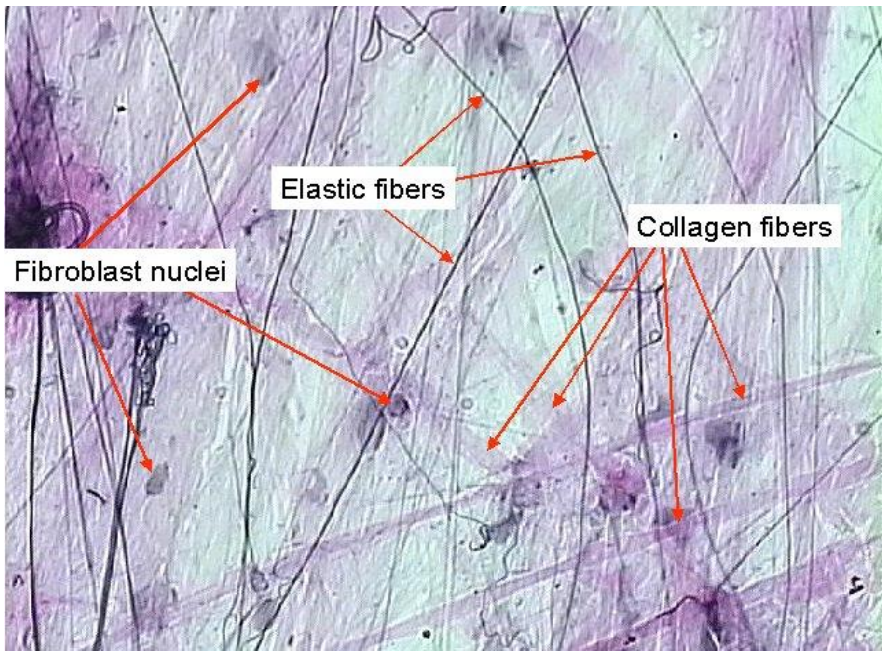
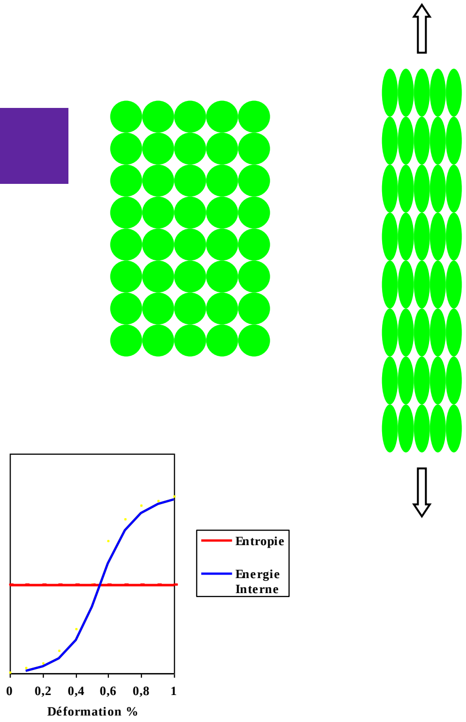
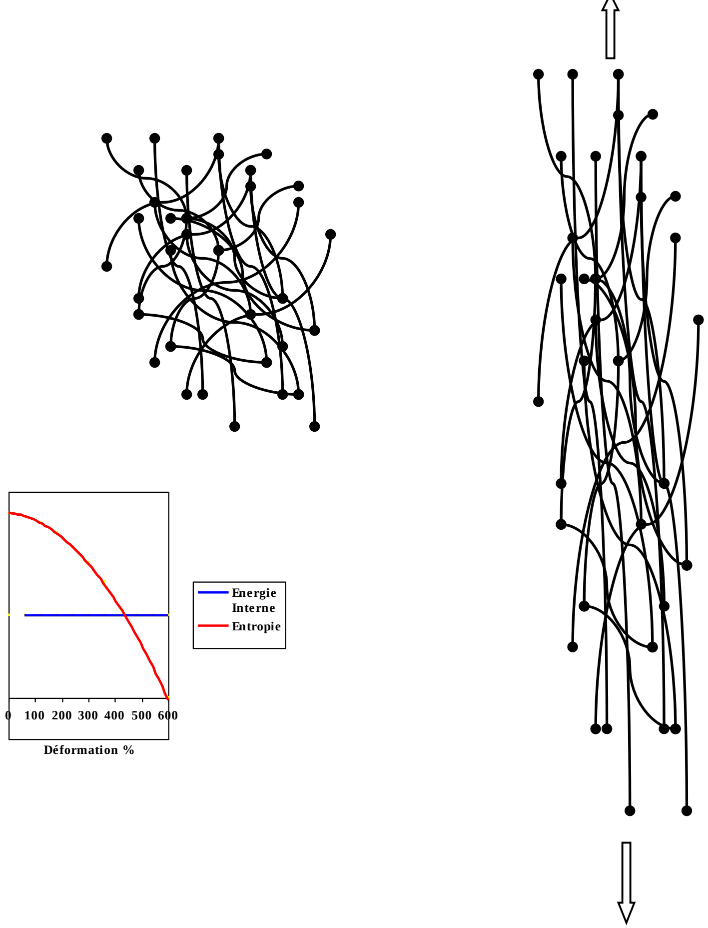
Soft biological tissues: many challenges for continuum mechanics







enthalpic
elasticity
(cristal)

entropic
elasticity
(biological tissue)

