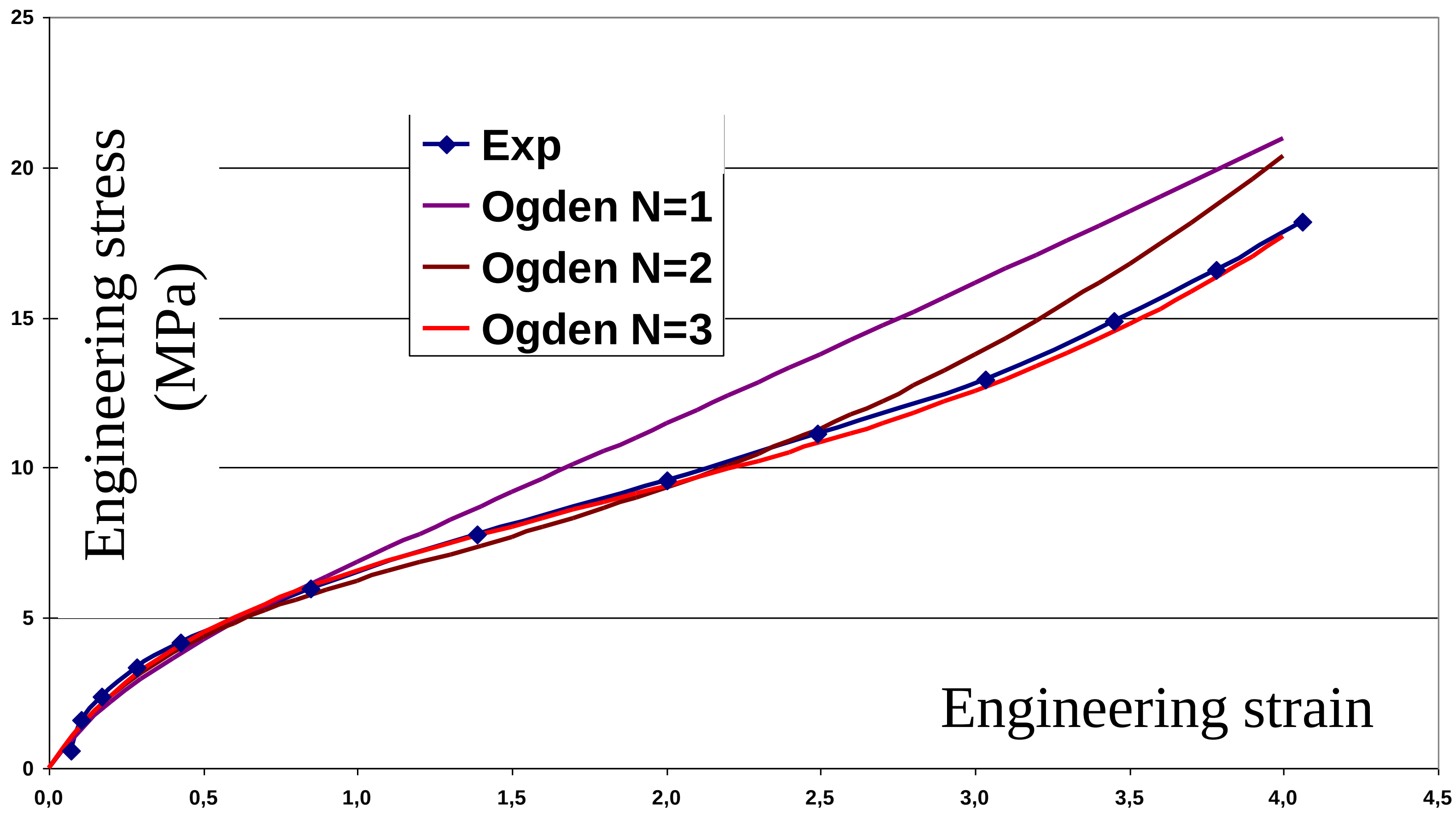
# Identification of parameters from all the data
# Prediction of the biaixial tensile test





avril@emse.fr

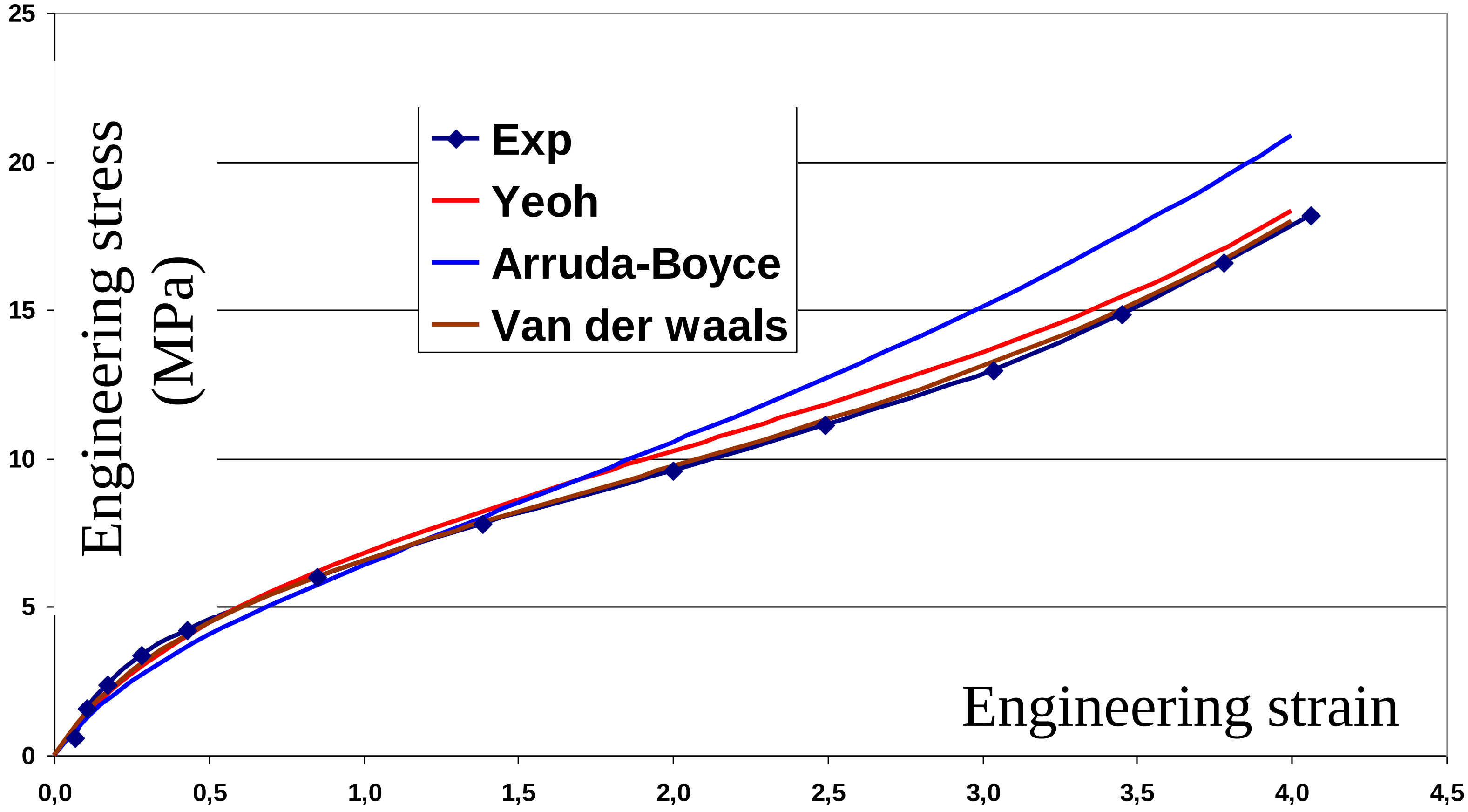
# Identification of parameters from all the data
# Prediction of the biaxial tensile test



