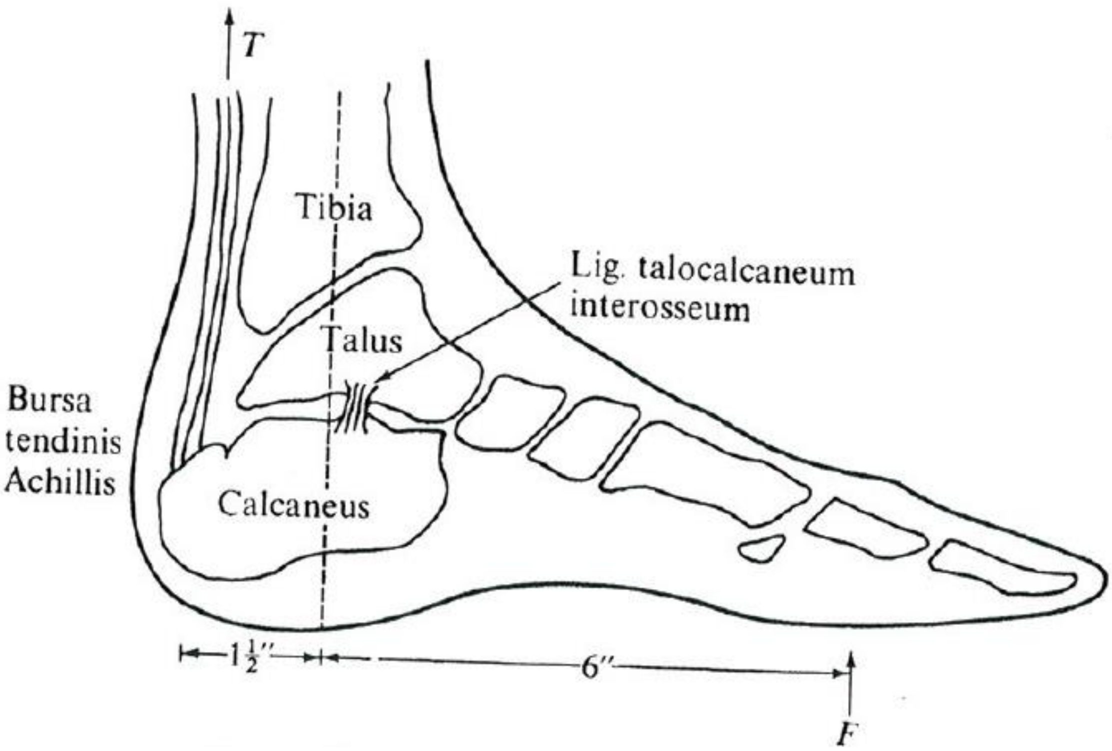
719.001 - Mechanics of biological tissues

Assignment #1.
Make groups of 2 or 3 and answer the following questions. Send the solution (1 per group) by email (pdf or scanned copy) to avril@emse.fr.
Deadline: 20th October 2021 (18:00)

Problem.
We consider a contraction of the gastronemius muscle in the calf that applies a tension in the Achilles tendon permitting to raise the heel.
Foot dimensions are given below in the schematic (1'' = 25mm).

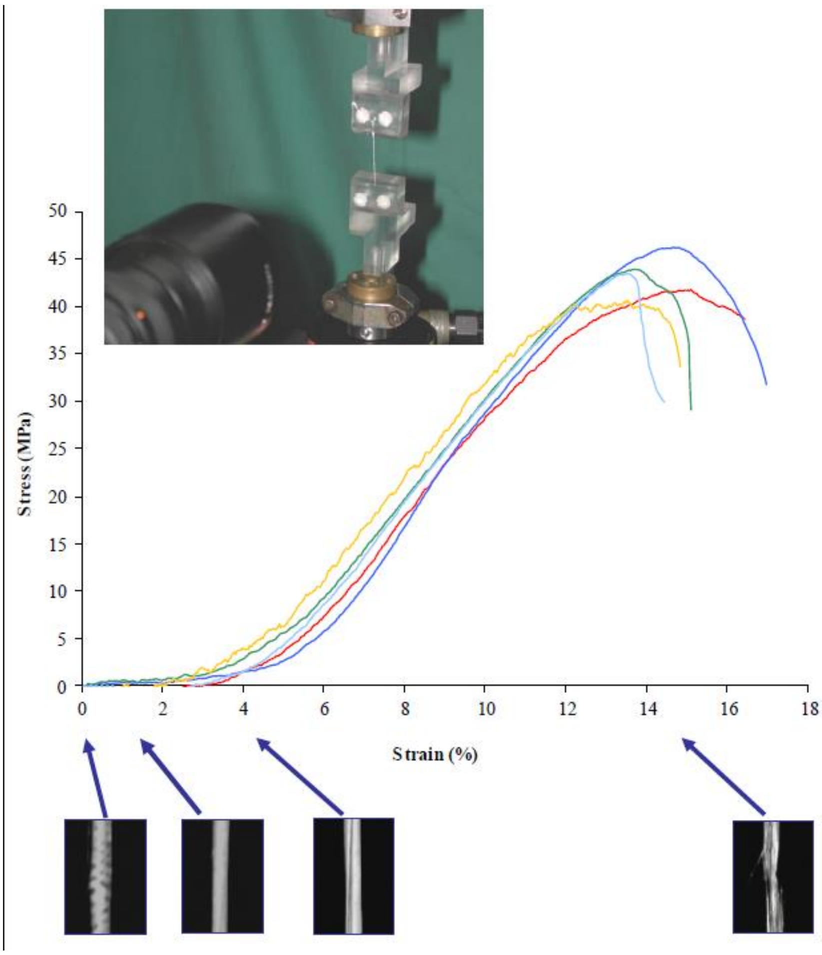
Remark. The dotted line in the schematic represents the line of the weight action applied by the rest of the body on the foot.