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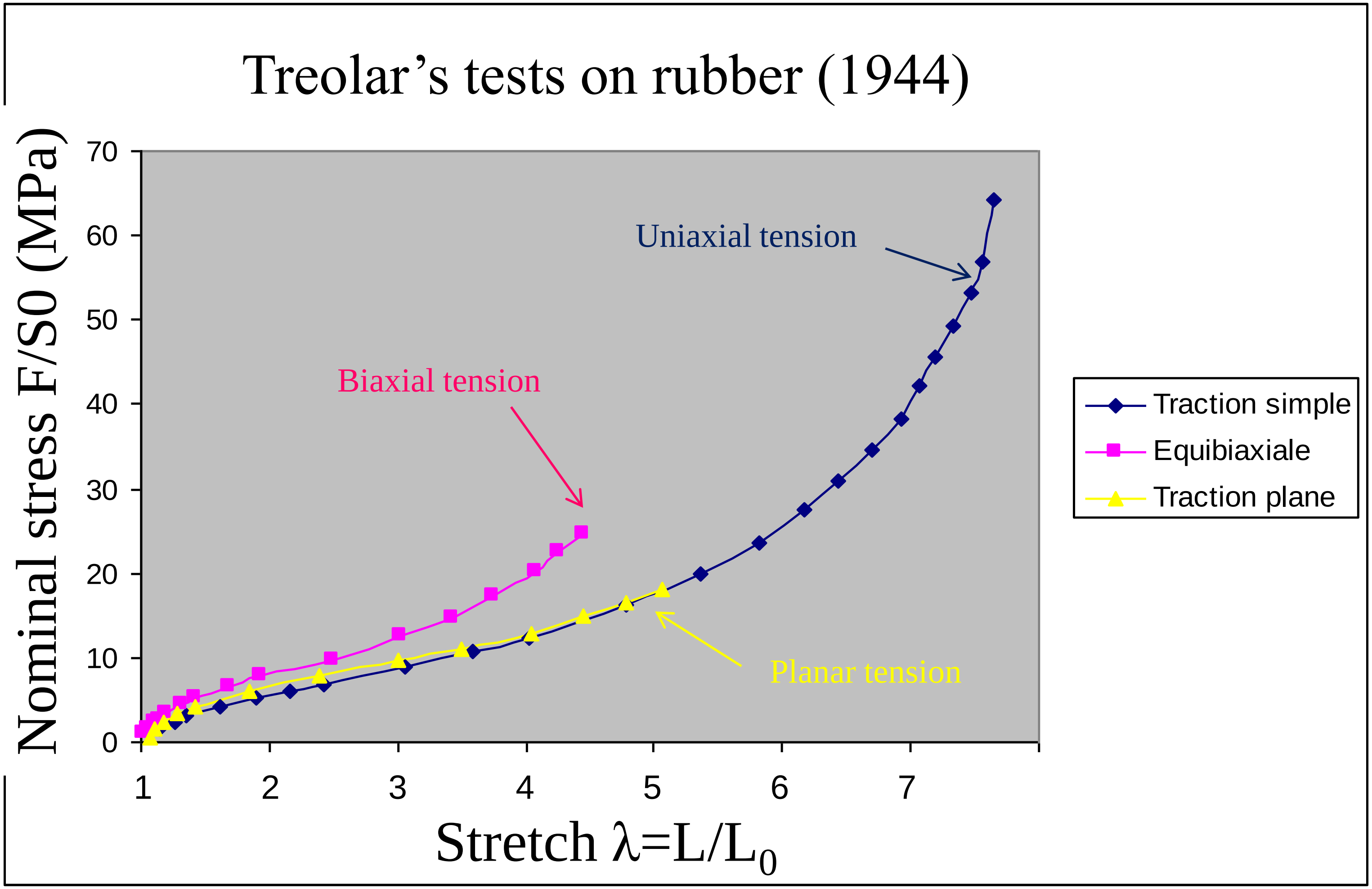
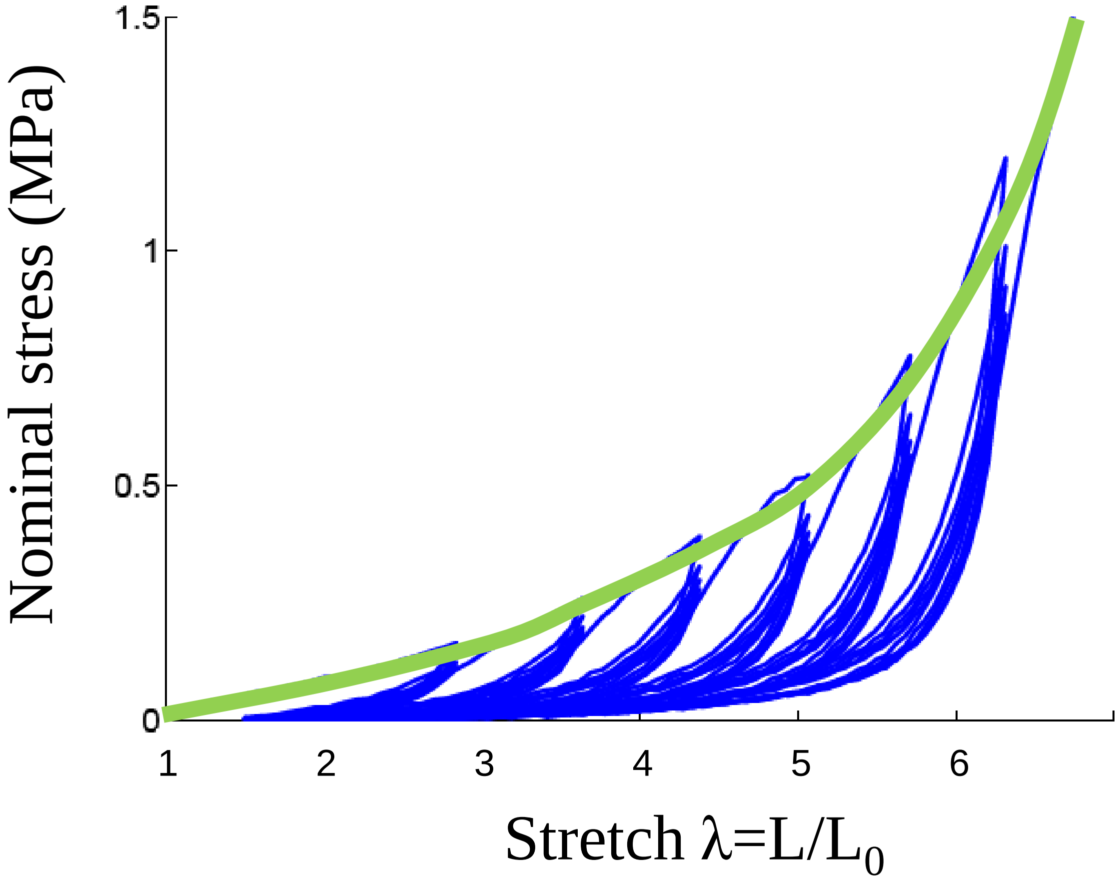
Hyperelasticity



