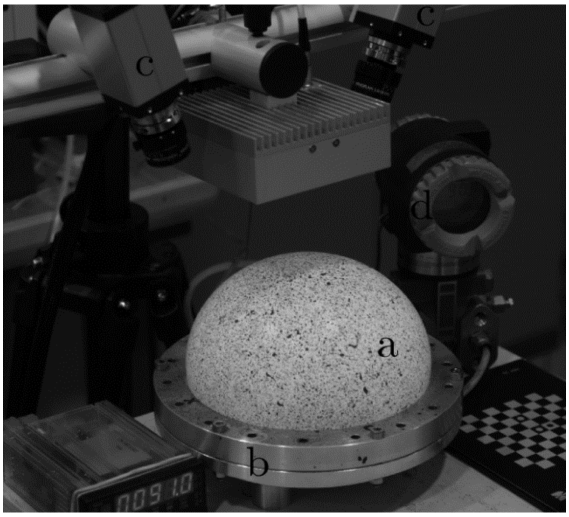
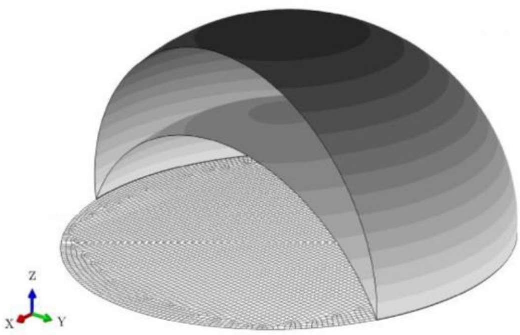
719.001 - Mechanics of biological tissues

Assignment #1.
Make groups of 2 or 3 and answer the following questions. Send the solution (1 per group) by email (pdf or scanned copy) to avril@emse.fr.
Deadline: 24th November 2021 (18:00)

Problem.
The goal of this assignment is to derive the hyperelastic parameters of a soft tissue membrane using a bulge inflation test as shown in the figure below. Initially the membrane is a disc of 30 mm diameter clamped all around. The thickness is 2 mm.





1. Assume the membrane is NeoHookean incompressible with a shear modulus G=0.2 MPa. The membrane is inflated until reaching a maximum deflection at the apex equal to 15 mm. Assuming that the shape in a hemisphere (only valid when the deflection is 15 mm), derive the pressure needed to reach such deflection (for that use the equations for the balloon inflation studied in class).

2. The displacement field was measured during the inflation of the membrane (made of aortic tissue) using the stereo image correlation method. A cloud of 644 points was defined in the field of view in initial position which is defined as the reference configuration. The coordinates in the reference configuration are (x_0, y_0, z_0). For each point, the digital image stereo-correlation method can reconstruct the coordinates (x, y z) for 43 loading steps. Each loading step corresponds to a pressure P applied in the system.

The problem data are available in Matlab format in the data_TP.mat file. You will find there (x_0, y_0, z_0), (x, y z) and P (given in kPa).
Plot the curve of the variation of P as a function of z at the apex of the membrane.
The inverse problem consists in finding a constitutive law for the membrane which would reproduce the same response curve. One solution would be to develop a finite element model of the bulge inflation test and then to update the parameters of the model soas to minimize the difference between the measured displacements and the displacements predicted by the model.
We will develop an alternative approach using the Laplace law.

3. From the available data, for every load step, derive the deformation gradient and the Green Lagrange strains at the apex of the inflated membrane.

Hint: use a least-squares minimization to find a linear transformation relating (x_0, y_0, z_0) to (x, y z) for a set of points around the apex.

4. Assume that the behavior of the tissue is NeoHookean incompressible with a shear modulus of 1 MPa. Write the expression of the stress as a function of the Green Lagrange strain and reconstruct the Cauchy stress at the apex of the inflated membrane for every loading step.

5. Derive the radius of curvature of the membrane at the apex of the inflated membrane for every loading step.

Hint: use a least-squares minimization to find the osculating sphere at the apex of the membrane.

6. Apply the Laplace law to predict the pressure related to the stress values obtained at question 4 and plot the values of the predicted pressure versus the maximum deflection at the apex. Compare the predicted pressured to the measured pressure.

7. Find the shear modulus that minimizes the deviations between the predicted pressures and the measured ones. Plot the obtained pressures and calculate the coefficient of determination R2. Comment on the ability of the Neohookean model to fit the data.

8. Propose another hyperelastic model to fit the data, plot the predicted pressures and the measured ones on the same graph, and compute the new R2 value.

Hint: Find below a possible solution