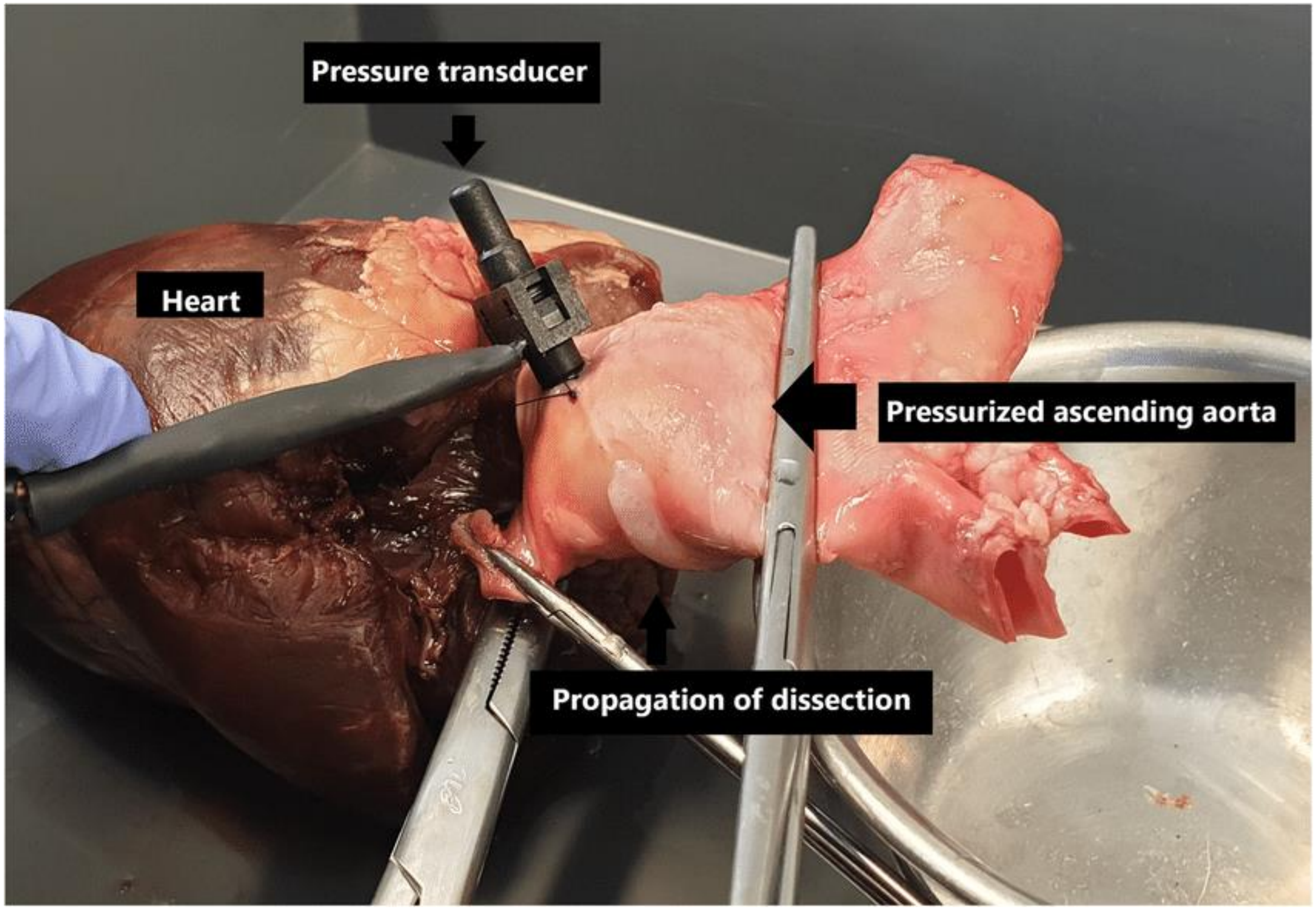


avril@emse.fr

# Identification of parameters from all the data
# Prediction of the biaxial tensile test



# Testing soft tissues: arteries

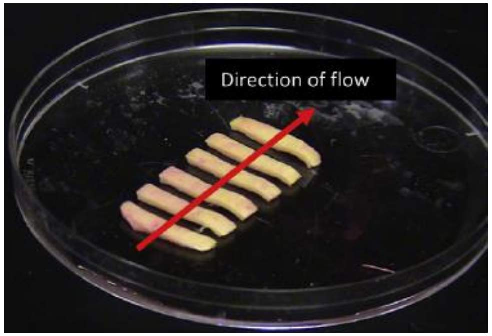


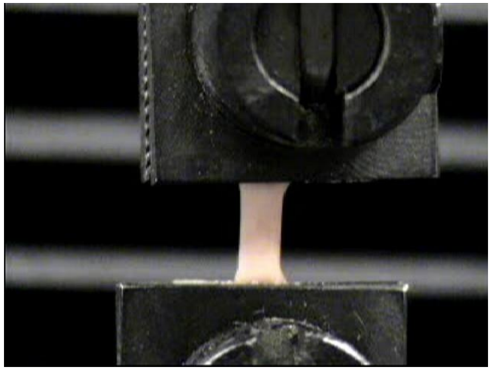


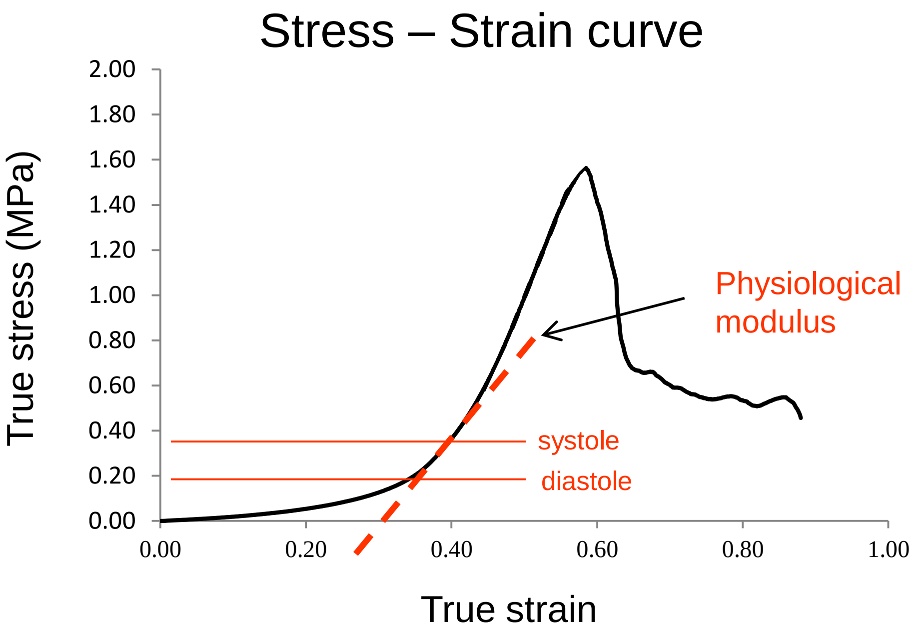
avril@emse.fr

# Tensile test - uniaxial tension

[Duprey, 2010]



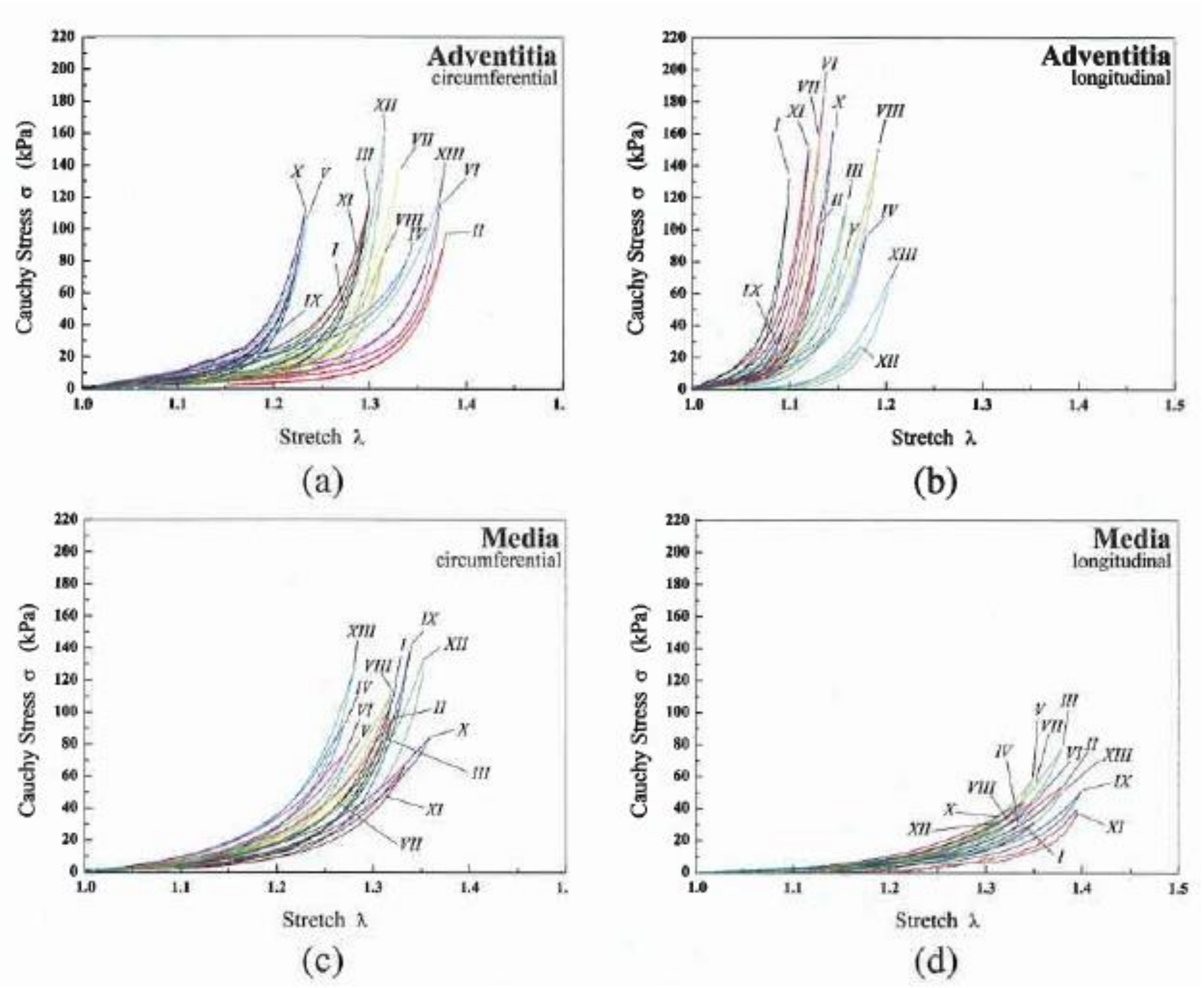







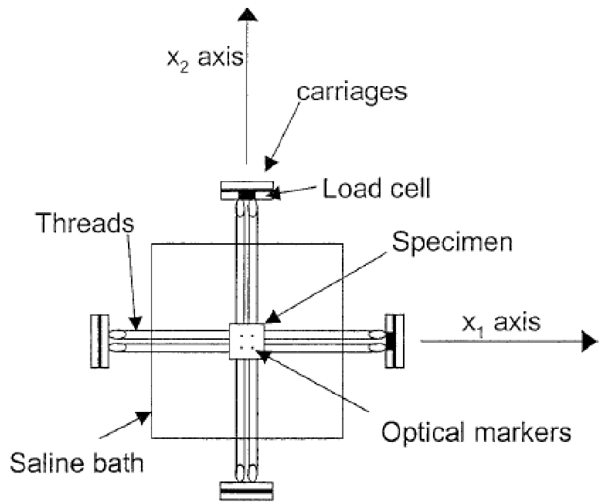
avril@emse.fr

# Tensile test - uniaxial tension

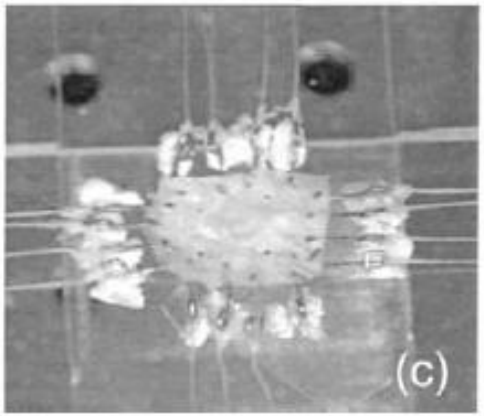



[Holzapfel, 2005]