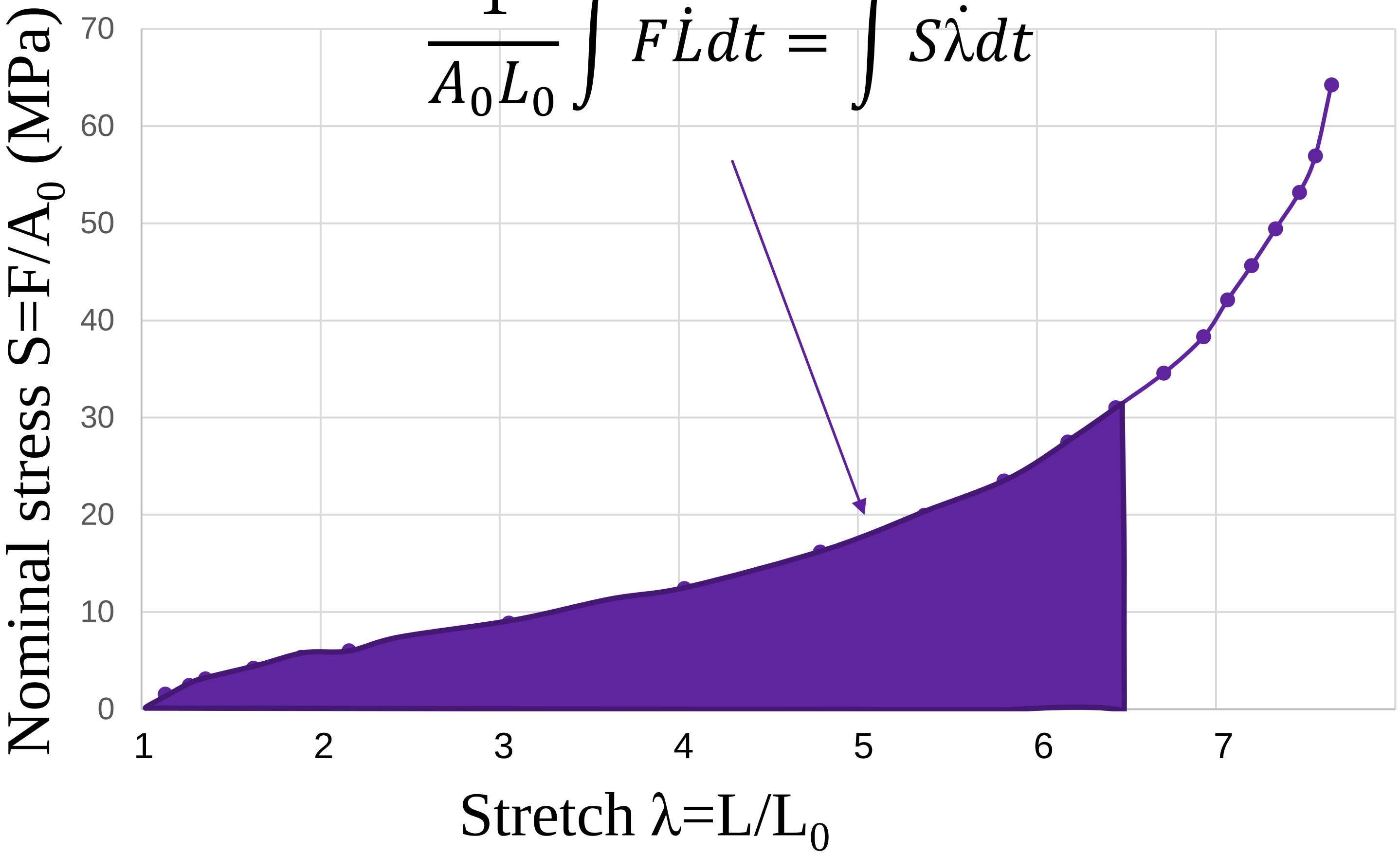
Pseudo-hyperelasticity:
Irreversible effects are neglected…



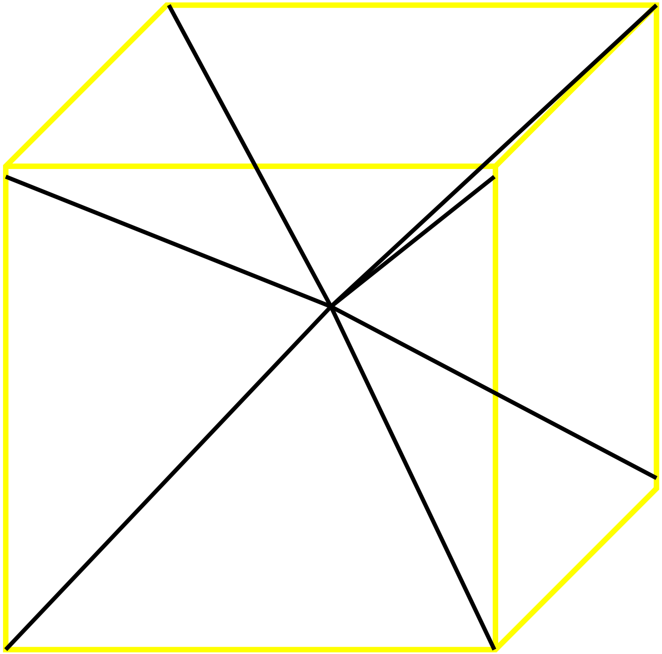
Strain energy density

Stored energy per unit volume

$$\frac{1}{A_0 L_0}\int F\dot{L}dt = \int S\dot{\lambda}dt$$



first invariant?



Deformation mapping in 3D

$$y_i = x_i + u_i(x_1, x_2, x_3, t)$$

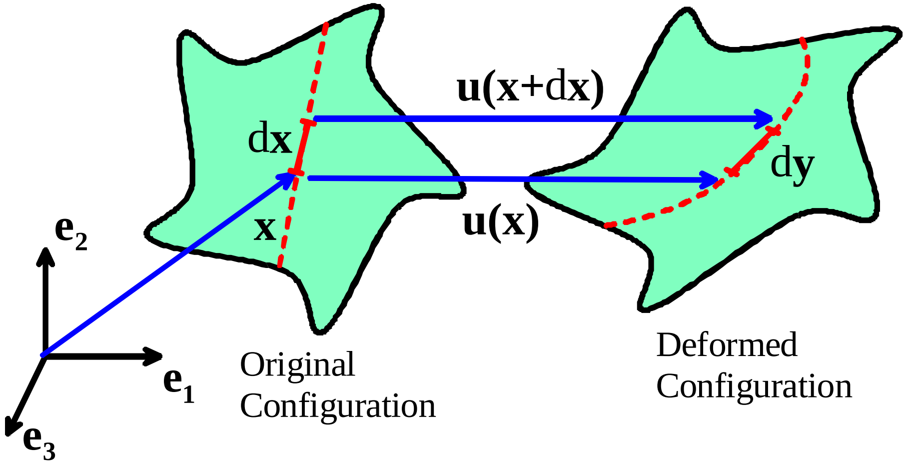
Deformation Gradient

$$\nabla \mathbf{y} = \nabla\left(\mathbf{x} + \mathbf{u}(\mathbf{x})\right) = \mathbf{F}$$

