Inverse characterization of local mechanical properties in aortic dissections

MR Bersi, C Bellini, P Di Achille, K Genovese, JD Humphrey, S Avril

CHALLENGES AND OBJECTIVES

Fundamentally understand local structure–function relationships in aortic dissections.

This requires an approach permitting:

1. **To reconstruct the regional distribution of mechanical properties of the aortic dissection.**

2. **To investigate their correlation with the underlying microstructure and its evolutions.**

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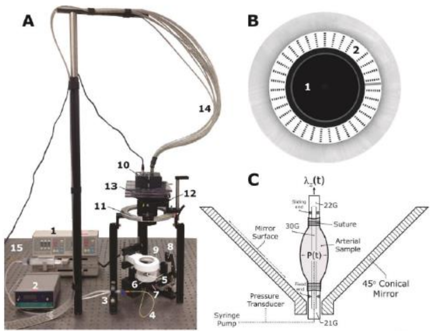
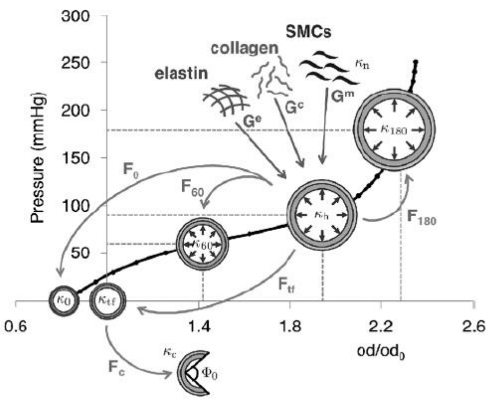
APPROACH

