Assignment: Intracranial Saccular Aneurysms
compiled with Professor Jonas Stålhand, Linköping University, Sweden

Class at Graz University of Technology, Austria, on
'Mechanics of Biological Tissues' – 719.001, Wintersemester 2020/21
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Introduction

Intracranial aneurysms (ICA) appear as sac-like out-pouching of the arterial wall in the brain vasculature, and are inflated by the blood that fills them, see Fig. 1. They are relatively frequent and affect 2-5% of the adult population. Most of them are asymptomatic and remain undetected, but there is a inherent risk of rupture and approximately 0.1-1% rupture per year. If the rupture leads to a subarachnoid haemorrhage, the fatality rate is 30-50%.

Figure 1: Saccular cerebral aneurysm in the circle of Willis. The aneurysm is visible as a small out-pouching of the arterial wall slightly to the left, at the middle of the image. Taken from: `neuropathology-web.org/chapter2/chapter2cCerebralhemorrhage.html`

The exact cause or the formation of ICA is not yet fully understood. Female gender, high blood pressure, smoking, heavy alcohol consumption, and hereditary factors all contribute to the risk of ICA. Mechanical factors also play an important role. It has been shown that the hemodynamic wall shear stress strongly correlate to regions where ICA form, although the blood pressure is probably the driving factor behind the aneurysm growth. Apoptosis of smooth muscle cells and elimination of elastin together with an increased collagen production by fibroblasts are also hall-marks for the aneurysm wall. As a consequence, the mechanical response of a fully matured ICA is very much dependent on the collagen structure.

Because of the rapid development of medical imaging in combination with screening of the population, we know that the number of detected ICA is increasing. The method to assess the rupture risk of an aneurysm and the need for surgical intervention is currently based on its size. In Sweden, e.g., surgical intervention is recommended for ICA when larger than 5 mm, a number used as a rule of thumb. This criterion is not robust since aneurysms below this limit are known to rupture, too.

In mechanics it is well-established that the rupture risk is associated with the wall stress rather than the size. When the stress exceeds the maximum tolerable value for the material, the structure breaks. This insight can be used to better assess rupture risk. In this assignment, you will use a (very) simplified mechanical model to study the response of ICA, and assess the rupture size.

Aneurysm Model

Assume that a ICA can be modeled by a spherical, isotropic and incompressible membrane. In its unloaded reference state, the aneurysm has a radius R and a wall thickness H, see Fig. 2. When it is subjected to an internal pressure P, it deforms to a new sphere with radius r and wall thickness h. Since soft tissues are assumed to be incompressible, the volume of the aneurysmal wall is preserved throughout the deformation and, therefore,

$$4\pi R^2 H = 4\pi r^2 h. \tag{1}$$

The sphere is a three-dimensional structure and the deformation gradient $\mathbf{F}$ is no longer a single value but a 3×3 matrix. For an isotropic sphere subjected to a uniform pressure, the deformation will be shear free and the deformation gradient becomes a diagonal matrix, i.e. $[\mathbf{F}] = \text{diag}[\lambda_\varphi, \lambda_\theta, \lambda_r]$, where the indices φ, θ and r indicate the azimuthal, polar and radial directions, respectively. Because of the spherical symmetry, the azimuthal and polar stretches are equal such that $\lambda = \lambda_\varphi = \lambda_\theta$. For a membrane we have

$$\lambda = \frac{r}{R}, \qquad \lambda_r = \frac{h}{H}. \tag{2}$$

Using these definitions, Eq. (1) can be rewritten as

$$\lambda^2 \lambda_r = 1. \tag{3}$$

Equation (3) provides us with an implicit definition of the third stretch λ_r which will be used later.

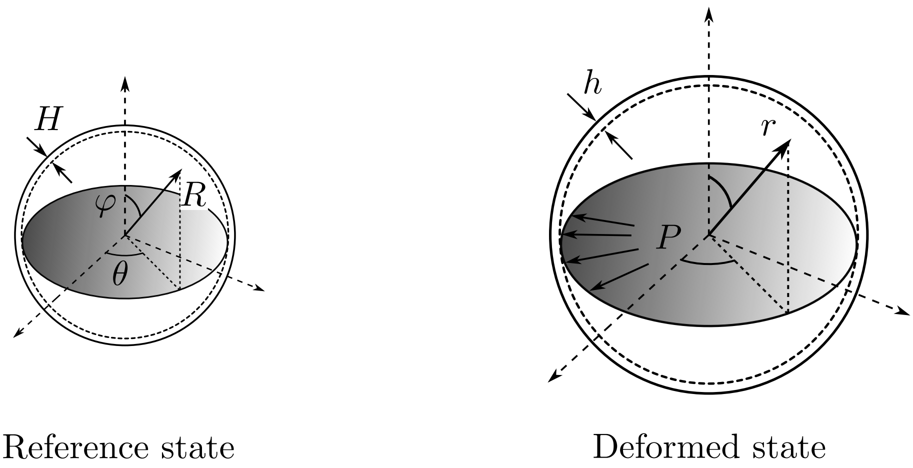

Figure 2: Idealized aneurysm in the unloaded and pressure-free reference state and the pressurized deformed state.