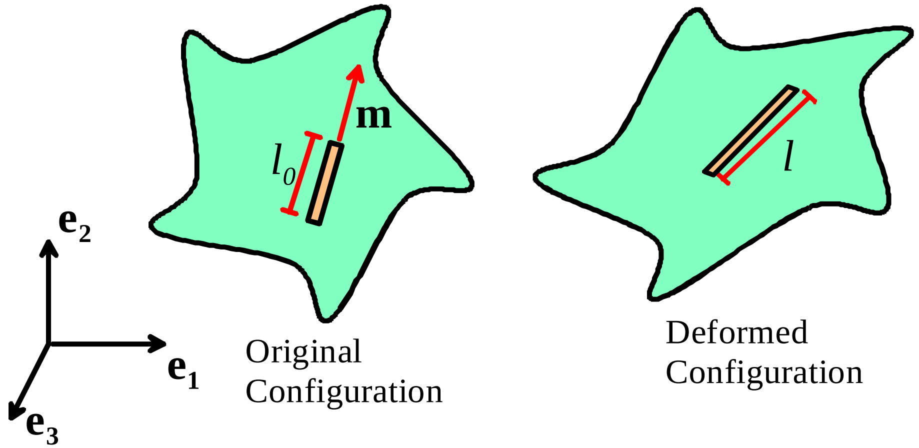
or $$\frac{\partial y_i}{\partial x_j} = \frac{\partial}{\partial x_j}\left(x_i + u_i\right) = \delta_{ij} + \frac{\partial u_i}{\partial x_j} = F_{ij}$$

$$d\mathbf{y} = \mathbf{F} \cdot d\mathbf{x}$$

$$dy_i = F_{ik} dx_k$$



Green Lagrange strain



$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^T \cdot \mathbf{F} - \mathbf{I}) \quad \text{or} \quad E_{ij} = \frac{1}{2}(F_{ki}F_{kj} - \delta_{ij})$$

$$\mathbf{m} \cdot \mathbf{E} \cdot \mathbf{m} = E_{ij} m_i m_j = \frac{l^2 - l_0{}^2}{2 l_0{}^2} = \frac{\delta l}{l_0} + \frac{(\delta l)^2}{2 l_0^2}$$

Stress measures

True / Cauchy

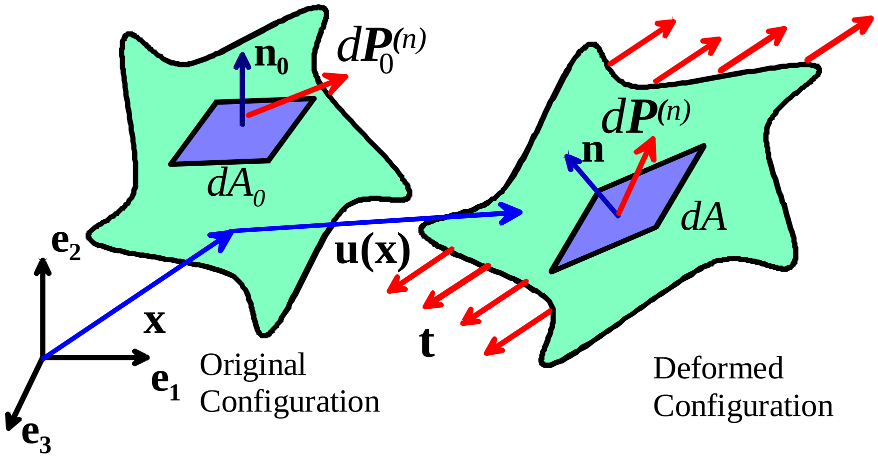
$$\boldsymbol{\sigma}$$

Nominal/ 1st Piola-Kirchhoff

$$\mathbf{S} = J\mathbf{F}^{-1} \cdot \boldsymbol{\sigma} \qquad S_{ij} = JF_{ik}^{-1}\sigma_{kj}$$

Material/2nd Piola-Kirchhoff

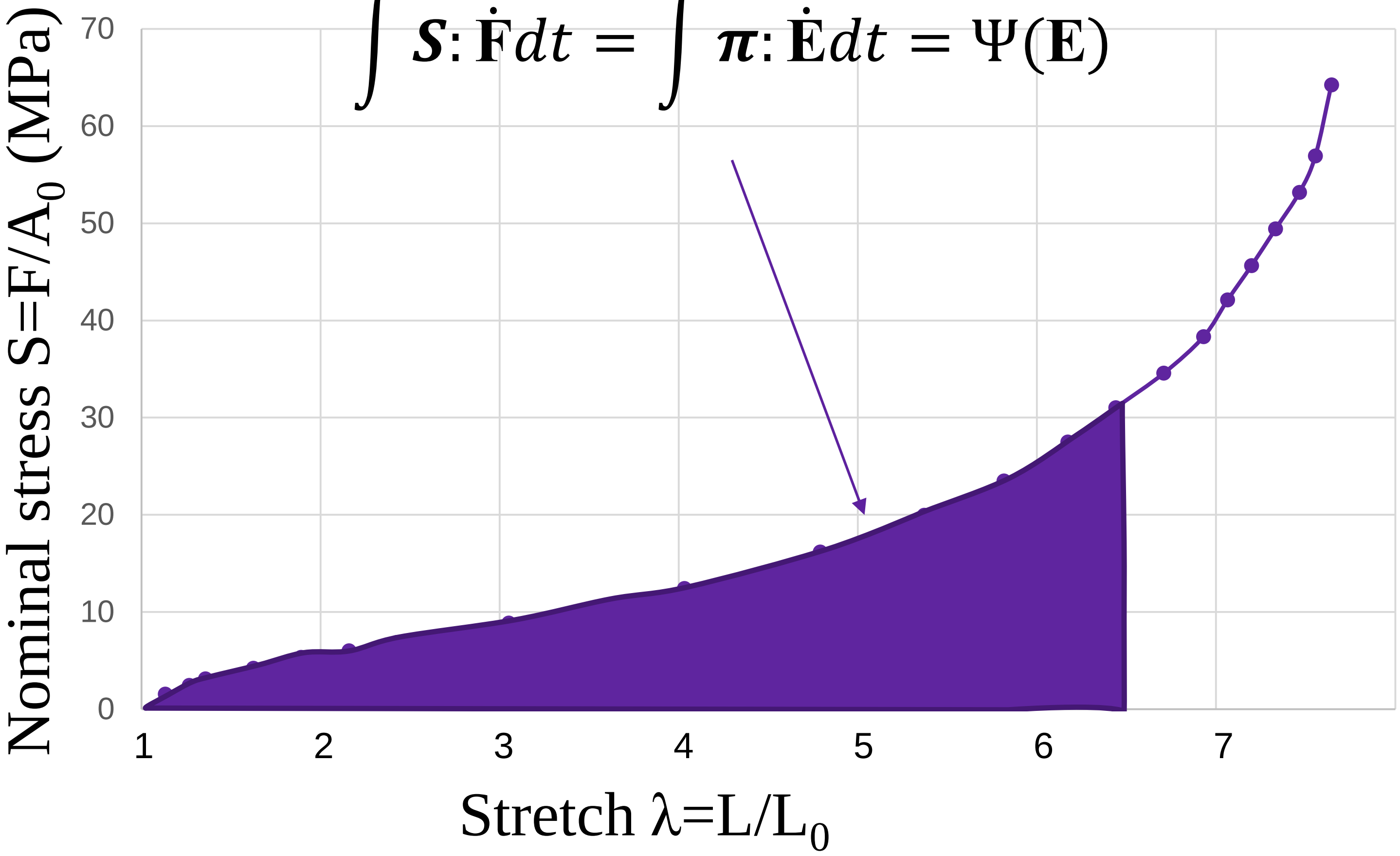
$$\boldsymbol{\pi} = J\mathbf{F}^{-1} \cdot \boldsymbol{\sigma} \cdot \mathbf{F}^{-T} \quad \pi_{ij} = JF_{ik}^{-1}\sigma_{kl}F_{jl}^{-1}$$