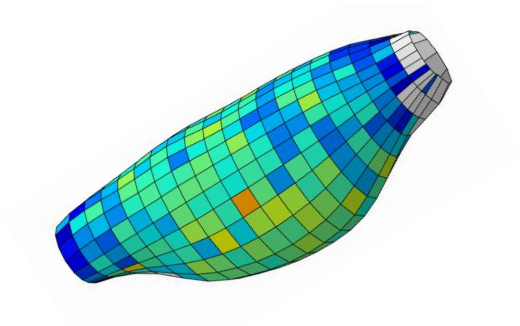
1. Experiments

2. Material model

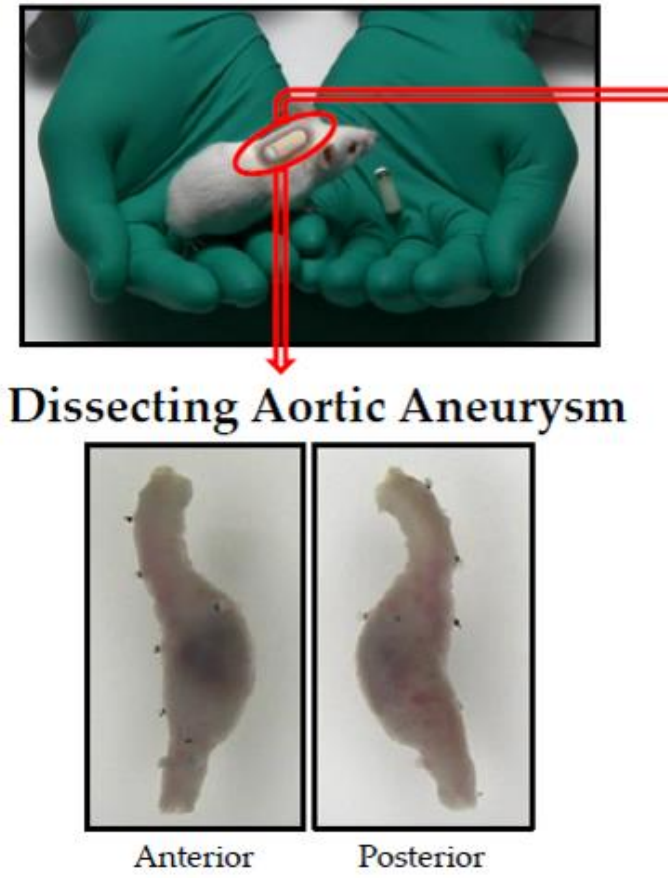
3. Inverse method

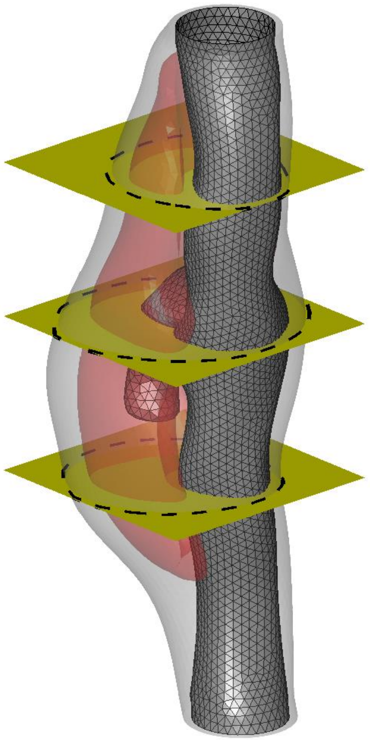


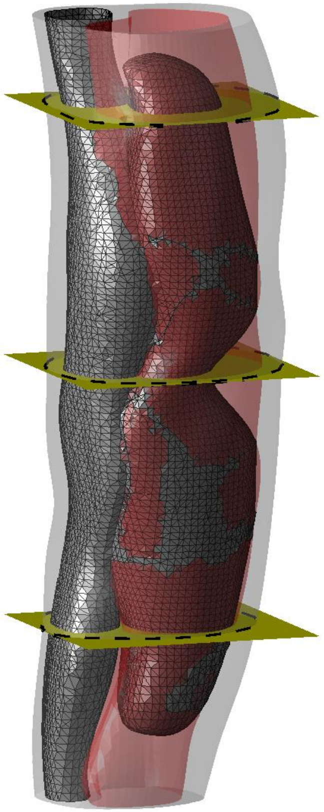




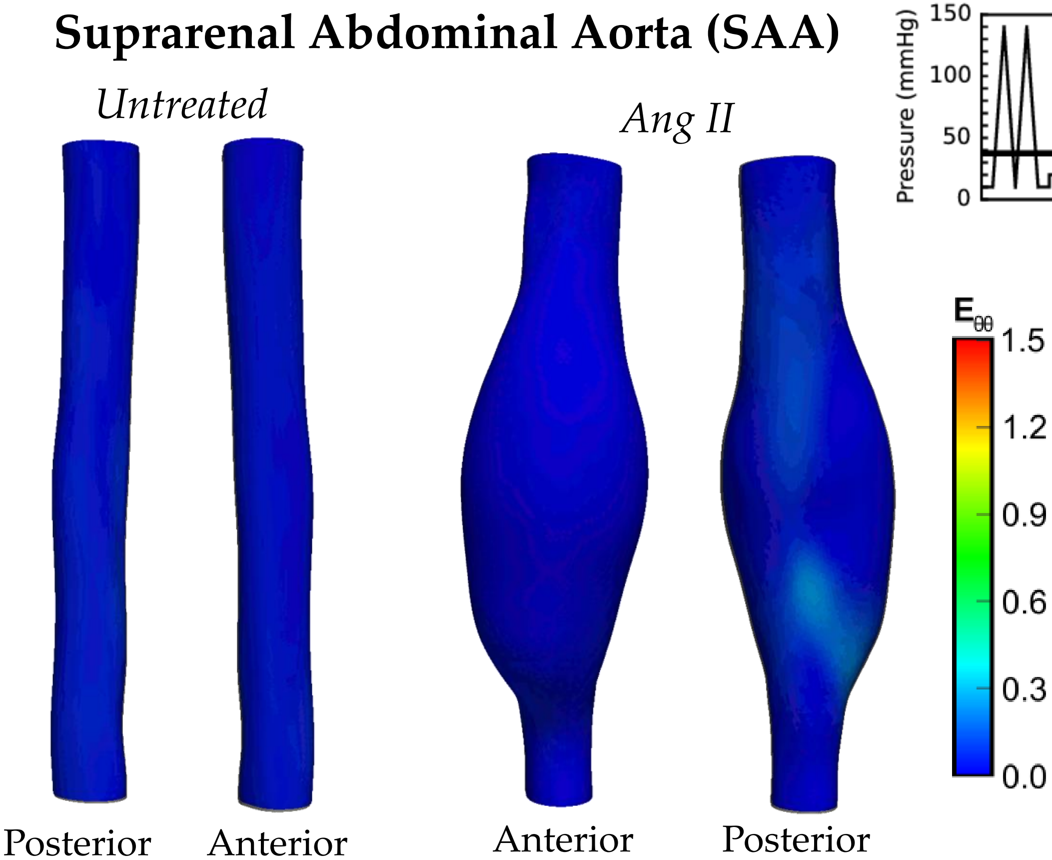
Dissected suprarenal aortas in the ANGII infused mice

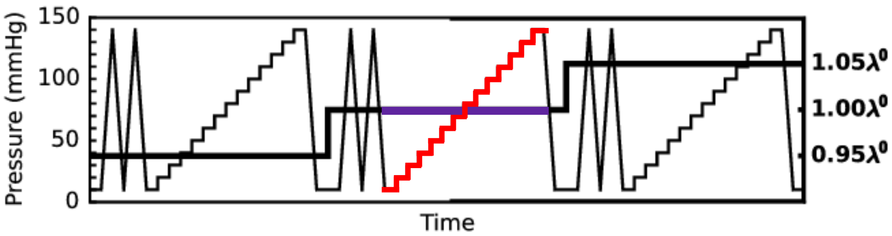


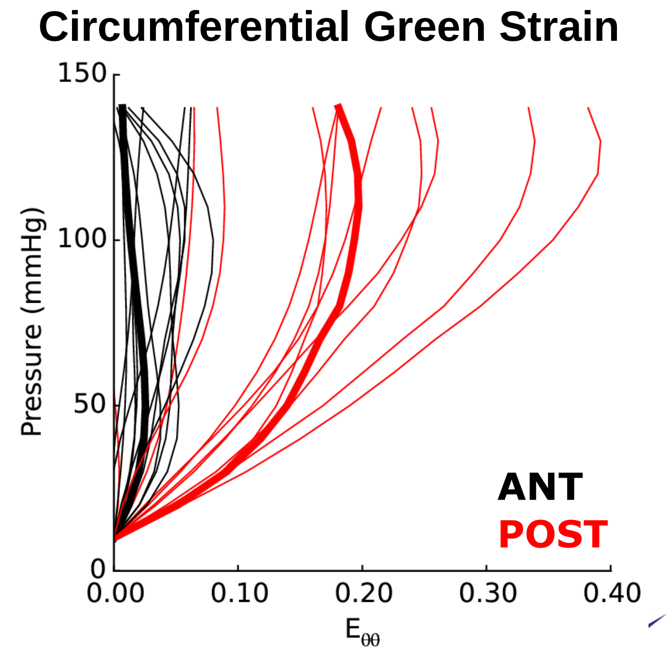




Measurement of surface deformation fields using Panoramic Digital Image Correlation (pDIC)