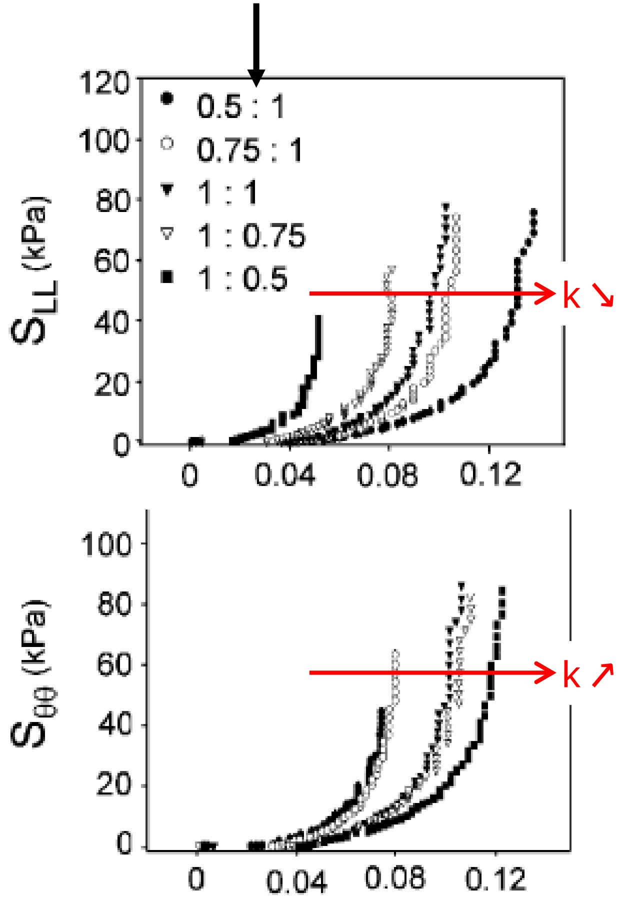
# Biaxial tension




[Sacks, 2000]




$$k = F_{\theta\theta} / F_{zz}$$

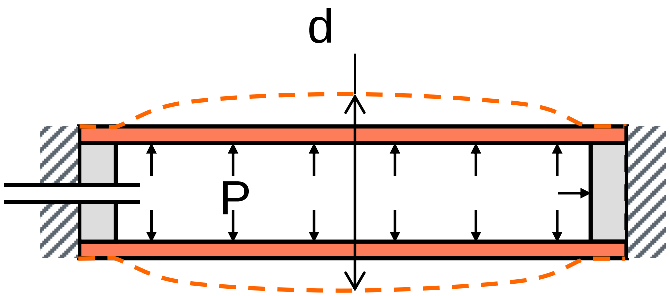



[VandeGeest, 2006]

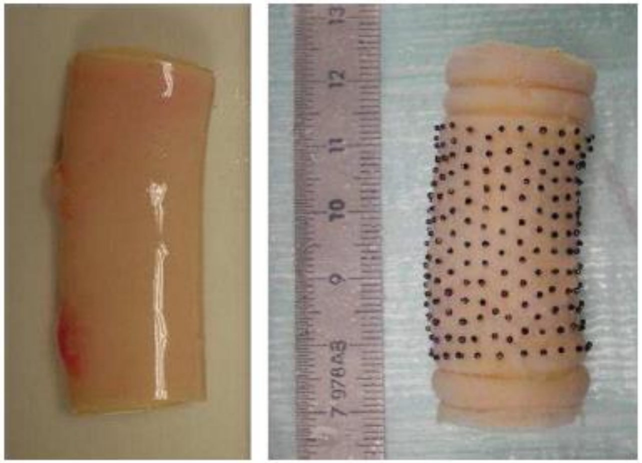
Human abdominal aorta


avril@emse.fr

# Tension inflation

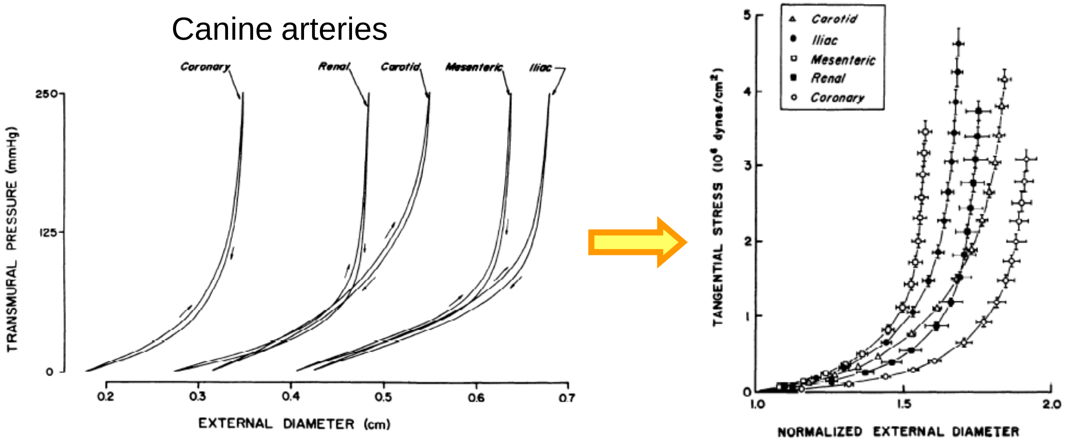



→ P - d curve

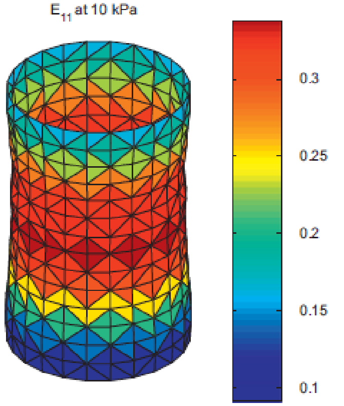


[Genovese,2009][Avril, 2010](CIS)

Canine arteries

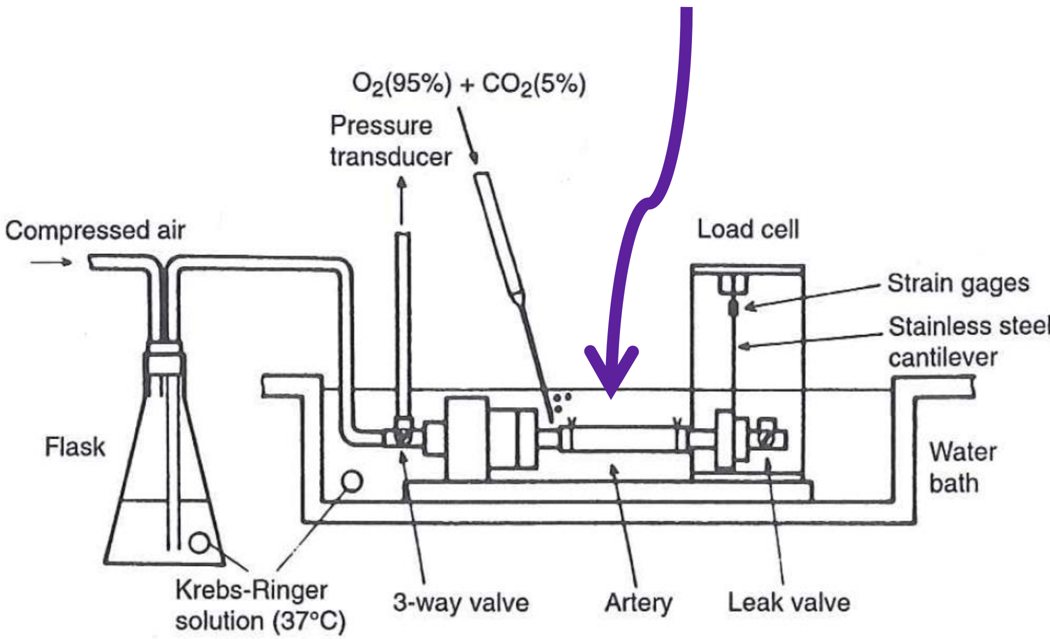


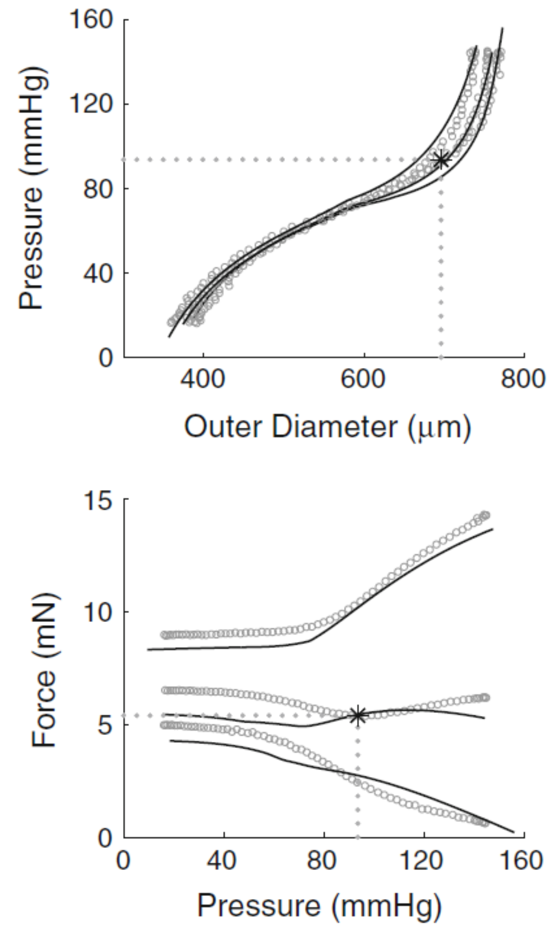

[Cox, 1978]