Hyperelasticity

Stored energy per unit volume

$$\int \boldsymbol{S} : \dot{\mathbf{F}} dt = \int \boldsymbol{\pi} : \dot{\mathbf{E}} dt = \Psi(\mathbf{E})$$



compressible hyperelastic behaviour

$$\boldsymbol{\sigma} = J\mathbf{F}.\frac{\partial\Psi}{\partial\mathbf{E}}.\mathbf{F}^T$$

incompressible hyperelastic behaviour (J=1)

$$\boldsymbol{\sigma} = \mathbf{F}.\frac{\partial\Psi}{\partial\mathbf{E}}.\mathbf{F}^T + c\mathbf{I}$$

Strain energy density: $\Psi = ?$

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$$\Psi(\mathbf{E}) = \Psi(\bar{I}_1, \bar{I}_2, J)$$

Polynomials of the first invariant

$$\Psi(\bar{I}_1, \bar{I}_2, J) = \sum_{i=1}^{N} C_{i0}\left(\bar{I}_1 - 3\right)^i + \sum_{i=1}^{N} \frac{1}{D_i}\left(J - 1\right)^{2i}$$

Initial shear modulus $\mu_0 = 2C_{10}$

Initial compressibility modulus $k_0 = \frac{2}{D_1}$

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$$\Psi(\mathbf{E}) = \Psi(\bar{I}_1, \bar{I}_2, J)$$

Polynomials of the first invariant

$$\Psi(\bar{I}_1, \bar{I}_2, J) = \sum_{i=1}^{N} C_{i0}\left(\bar{I}_1 - 3\right)^{i} + \sum_{i=1}^{N} \frac{1}{D_i}\left(J - 1\right)^{2i}$$

Particular case 1: Neo-Hookean behaviour (N=1)

$$\Psi\left(\bar{I}_1, \bar{I}_2, J\right) = C_{10}\left(\bar{I}_1 - 3\right) + \frac{1}{D_1}\left(J_{el} - 1\right)^2$$

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$$\Psi(\mathbf{E}) = \Psi(\overline{I}_1, \overline{I}_2, J)$$

Polynomials of the first invariant

$$\Psi(\overline{I}_1, \overline{I}_2, J) = \sum_{i=1}^{N} C_{i0}\left(\overline{I}_1 - 3\right)^i + \sum_{i=1}^{N} \frac{1}{D_i}\left(J - 1\right)^{2i}$$

Particular case 2: Yeoh behaviour (N=3)

$$\Psi\left(\overline{I}_1, \overline{I}_2, J\right) = \sum_{i=1}^{3} C_{i0}\left(\overline{I}_1 - 3\right)^i + \sum_{i=1}^{3} \frac{1}{D_i}\left(J_{el} - 1\right)^{2i}$$

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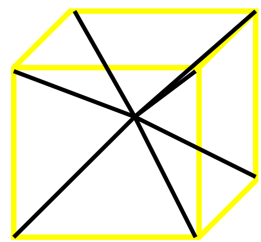
$$\Psi(\mathbf{E}) = \Psi(\bar{I}_1, \bar{I}_2, J)$$

Other functions of the first invariant

ARRUDA-BOYCE

Model with 8 chains



Statistical mechanics
Gaussian chains

$$\rho_0 \psi = Nk\theta\sqrt{n}\left[\beta\lambda_{chain} - \sqrt{n}\ln\left(\frac{\sinh\beta}{\beta}\right)\right]$$

$$\lambda_{chain} = \sqrt{\frac{\bar{I}_1}{3}} \text{ and } \beta = \ell^{-1}\left(\frac{\lambda_{chain}}{\sqrt{3}}\right)$$

$$\Psi\left(\bar{I}_1, \bar{I}_2, J\right) = \mu\sum_{i=1}^{5}\frac{C_i}{(\lambda_m)^{2i-2}}\left(\bar{I}_1^{\,i} - 3^i\right) + \frac{1}{D}\left[\frac{\left(J_{el}^{\,2} - 1\right)}{2} - \ln\left(J_{el}\right)\right]$$

$$C_1 = \frac{1}{2}; \quad C_2 = \frac{1}{20}; \quad C_3 = \frac{11}{1050}; \quad C_4 = \frac{19}{7050}; \quad C_5 = \frac{519}{673750}$$

$$\Psi(\mathbf{E}) = \Psi(\bar{I}_1, \bar{I}_2, J)$$

Forms written with the principal stretches

OGDEN

$\lambda_1, \lambda_2, \lambda_3$ Principal stretches $\bar{\lambda}_i = J^{-\frac{1}{3}} \lambda_i$

$$\Psi(\bar{I}_1, \bar{I}_2, J) = \sum_{i=1}^{N} \frac{2\mu_i}{\alpha_i^2} \left[\bar{\lambda}_1^{\alpha_i} + \bar{\lambda}_2^{\alpha_i} + \bar{\lambda}_3^{\alpha_i} - 3 \right] + \sum_{i=1}^{N} \frac{1}{D_i} (J_{el} - 1)^{2i}$$

$$k_0 = \frac{2}{D_1} \qquad \mu_0 = \sum_{i=1}^{N} \mu_i$$

$$N = 2; \quad \alpha_1 = 2; \quad \alpha_2 = -2; \quad \Rightarrow \quad \text{Mooney} - \text{Rivlin}$$

$$N = 1; \quad \alpha_1 = 2; \quad \Rightarrow \quad \text{Neo} - \text{Hookean}$$