Variable	Value	Description
R	3.0 mm	Referential radius
H	278 μm	Referential wall thickness

Table 1: Values for the referential geometry of the aneurysm model[1].

Question I (3pt)

write down the force equilibrium for an aneurysm of radius r and wall-thickness h subjected to an internal pressure P and derive an expression for the in-plane principal stresses (this expression is referred to as Laplace's law in the literature). Because the aneurysm is assumed to be a membrane, you may take the wall stress σ constant in the radial direction and set the external pressure to zero.

Question II (3pt)

In experiments it is customary to measure the aneurysm dimensions in the referential state. Rewrite the expression for the wall stress derived in Question I in terms of P, R, H, and F, for specific values see Table 1.

Question III (3pt)

Since the aneurysm is modeled as an isotropic material, let us assume a neo-Hookean strain-energy function. This is a very common strain energy for rubber-like materials and is often used for elastin-rich tissues. Its functional form is

$$\Psi = \frac{\mu}{2}(\lambda_\varphi^2 + \lambda_\theta^2 + \lambda_r^2 - 3), \tag{4}$$

[1] Steiger, et al., *Heart Vessels* (1989) 5:41-46

where $\mu > 0$ is a material constant. By using this strain-energy function, derive an expression for the Cauchy stresses in the membrane as a function of λ.

Question IV (3pt)

Download the data file `aneurysmdata.txt` from the course homepage which includes experimental data from an *in vitro* inflation test of an aneurysm. Use the data to compute the material parameter μ in Eq. (4) by a linear regression.

Question V (3pt)

A long standing question regarding ICA has been how structures consisting of large amounts of stiff collagen with low resilience can continue to grow and eventually rupture. One hypothesis put forward which explains this has to do with a limit point instability. This means that the pressure $P = P(\lambda)$ initially increases but reaches an extrema (maximum) after which it drops. Show that this is the case when using the strain energy in Question III and compute the critical stretch $\lambda = \lambda_{\rm crit}$ when this happens.

Question VI (5pt)

The limit point instability hypothesis is based on experiments where collagen patches were glued onto a thin elastomeric (rubber) sheet[2]. A serious critique has been raised regarding the validity for ICA; the limit point instability may originate from the elastomer and need not be an intrinsic property of the aneurysm material. In fact, most soft biological tissues are better described by an exponential response than by a rubber-like response. Revisit Question IV and compute the material constants using an exponential strain-energy function, see class.

Hint: the exponential function can be made linear in the material constant by applying the natural logarithm. Study the pressure-stretch response and see if the instability remains. Is the the limit point hypothesis reasonable? If not, can you think of another reason why aneurysms grow?

Question VII (5pt)

The maximum wall stress before rupture has been reported to be $\sigma_{\rm max} = 500\,{\rm kPa}$[3]. Use the exponential strain-energy function and estimate the maximum pressure $P_{\rm max}$ and the corresponding stretches $\lambda_{\rm max}$ and $\lambda_{r\,{\rm max}}$ at which the aneurysm will rupture. In addition, determine the size of the aneurysm at rupture.

[2] Austin et al., *Neurosurgery* (1989) 24:722-730
[3] Steiger et al., *Heart Vessels* (1989) 5:41-46

Reporting

The assignment is to be presented in terms of a short written report. Write the report using a word processor (e.g., LaTeX, Mircosoft Word, Open Office Writer, or Google Docs) with a standard font, and size 11-12 pt. Page margins should be normal. The **report must not exceed 5 pages** including derivations, figures, results, tables, references, etc. In addition, attach any code used to solve the assignment. This attachment is not subject to the page limit. Send the report **as a pdf** to the e-mail address `holzapfel@tugraz.at`. Name the file as:

`assignment_{your first name}_{your family name}.pdf`,

e.g., assignment_Gerhard_Holzapfel.pdf. The deadline for submitting the report is November 24, 2020 (last class).