1. Derive the tension force T and the axial stress σ in an Achilles tendon having a cross section area of 1.6 cm^2 when a 70 kg person raises the heel.

2. The person jumps and lifts up to an elevation of 70 cm. Use the conservation of mechanical energy (kinetic energy $mv^2/2$ + potential energy mgz) to find the velocity of the person at take-off.

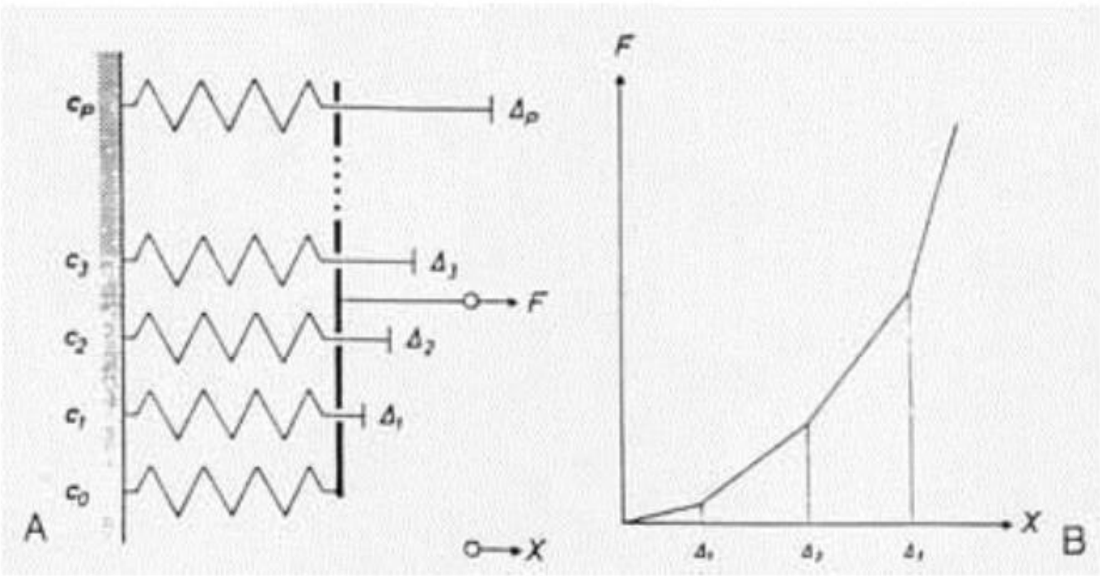
3. We assume it takes about 0.6 s to stretch the Achilles tendon and make the foot take off. Use the fundamental principle of dynamics (force = mass . acceleration) to derive the reaction force on the ground (F) and the axial stress σ in the Achilles tendon when the person jumps.
4. Scientists have characterized the tensile properties of the Achilles tendon and obtained the following response



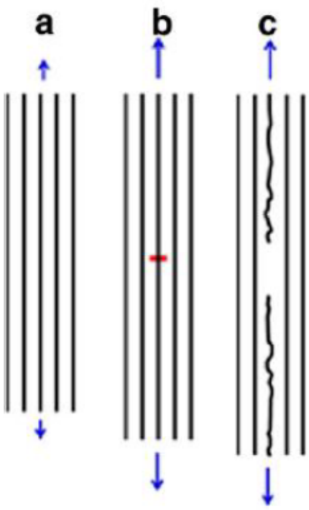
How does the stress in a jump compares to the strength of the Achilles tendon? How likely is an injury when we jump?

5. A simple model illustrating the dependence of tendon nonlinear stress/strain relationships is shown below. Each spring represents collagen fibrils (assumed average length: 60 μm, assumed averaged diameter: 200 nm). When a fibril i is crimped, $X<\Delta i$. The average elastic modulus of a collagen fibril is 2.5 GPa and its average strength is 100 MPa (L. Yang thesis, University of Twente). The effective tangent elastic modulus of a tendon varies linearly from 5 MPa to 50 MPa when the

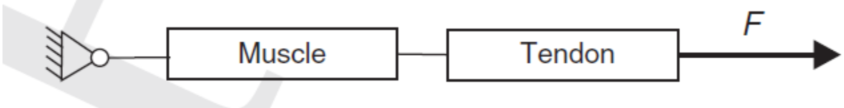
ligament is stretched up to 10% of its initial length. Determine the proportion of uncrimped fibrils. Determine the stress in the fibrils.



6. What is the proportion of fibrils that may break when the person jumps. Does the fibrous structure make the tendon more or less likely to break as compared to a homogeneous material?



7. Kinetic energy was needed for take-off (question 2). The same kinetic energy will be obtained also at landing. Make a schematic to explain how a fraction of this kinetic energy can be stored as potential energy in the Achilles tendon. You can use the following type of representation in your schematic



8. How much (approximately) potential energy can be stored in a stretched tendon given the curves of question 4 and standard dimensions of the Achilles tendon? How high can the person jump if all this potential energy could be transformed in kinetic energy?

9. Explain why only a fraction of the energy stored in the stretched Achilles tendon can be transformed in kinetic energy. How much is this fraction? How can training improve this fraction to jump higher?