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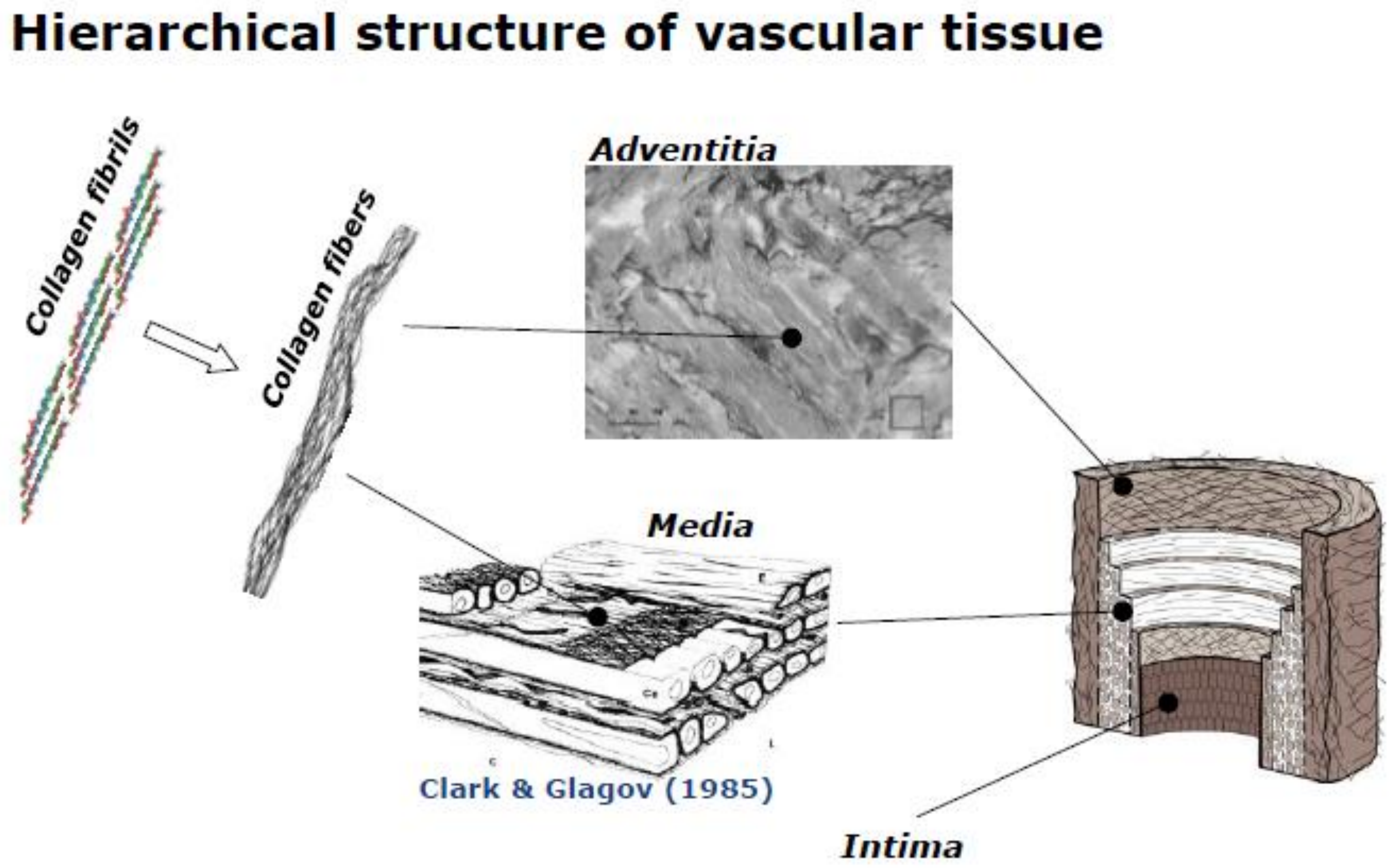
Anisotropy induced by collagen fibers