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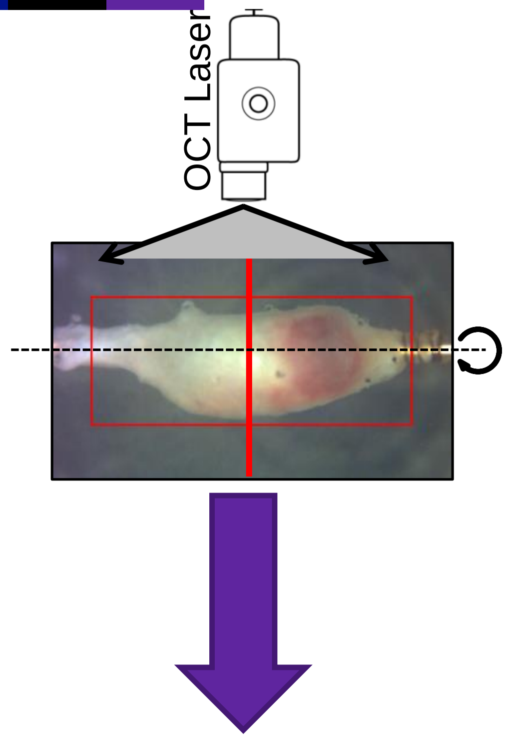
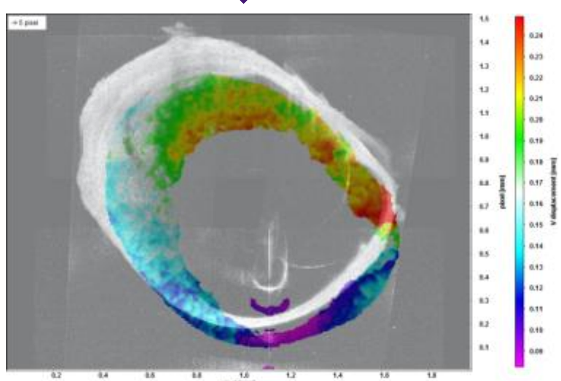
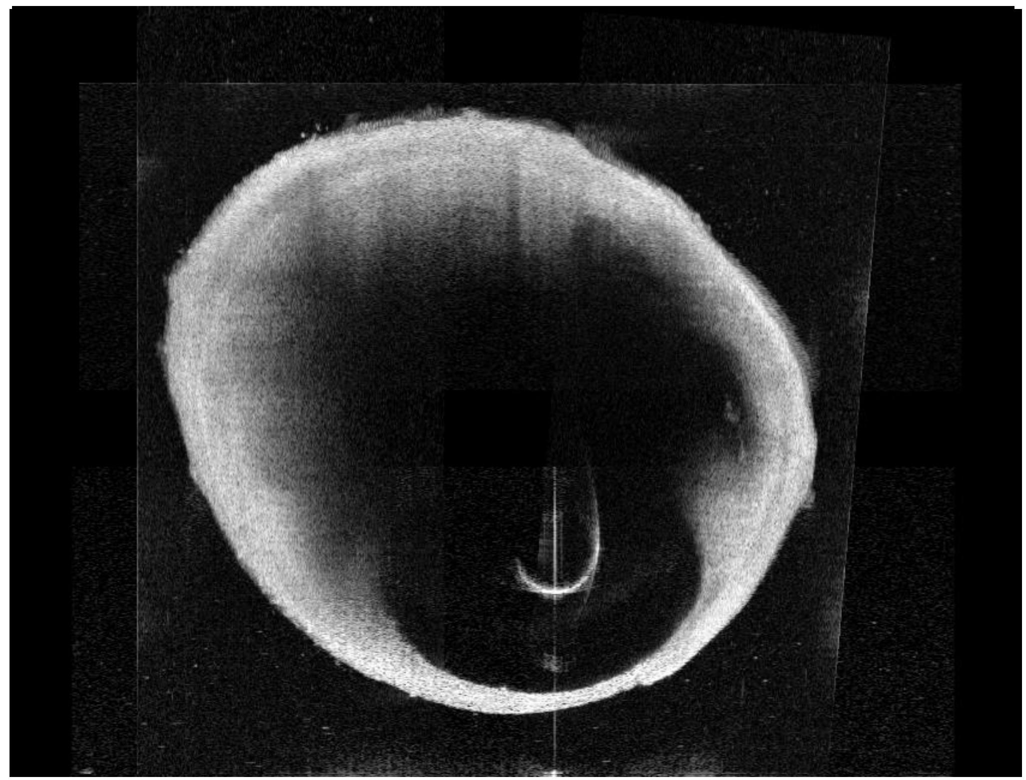
Measurement of bulk deformation fields by Digital Volume Correlation on OCT images







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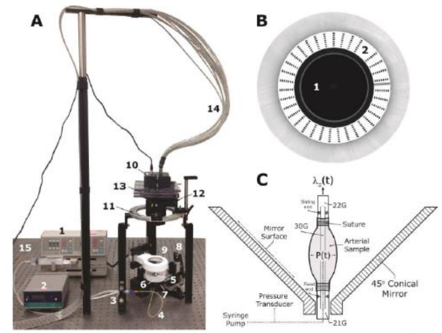

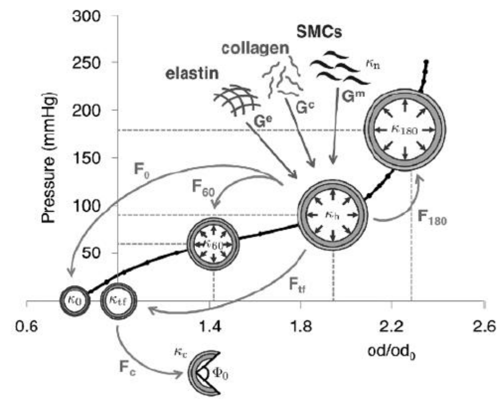

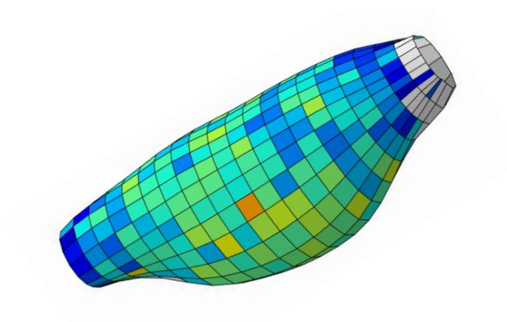

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CONSTITUTIVE MODEL

Bellini, et al., Ann. Biomed. Eng., **42**(3), pp. 488–502, 2014

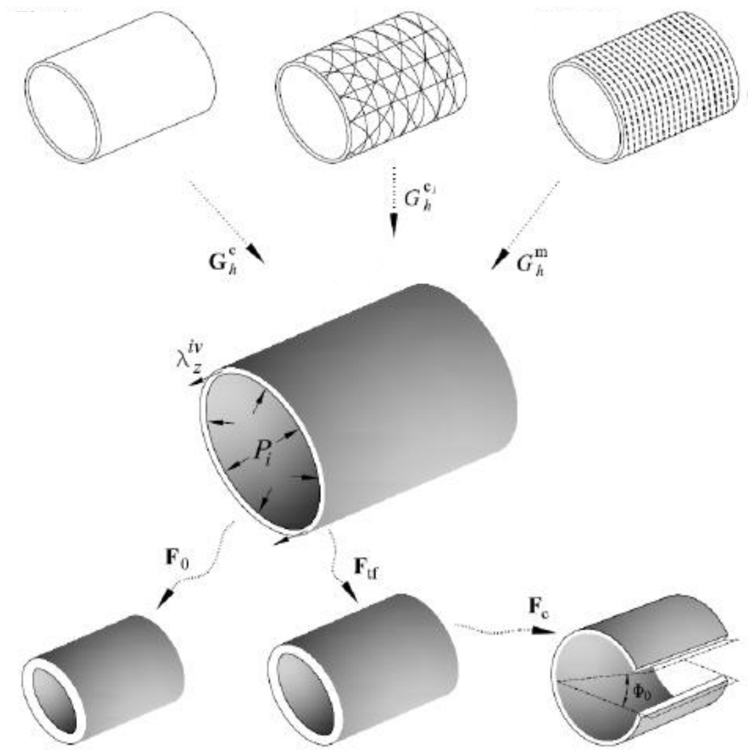
Strain energy functions:

$$W = \phi^{\mathrm{e}} W^{\mathrm{e}}(\mathbf{F}^{\mathrm{e}}) + \phi^{\mathrm{m}} W^{\mathrm{m}}(\lambda^{\mathrm{m}}) + \sum_{j=1}^{4} \phi^{\mathrm{c}_j} W^{\mathrm{c}_j}(\lambda^{\mathrm{c}_j})$$

$$W^{\mathrm{e}}(\mathbf{F}^{\mathrm{e}}) = \frac{c^{\mathrm{e}}}{2}\left[\mathrm{tr}\left((\mathbf{F}^{\mathrm{e}})^{\mathrm{T}}\mathbf{F}^{\mathrm{e}}\right) - 3\right]$$

$$W^{\mathrm{m}}(\lambda^{\mathrm{m}}) = \frac{c_2^{\mathrm{m}}}{4c_3^{\mathrm{m}}}\left[e^{c_3^{\mathrm{m}}((\lambda^{\mathrm{m}})^2-1)^2} - 1\right]$$

$$W^{\mathrm{c}}(\lambda^{\mathrm{c}_j}) = \frac{c_2^{\mathrm{c}}}{4c_3^{\mathrm{c}}}\left[e^{c_3^{\mathrm{c}}((\lambda^{\mathrm{c}_j})^2-1)^2} - 1\right]$$



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PARAMETERS TO BE IDENTIFIED

$$W = \phi^{\mathrm{e}} W^{\mathrm{e}}(\mathbf{F}^{\mathrm{e}}) + \phi^{\mathrm{m}} W^{\mathrm{m}}(\lambda^{\mathrm{m}}) + \sum_{j=1}^{4} \phi^{\mathrm{c}_j} W^{\mathrm{c}_j}(\lambda^{\mathrm{c}_j})$$

$$W^{\mathrm{e}}(\mathbf{F}^{\mathrm{e}}) = \frac{c^{\mathrm{e}}}{2}\left[\mathrm{tr}\left((\mathbf{F}^{\mathrm{e}})^{\mathrm{T}}\mathbf{F}^{\mathrm{e}}\right) - 3\right]$$

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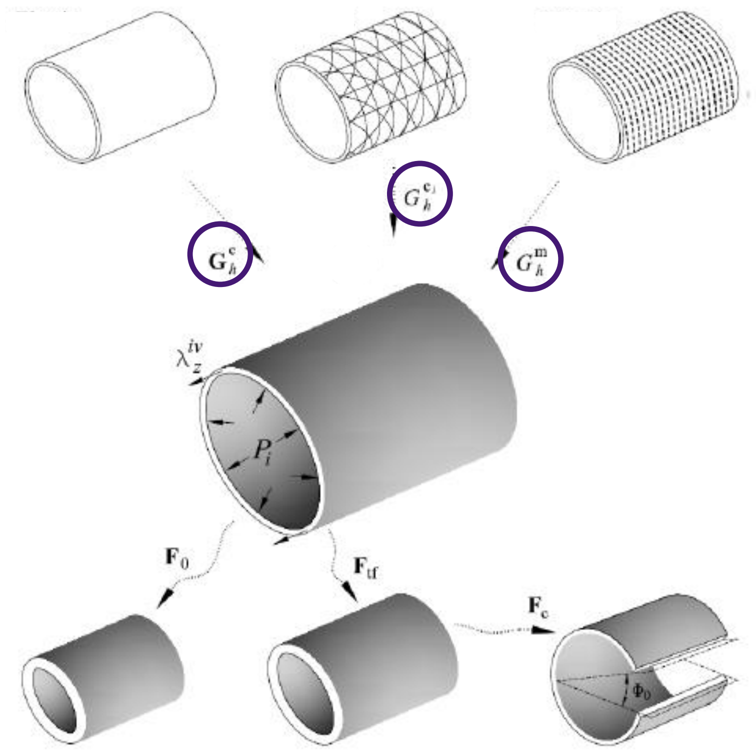

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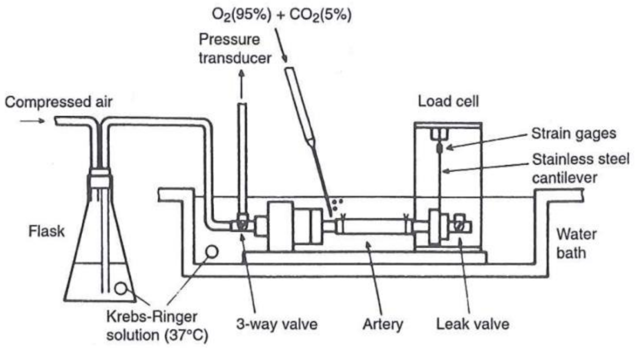
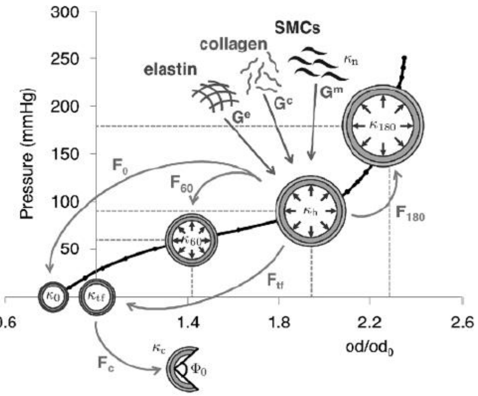
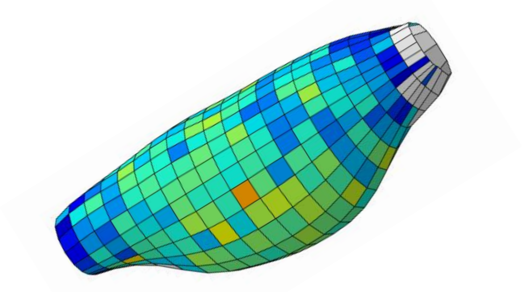
APPROACH