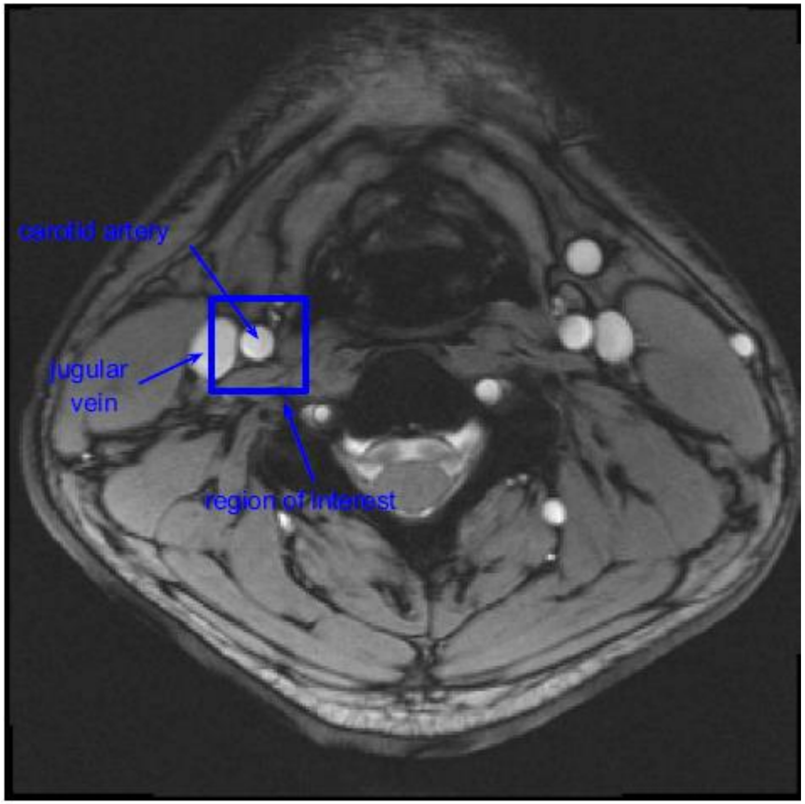
# Tension inflation =

# Functional biomechanical behavior

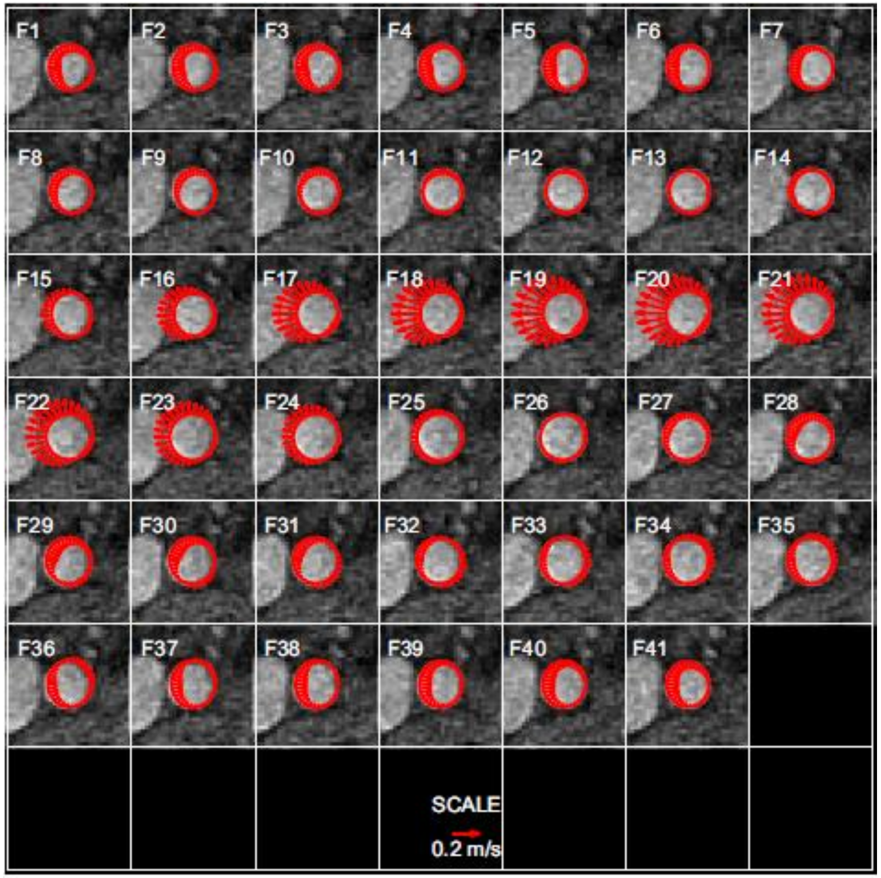


# In vivo measurements



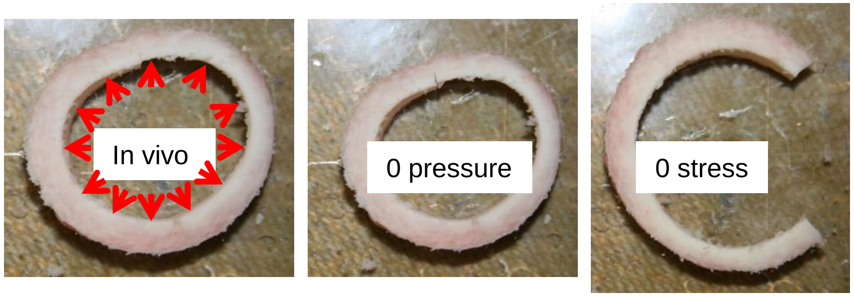

[Avril, 2010]



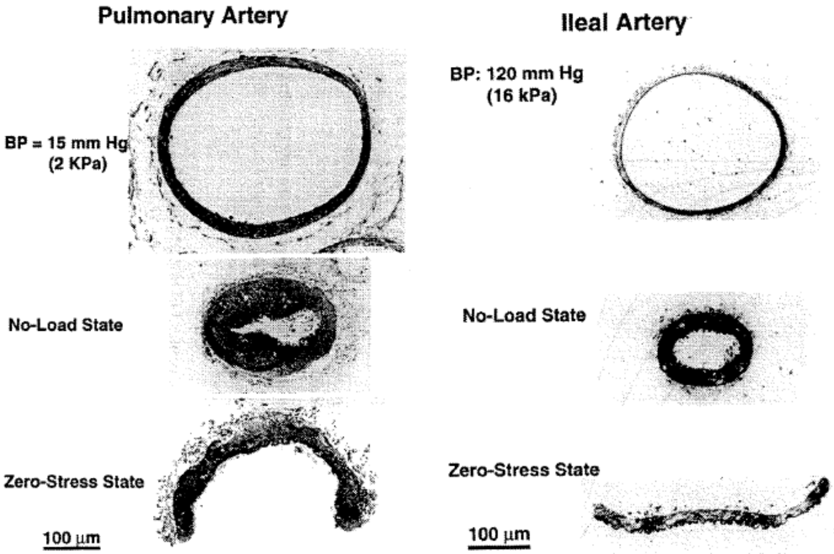

[Avril, 2011]


MINES Saint-Étienne

# Residual stresses




bovine aorta




[Fung, 1993]

# Incompressibility

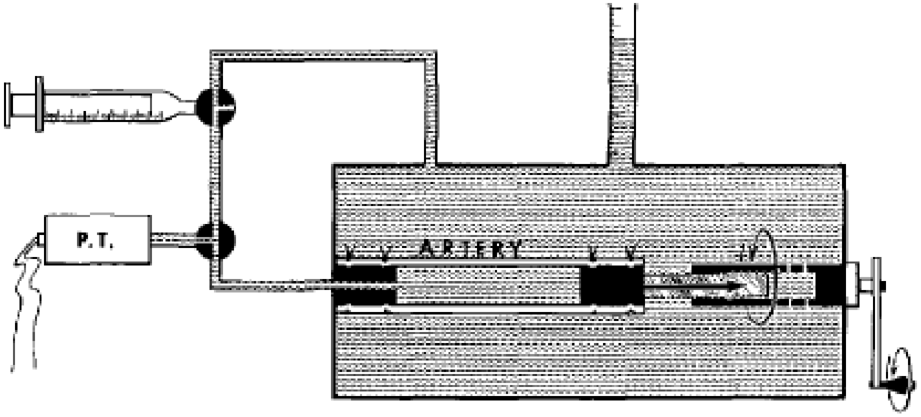



FIGURE 1

*Diagram of experimental set-up. P.T. = pressure transducer. During an experiment the capillary tube shown at the top of the figure was directed horizontally so that changes in fluid level in the tube would not change the static pressure level. Note from the directional arrows that the end of the artery moves to the right without twisting as the crank is advanced.*

[Carew, 1968]