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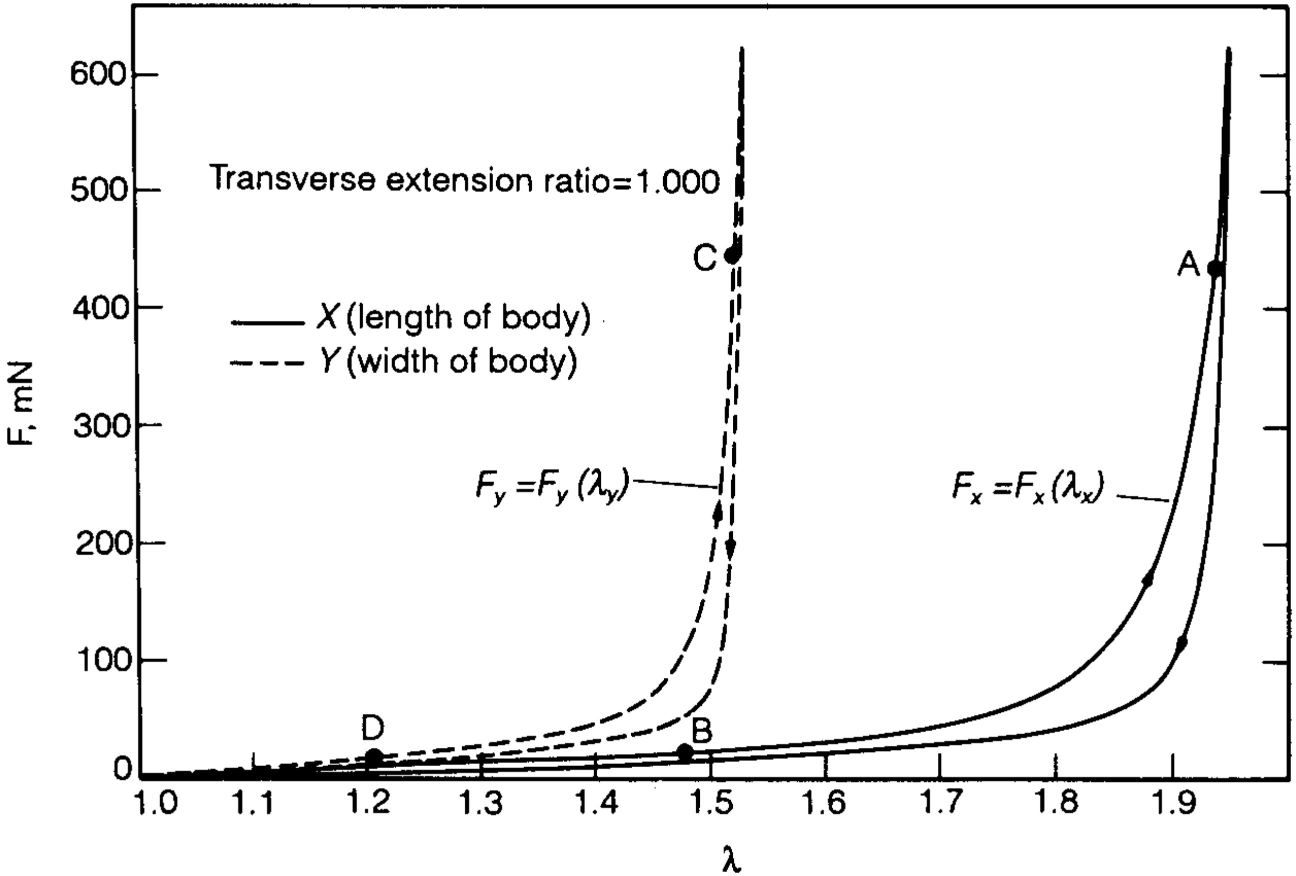
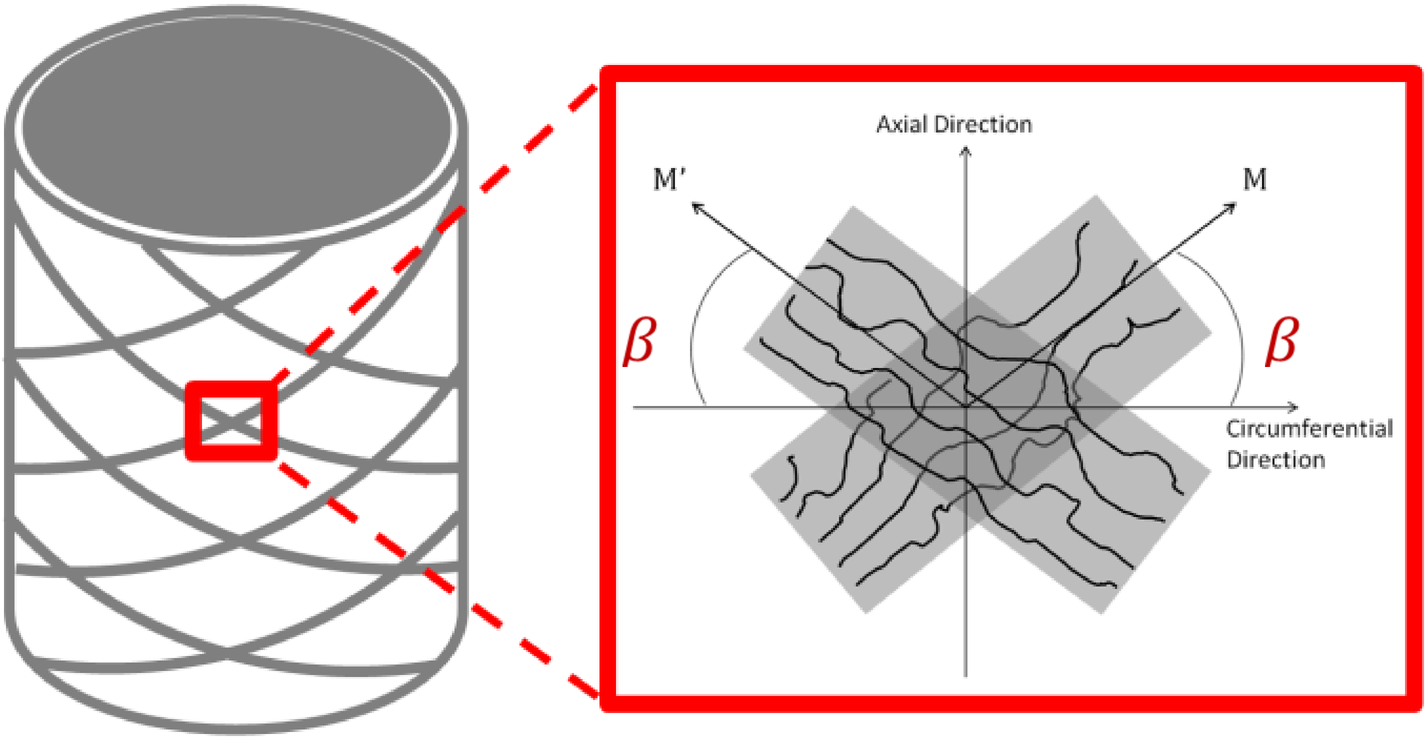
Anisotropy induced by collagen fibers



Anisotropic strain energy density

$$\Psi_f(I_4, I_6) = \frac{k_1}{2k_2} \sum_{i=4,6} \{exp[k_2(I_i - 1)^2] - 1\},$$

(Holzapfel et al., 2000)



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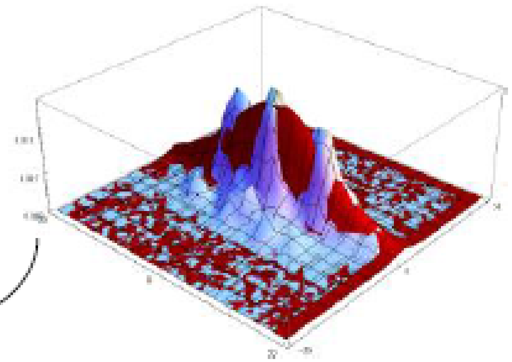
Micro-fiber approach



$$\sigma = \frac{2}{\pi}\int_{\phi=0}^{\pi/2}\int_{\theta=0}^{\pi/2}\rho(\phi,\theta)\sigma(\lambda)\mathrm{dev}(\mathbf{m}\otimes\mathbf{m})\cos\phi\,\mathrm{d}\phi\,\mathrm{d}\theta + p\mathbf{I}$$