1. Experiments
2. Material model
3. **Inverse method**







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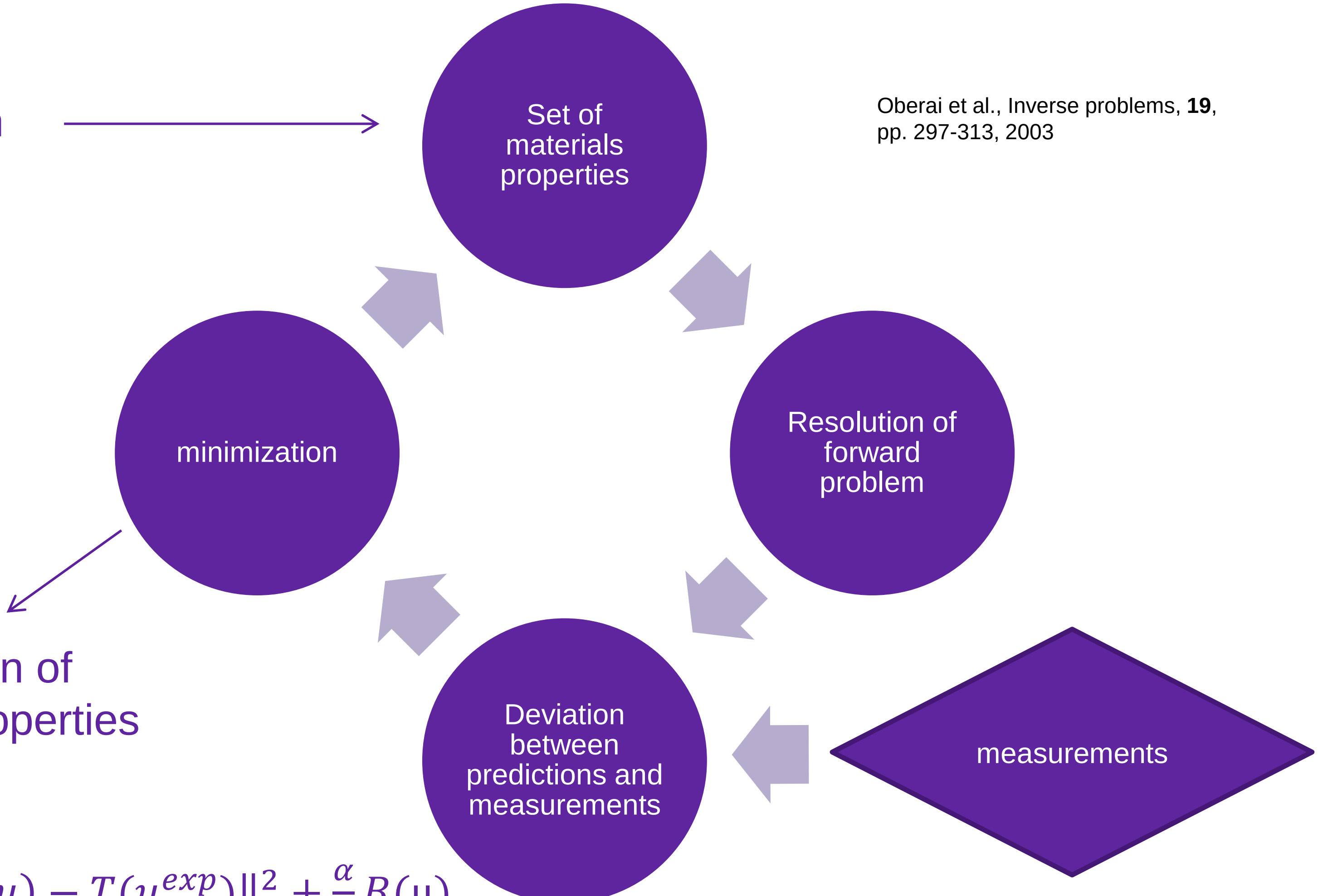
Inverse approach – traditional approach

Oberai et al., Inverse problems, **19**, pp. 297-313, 2003

initialization



Identification of material properties

$$J(\mu)= \|T(u) - T(u^{exp})\|^2 + \frac{\alpha}{2} B(\mu)$$

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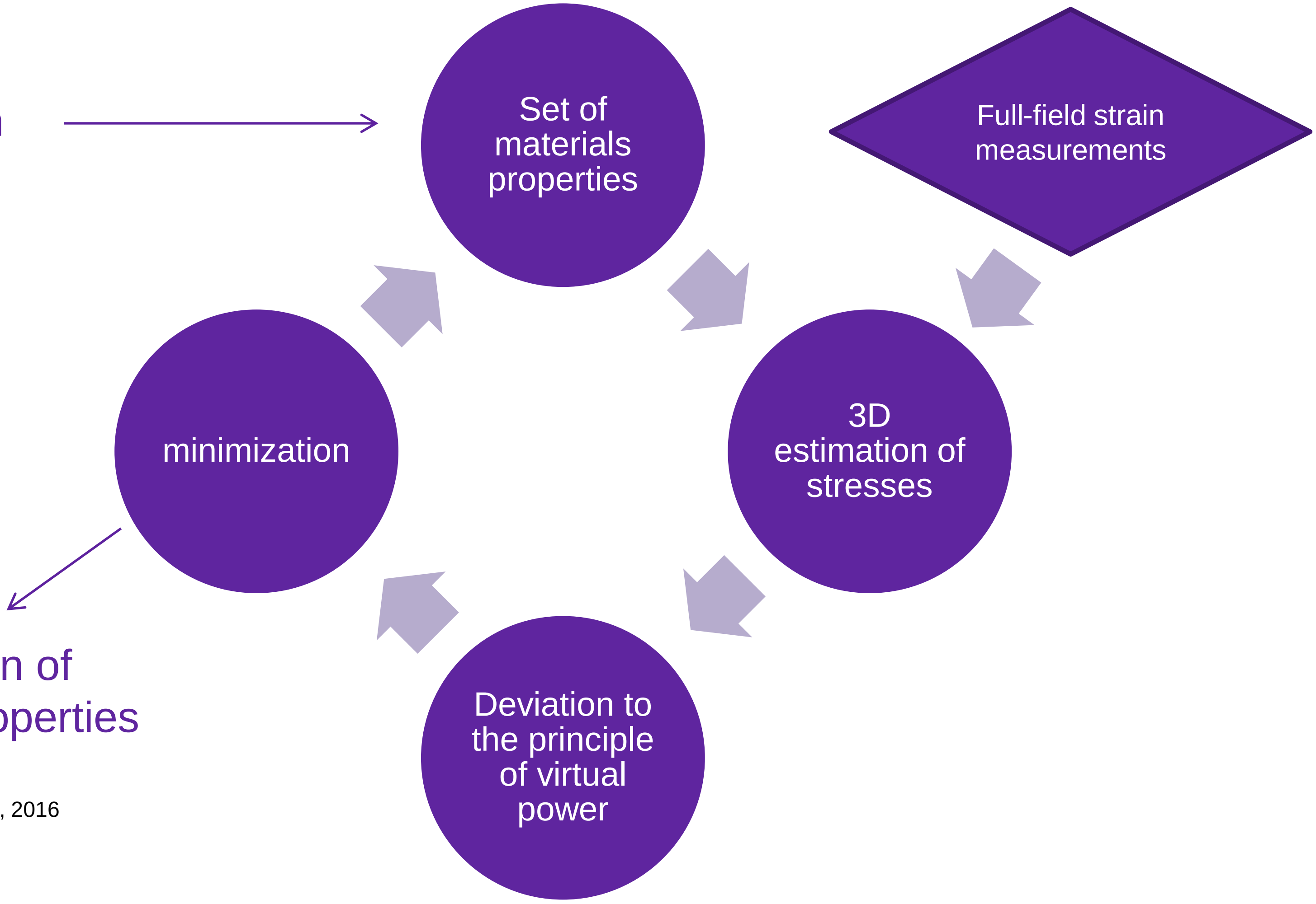
Alternative inverse approach: the virtual fields method