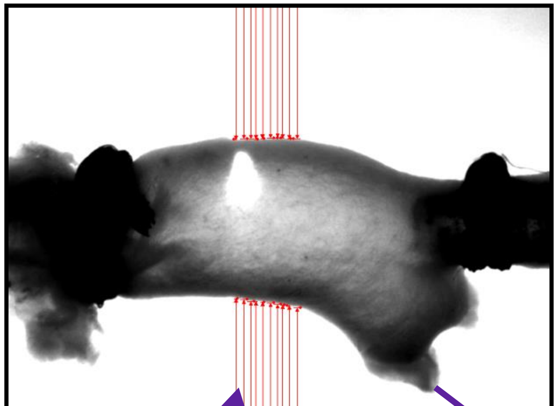
$$K = \frac{p_{tissu}}{\Delta V / V_0} = \frac{tr\left(\underline{\underline{\boldsymbol{\sigma}}}\right)/3}{\Delta V / V_0} \quad >> \quad G = \frac{E}{2(1+\upsilon)}$$


avril@emse.fr

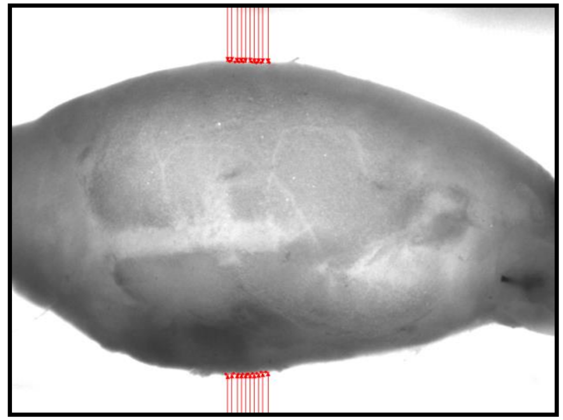

MINES Saint-Étienne

# Advanced investigations

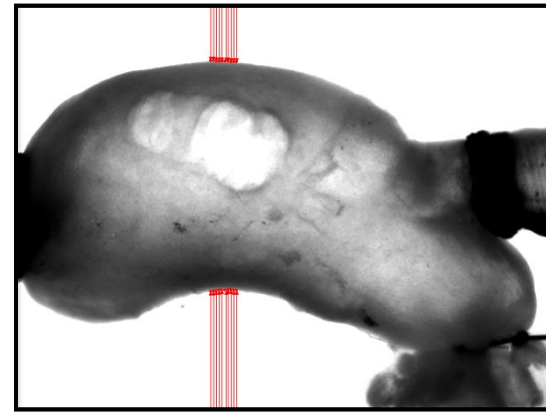
Control

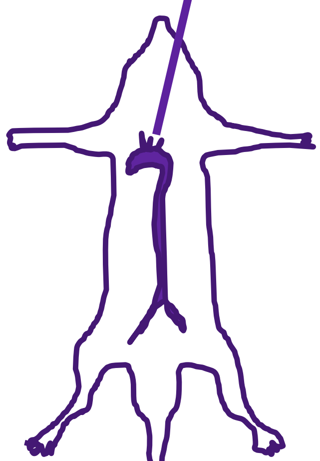


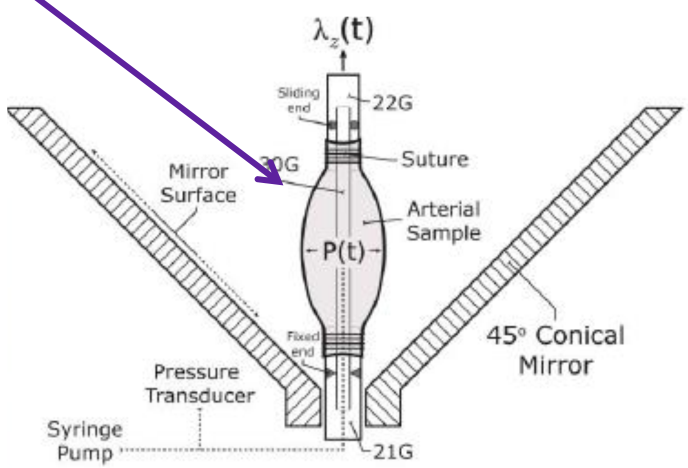
Fibulin 4 SMC KO

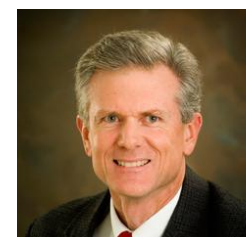