$\sigma(\lambda)$ *Cauchy stress of a bundle of collagen fibrils*

$\rho(\phi,\theta)$ *Orientation density function (Bingham)*

$\mathbf{m} = \mathbf{FM}/|\mathbf{FM}|$ *Spatial fiber bundle direction*

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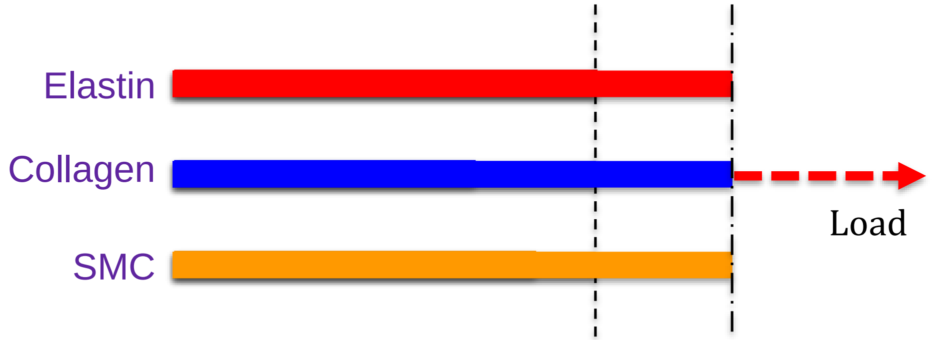
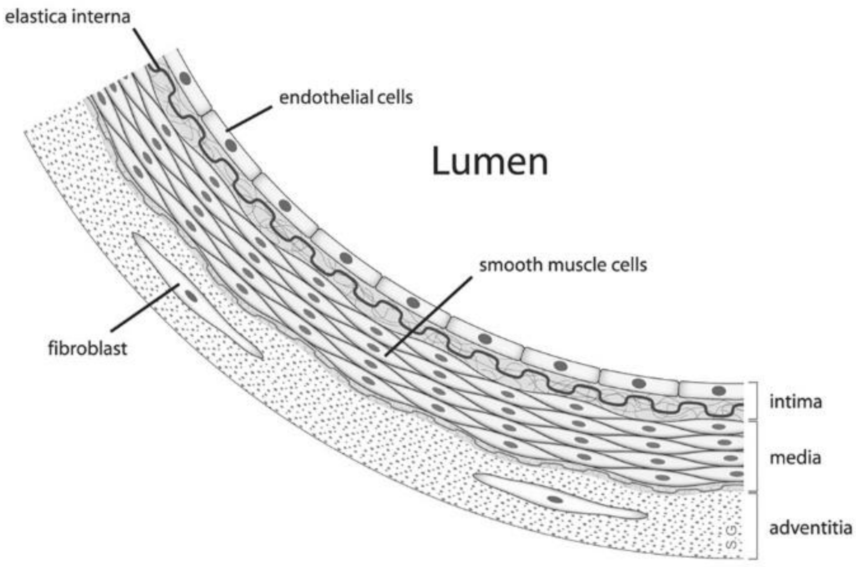
Layer-specific constitutive model

Strain-energy function based on the constrained mixture theory

$$W = \varrho_t^{\mathrm{e}}\left(\overline{W}^{\mathrm{e}}(\overline{I}_1^{\mathrm{e}}) + U(J_{\mathrm{el}}^{\mathrm{e}})\right) + \sum_{j=1}^{n} \varrho_t^{\mathrm{c}_j} W^{\mathrm{c}_j}(I_4^{\mathrm{c}_j}) + \varrho_t^{\mathrm{m}} W^{\mathrm{m}}(I_4^{\mathrm{m}})$$

Deposition stretch of each constituent:



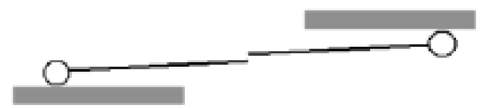


Humphrey & Rajagopal, Math Models Methods Appl Sci. (2002) ; Bellini et al, ABME (2014), Mousavi & Avril, BMMB (2017)

Damage and plasticity

Collagen fiber stress (2nd Piola-Kirchhoff)

Damage due to slide-apart glycan duplex



$$d = 1 - \exp[-a(\lambda_{st}/\lambda_{st\,0} - 1)^2]$$

Damage

$$S_c = (1 - d) c_f \langle \lambda/\lambda_{st} - 1 \rangle$$

c_f *Elastic fiber stiffness*

λ *Total fiber stretch*

$$Y = Y_0 + \eta \dot{\lambda}_{st}$$

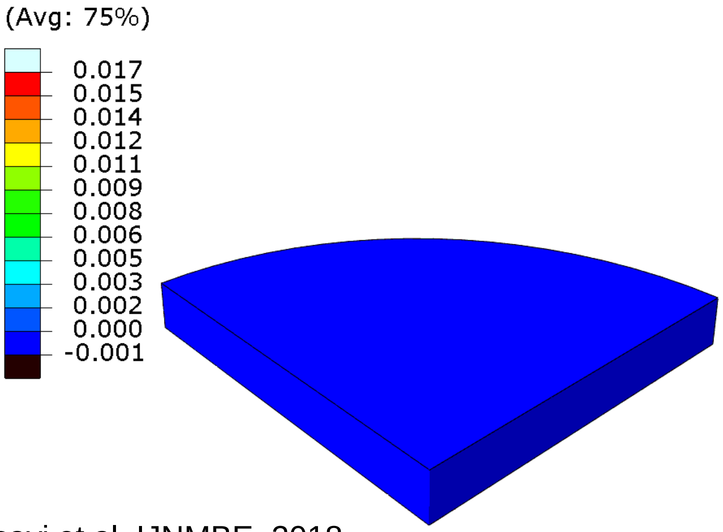
Plastic deformation

Plastic sliding of the glycan duplex

Gasser, Acta Biomat (2011)

Abaqus finite-element implementation and verification

- ✓ FE software ABAQUS coupled with UMAT
- ✓ Hexahedral and tetrahedral elements
- ✓ Structural mesh (r, θ, z)
- ✓ Two different layers (media and adventitia)



Mousavi et al, IJNMBE, 2018

Time dependent material properties: different approaches