initialization



Identification of material properties

Bersi et al., J Biomech Eng, 2016

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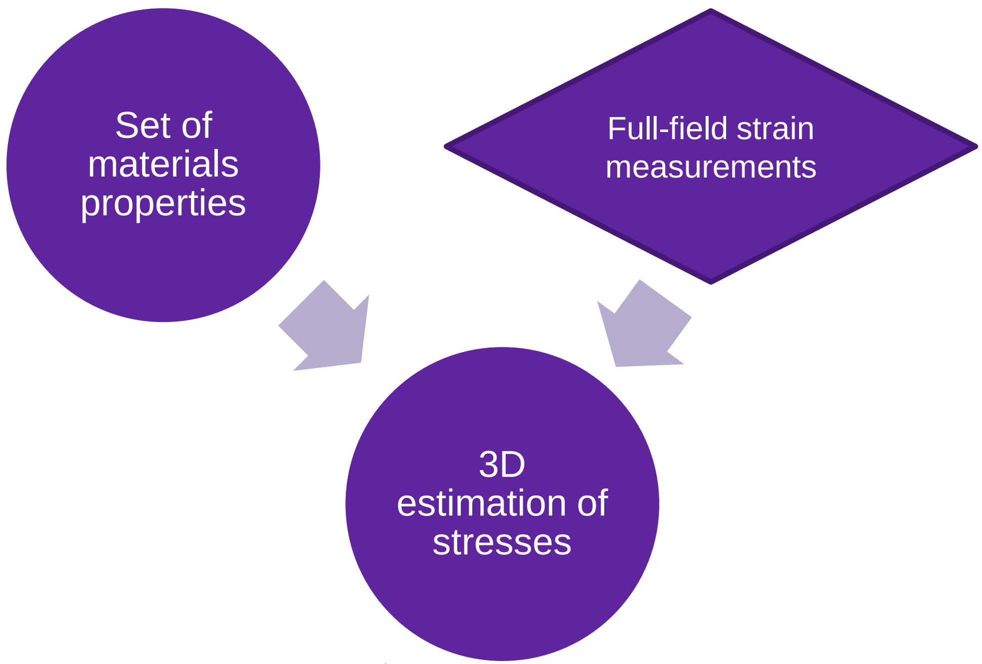
Full-field stress reconstruction

initialization →



$$W = \phi^{e} W^{e}(\mathbf{F}^{e}) + \phi^{m} W^{m}(\lambda^{m}) + \sum_{j=1}^{4} \phi^{c_j} W^{c_j}(\lambda^{c_j})$$

$$W^{e}(\mathbf{F}^{e}) = \frac{c^{e}}{2}\left[\mathrm{tr}\left((\mathbf{F}^{e})^{T}\mathbf{F}^{e}\right) - 3\right]$$

$$W^{m}(\lambda^{m}) = \frac{c_2^{m}}{4c_3^{m}}\left[e^{c_3^{m}((\lambda^{m})^2-1)^2} - 1\right]$$

$$W^{c}(\lambda^{c_j}) = \frac{c_2^{c}}{4c_3^{c}}\left[e^{c_3^{c}((\lambda^{c_j})^2-1)^2} - 1\right]$$

Simple application of the constitutive model for each element

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Minimization of the equilibrium gap using the principle of virtual power

minimization

$$J = \sum_{p} \sum_{\lambda} \Bigg(\underbrace{- \int_{\omega(t)} \underline{\underline{\sigma}} : \left(\underline{\nabla} \otimes \underline{\xi}^* \right) d\omega}_{P^*_{int}} + \underbrace{\oint_{\partial\omega(t)} \underline{T} : \underline{\xi}^* \, ds}_{P^*_{ext}} \Bigg)^2$$

Bersi et al., J Biomech Eng, 2016

Resolution:

$$\min_{c_3^1, c_3^{2,3}, c_3^4, \alpha, \beta} \left[\min_{c^e, c_2^1, c_2^{2,3}, c_2^4} \left[\frac{J(u)}{A} + \frac{J(v)}{B} \right] \right]$$

Linear least-squares

Genetic algorithm

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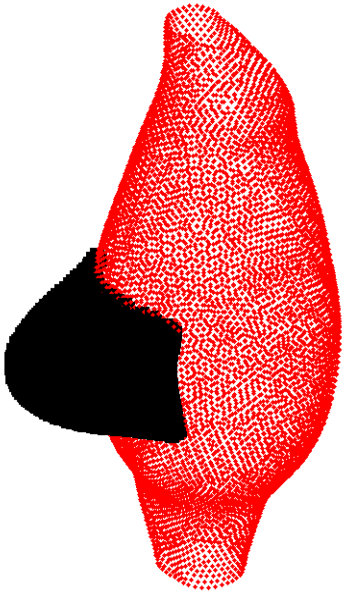
Example of virtual field

1. A local virtual radial "bulge": **u**(**x**) = [f(**x**-**x**$_0$) /r^2] **e**$_r$



$$\underbrace{-\int_{\omega(t)} \underline{\underline{\sigma}}: \left(\underline{\nabla} \otimes \underline{\xi}^*\right) d\omega}_{P^*_{int}} + \underbrace{\oint_{\partial\omega(t)} \underline{T}: \underline{\xi}^* \, ds}_{P^*_{ext}} = 0$$

After deriving virtual strains (infinitesimal) local internal virtual work is derived at every Gauss point and integrated across the volume

The virtual field is normalized such as the external virtual work equals P.