Fibrillin 1 *mgR/mgR*



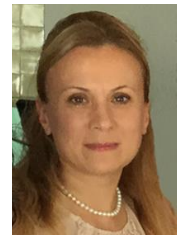




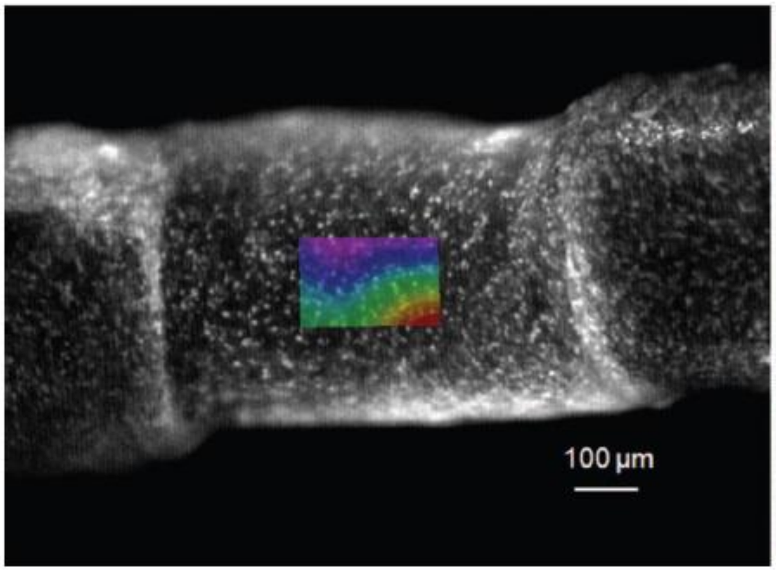



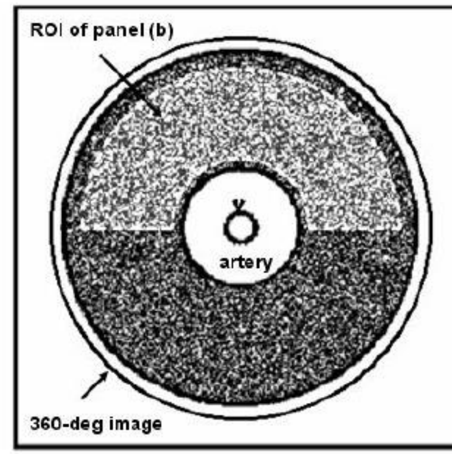
# MEASUREMENT OF THE RESPONSE USING DIGITAL IMAGE CORRELATION

classical

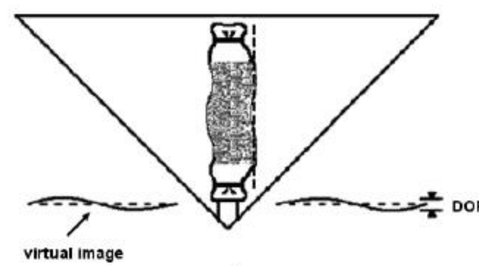


panoramic



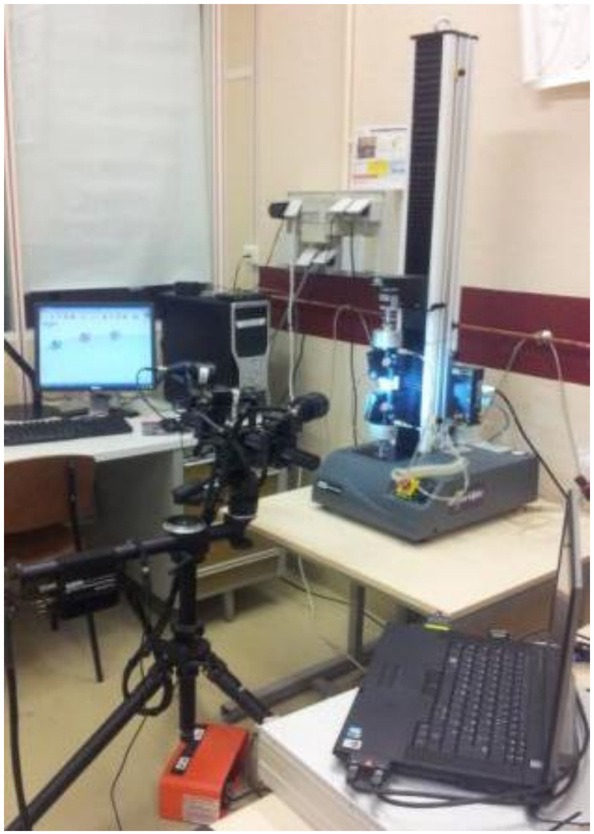


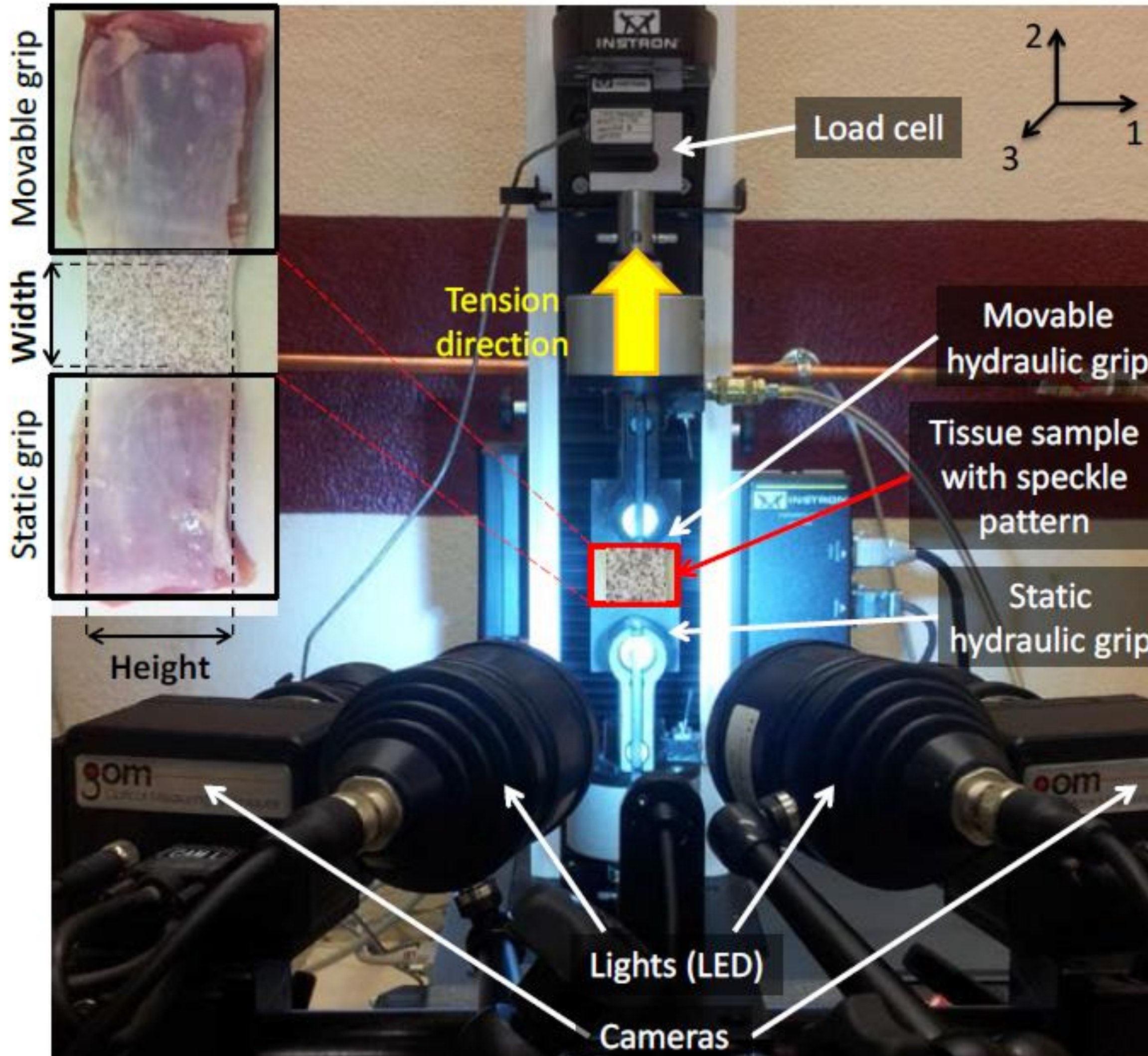



Badel et al. CMBBE, **15**, p 37-48, 2012.

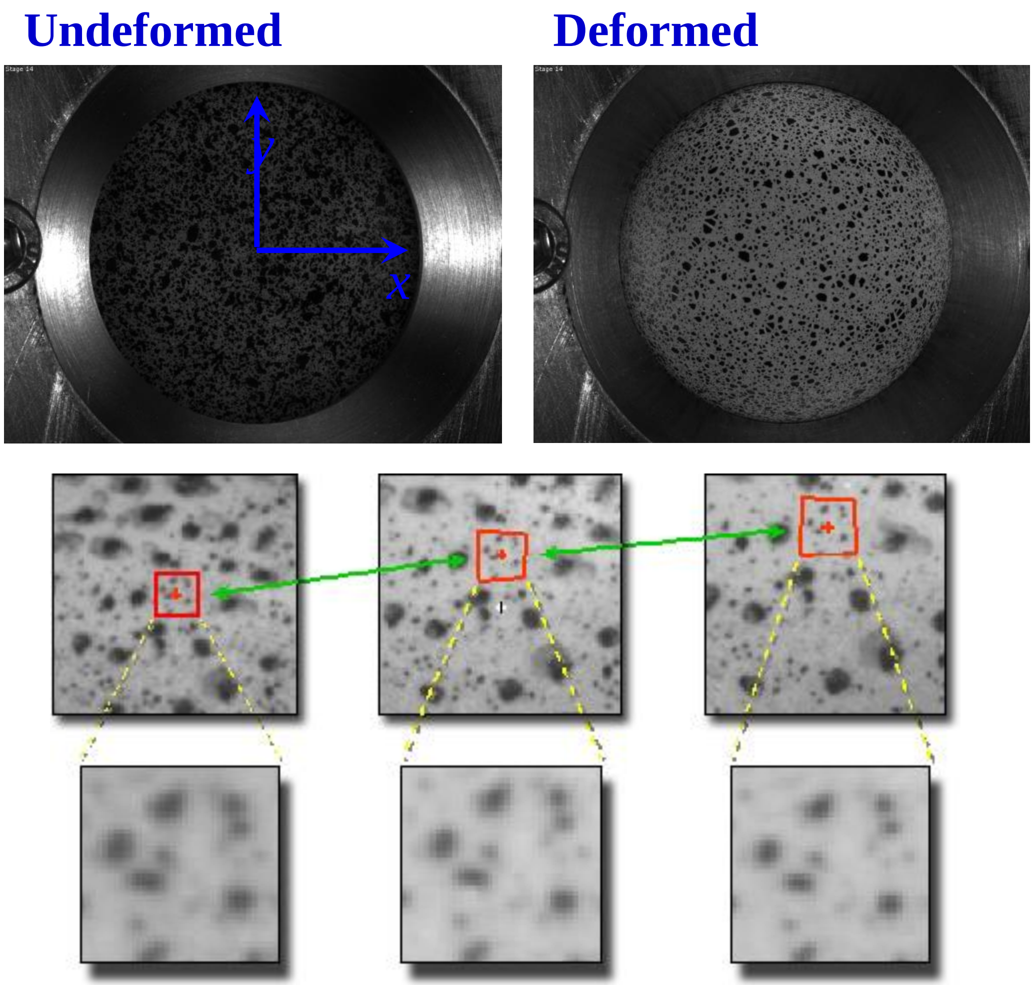
Genovese. Optics Lasers Eng, **47**, p 995-1008, 2009.

# DIC tend to be used for all kind of soft tissues

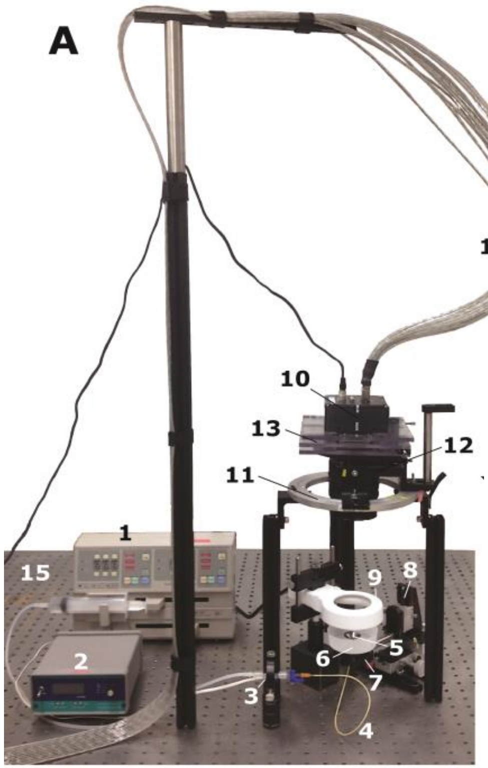


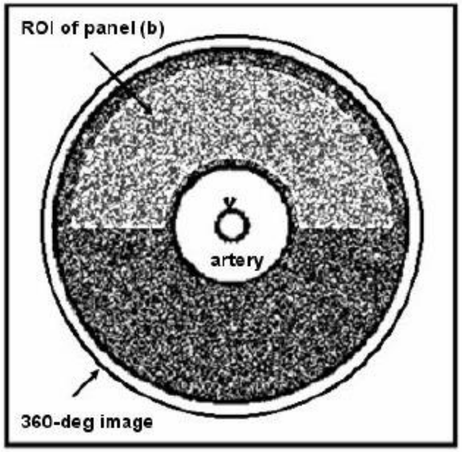


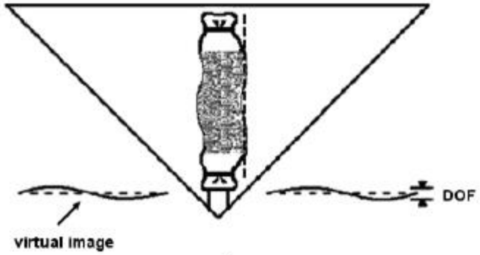
# Principle of DIC

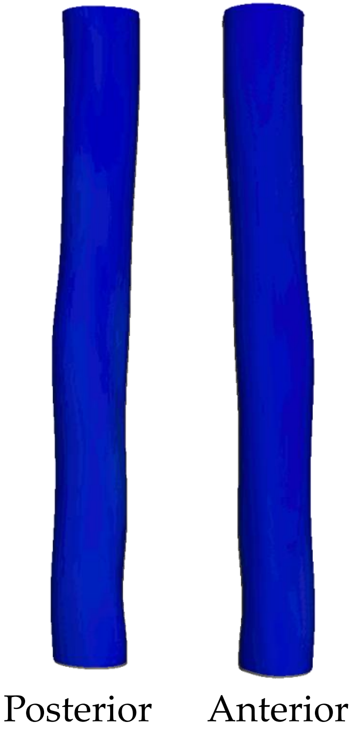


# The pDIC technique

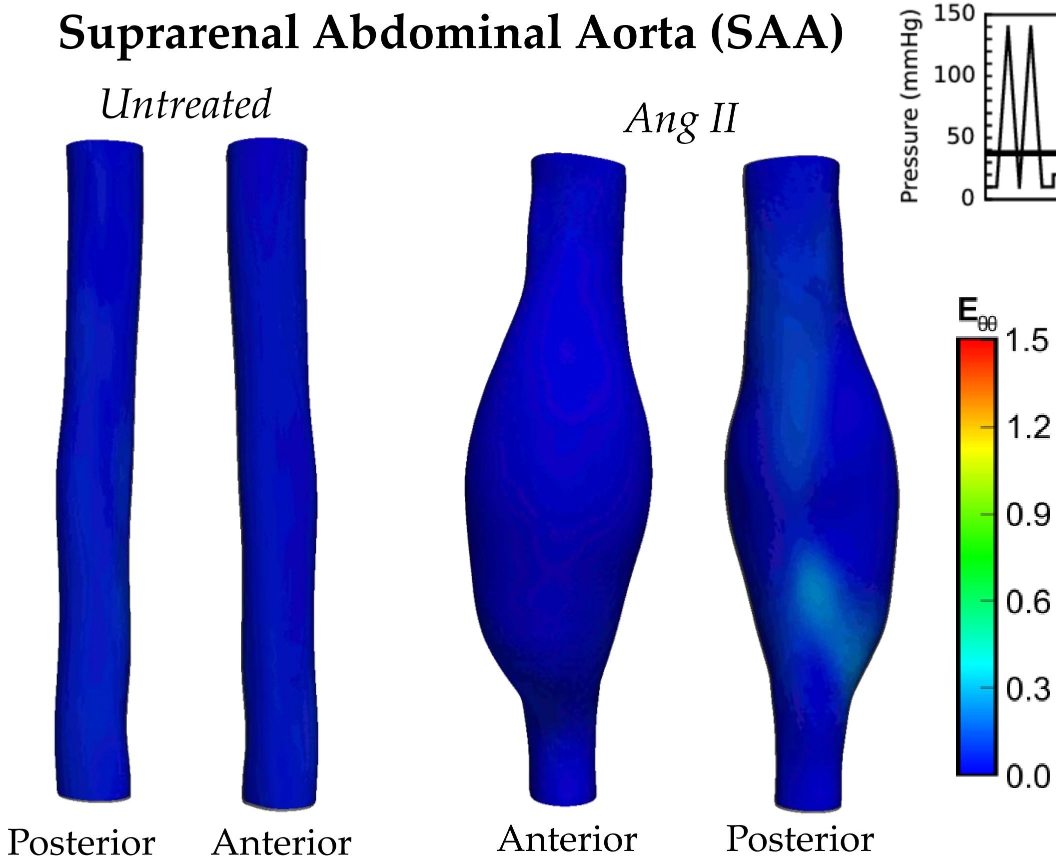


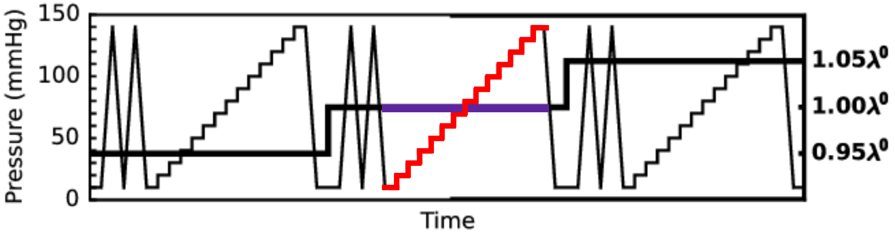



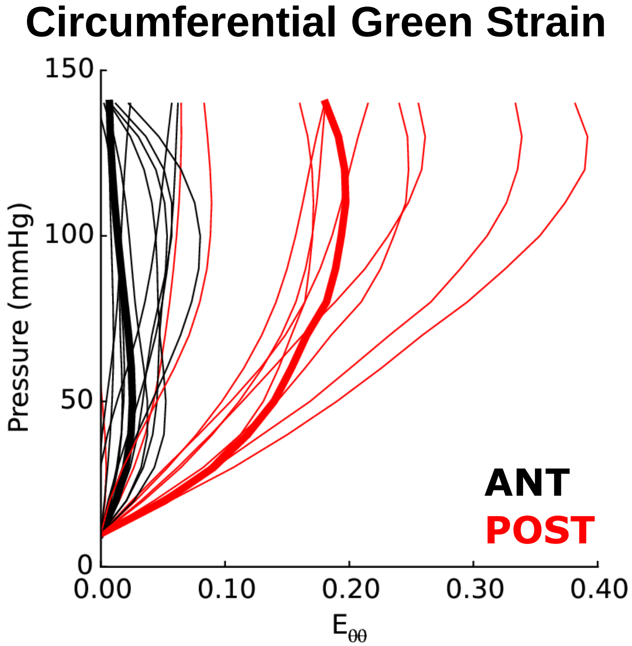



# Measurement of surface deformation fields using Panoramic Digital Image Correlation (pDIC)





avril@emse.fr

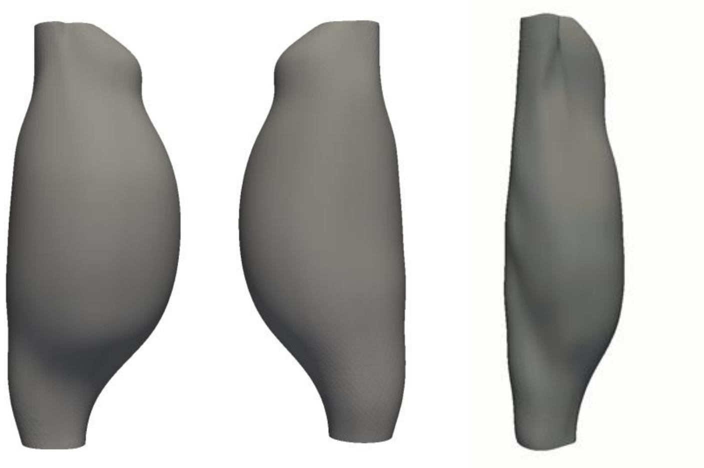
MINES Saint-Étienne

# pDIC measurements

## Fibulin 4 SMC KO

*ventral* *dorsal* *inflation*

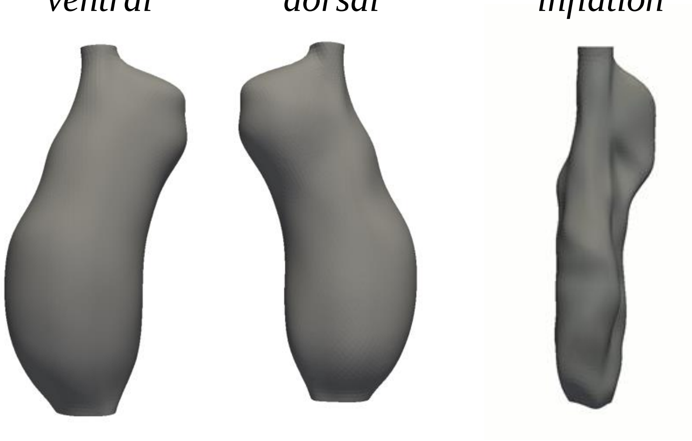
6852



## Fibrillin 1 *mgR/mgR*

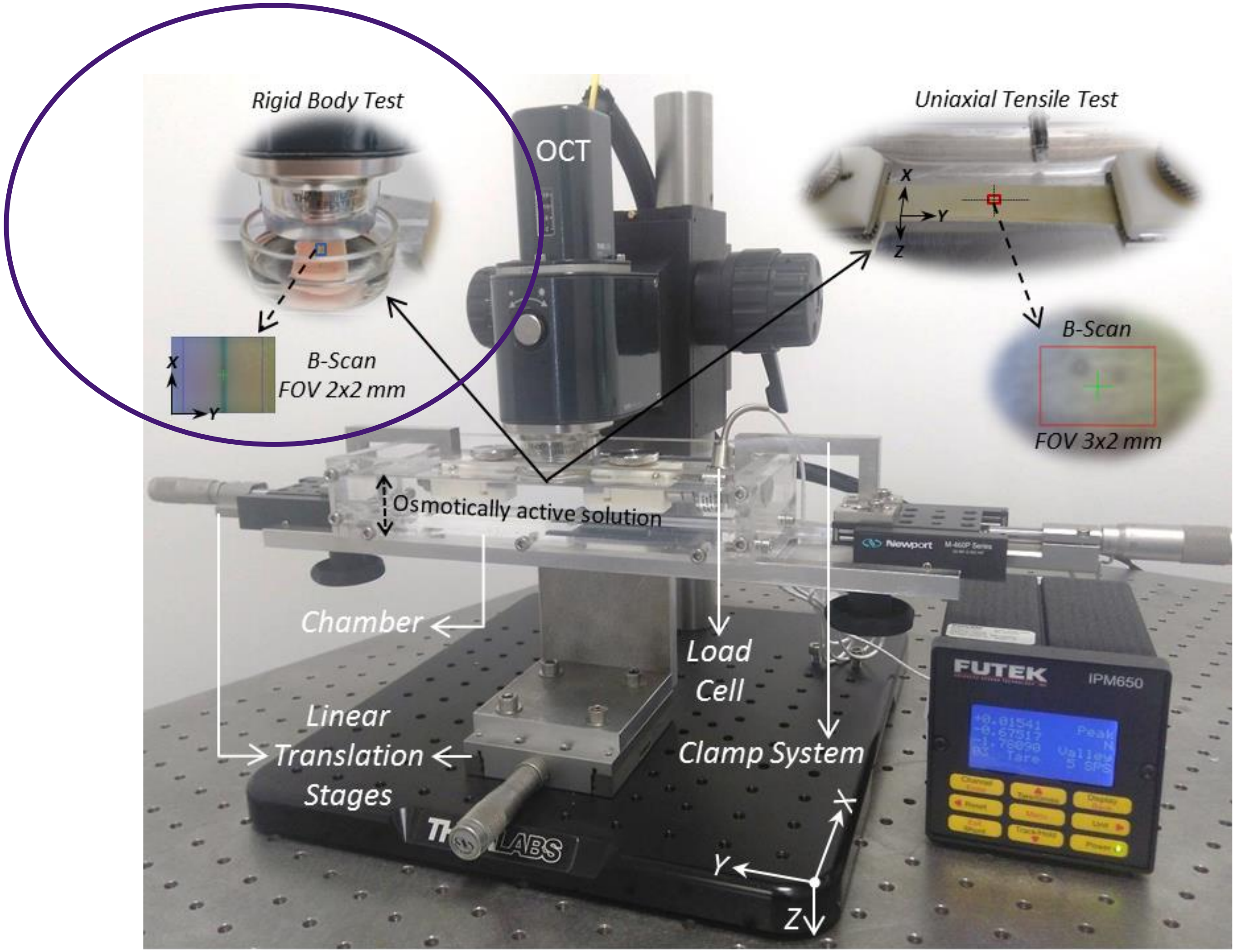
*ventral* *dorsal* *inflation*

CS38




avril@emse.fr

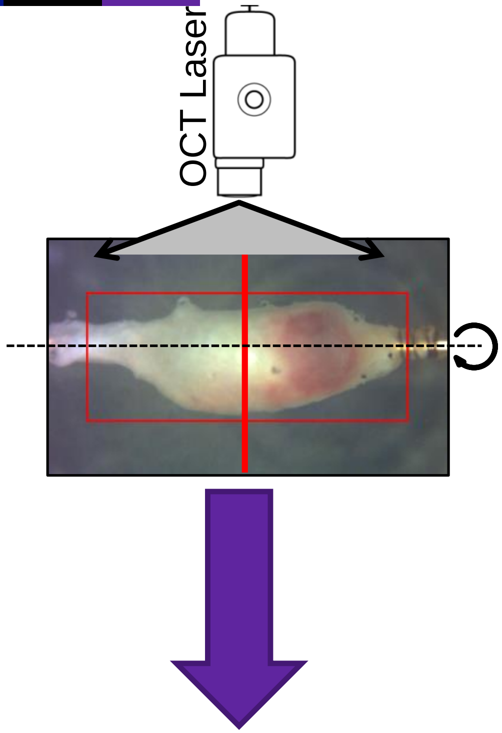
# OCT-DVC applied to arterial mechanics

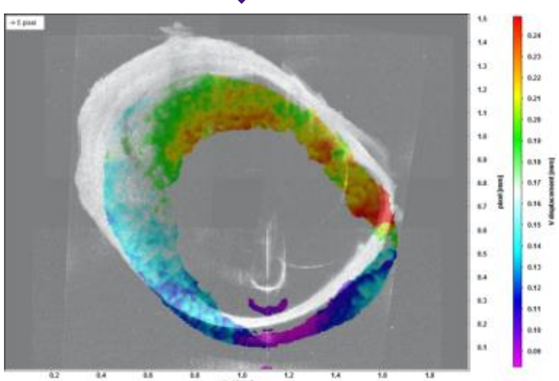