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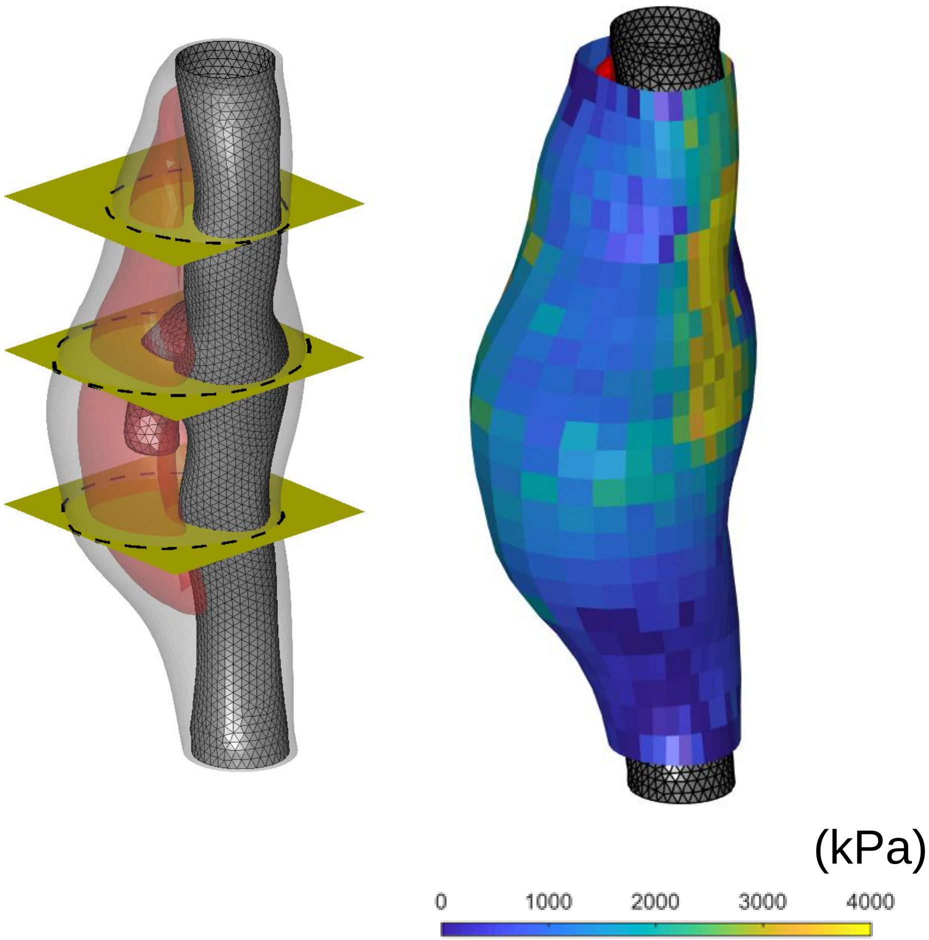
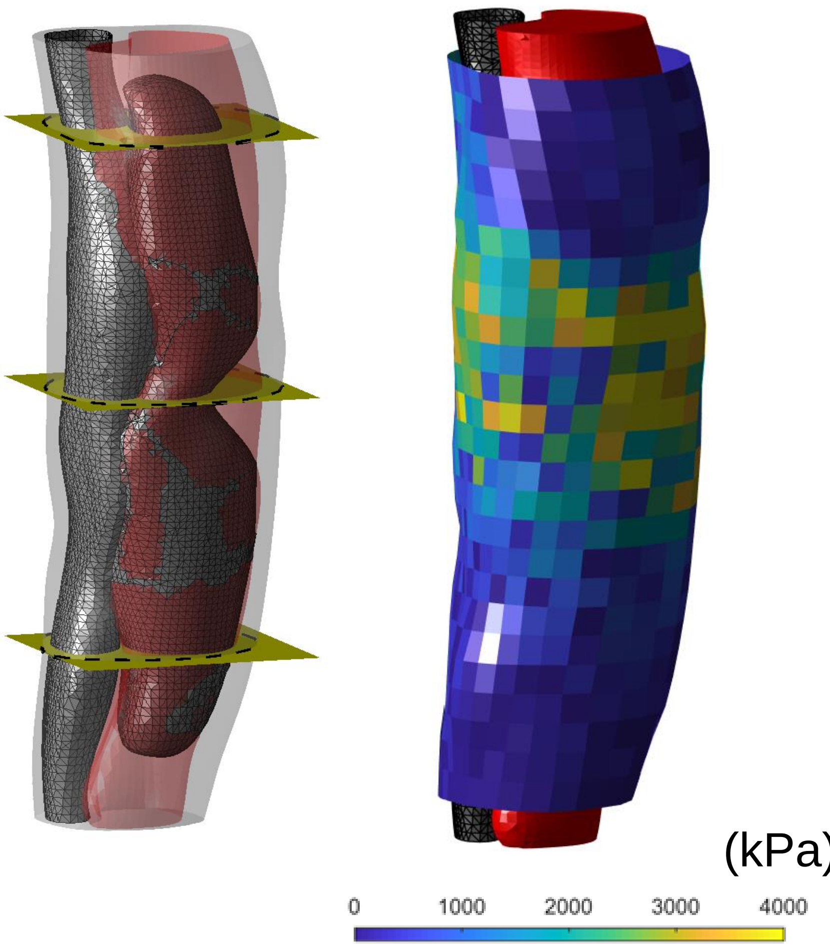
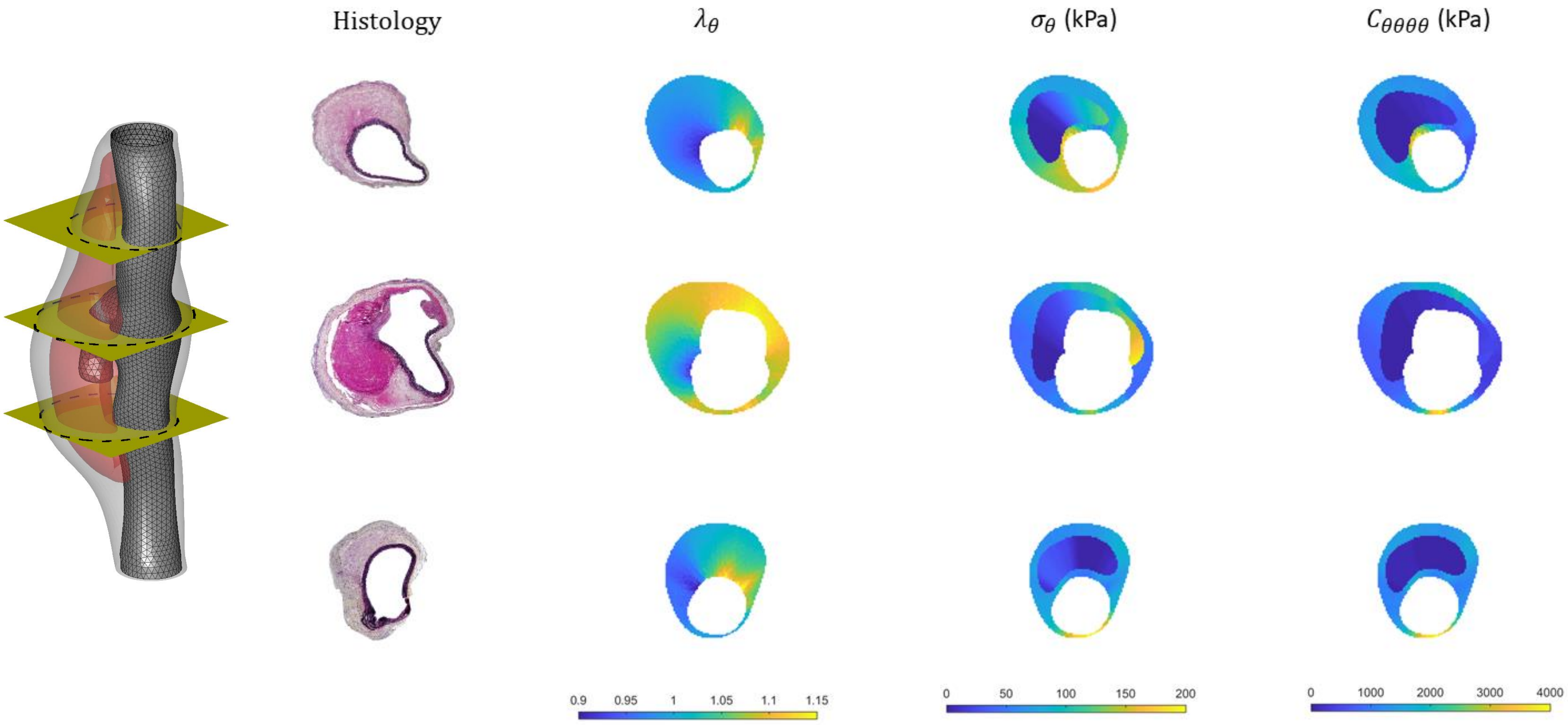
Results - Highlights