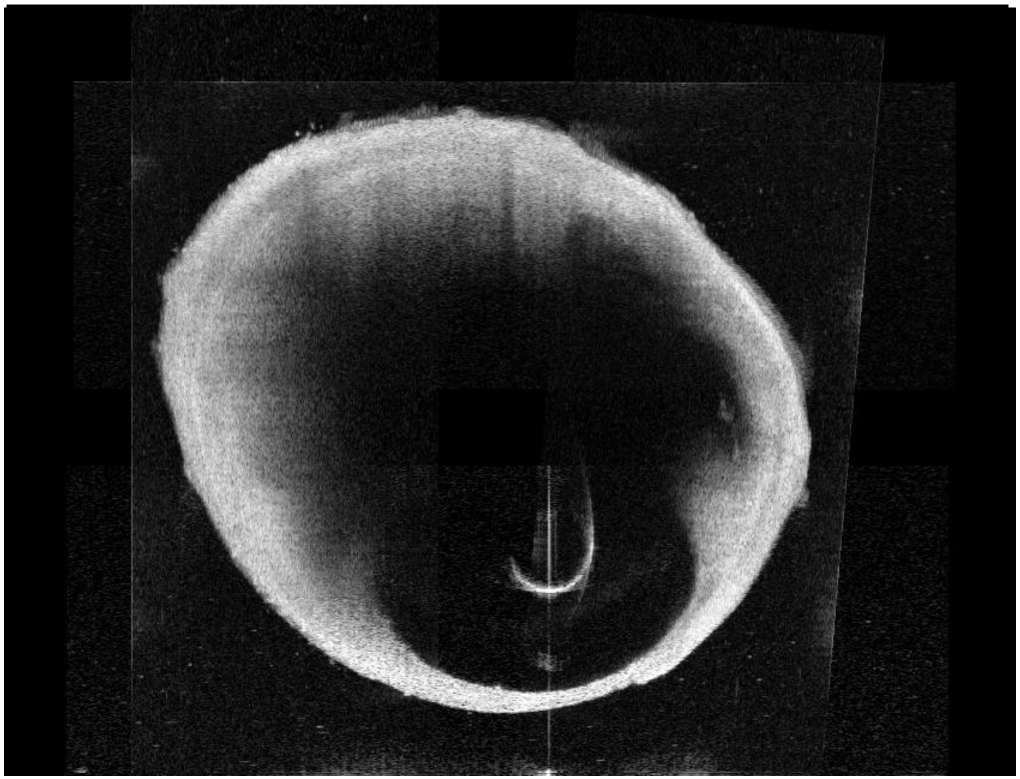



MINES Saint-Étienne

# Measurement of bulk deformation fields by Digital Volume Correlation on OCT images

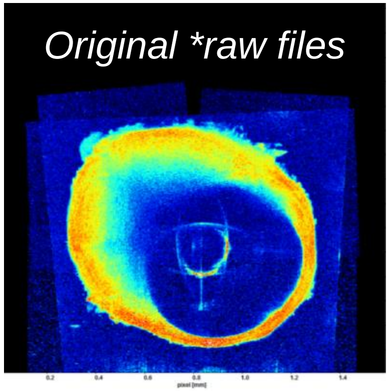


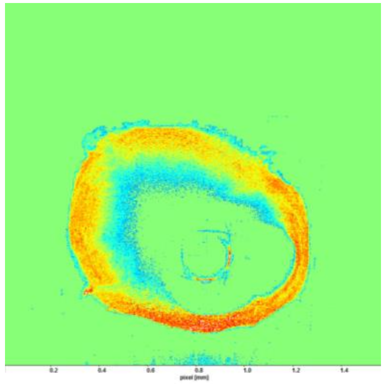
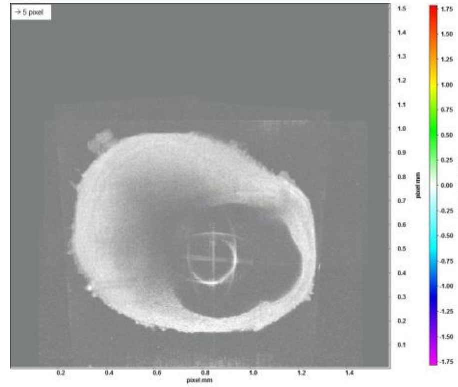
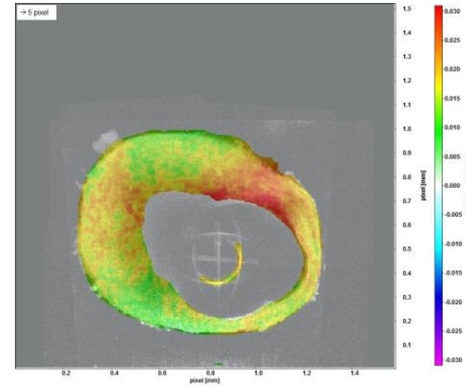






MINES Saint-Étienne

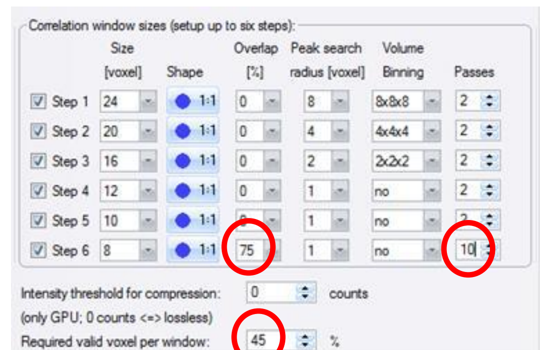
# Image Processing Methodology – Inflation Test



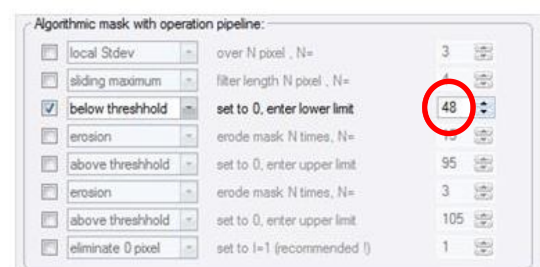
Title: REF
Width: 667 pixels
Height: 640 pixels
Depth: 100 pixels
Voxel size: 1x1x1 pixel^3 (2.38um)

*Original Image Parameters*




Pressures: 20, 40, 60, 80, 100, 120, 140
Lambda (λ) 1 – 2 – 3

- ***Algorithmic Mask – Threshhold: 48***
- ***Correlation window size [voxel]: 24 (8x8x8)***
- ***Overlap : 75%***
- ***Passes : 10***
- ***Required valid voxel per window: 44%***





MINES Saint-Étienne