Obtained linearized circumferential stiffness

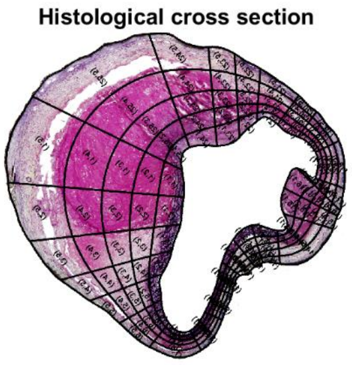


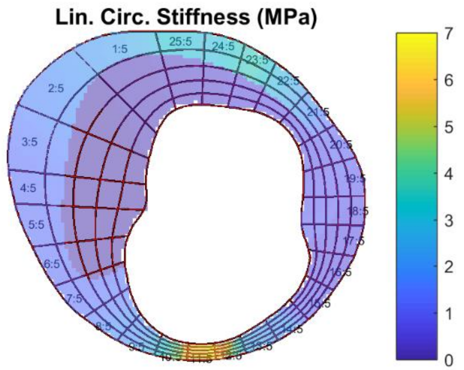


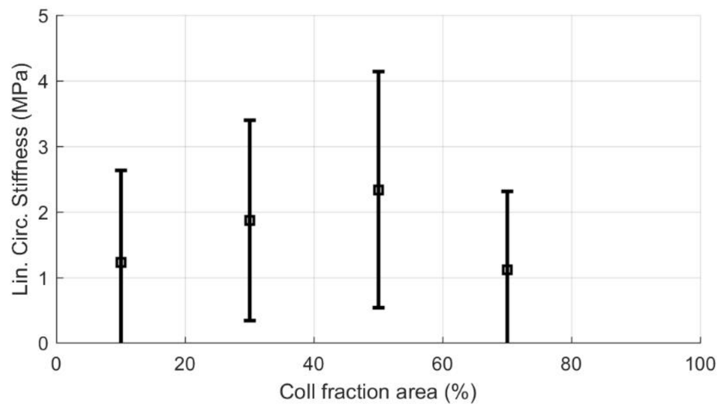
Cross sectional results

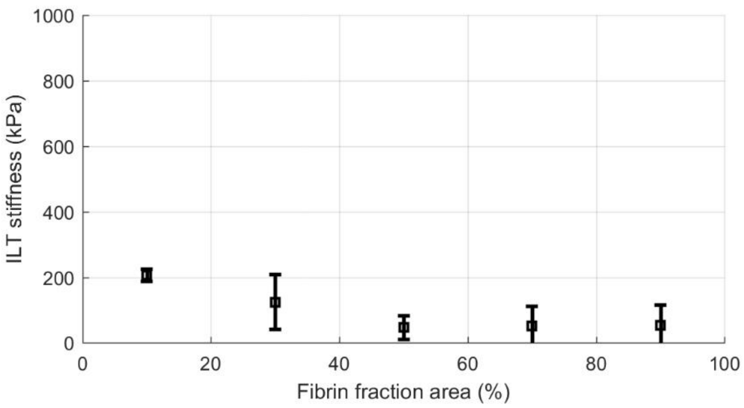


Structure – function analysis









SUMMARY

- Inverse approach permitting to reconstruct the **regional distribution of mechanical properties** of the aorta.

- Towards correlations between mechanical properties and underlying microstructures during aneurysm growth.

- 100 to 1000 independent local responses which could be used to set up **statistical mechanobiological models using Bayesian inference**.

- **Machine learning** could be used to predict median responses from such statistical models

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