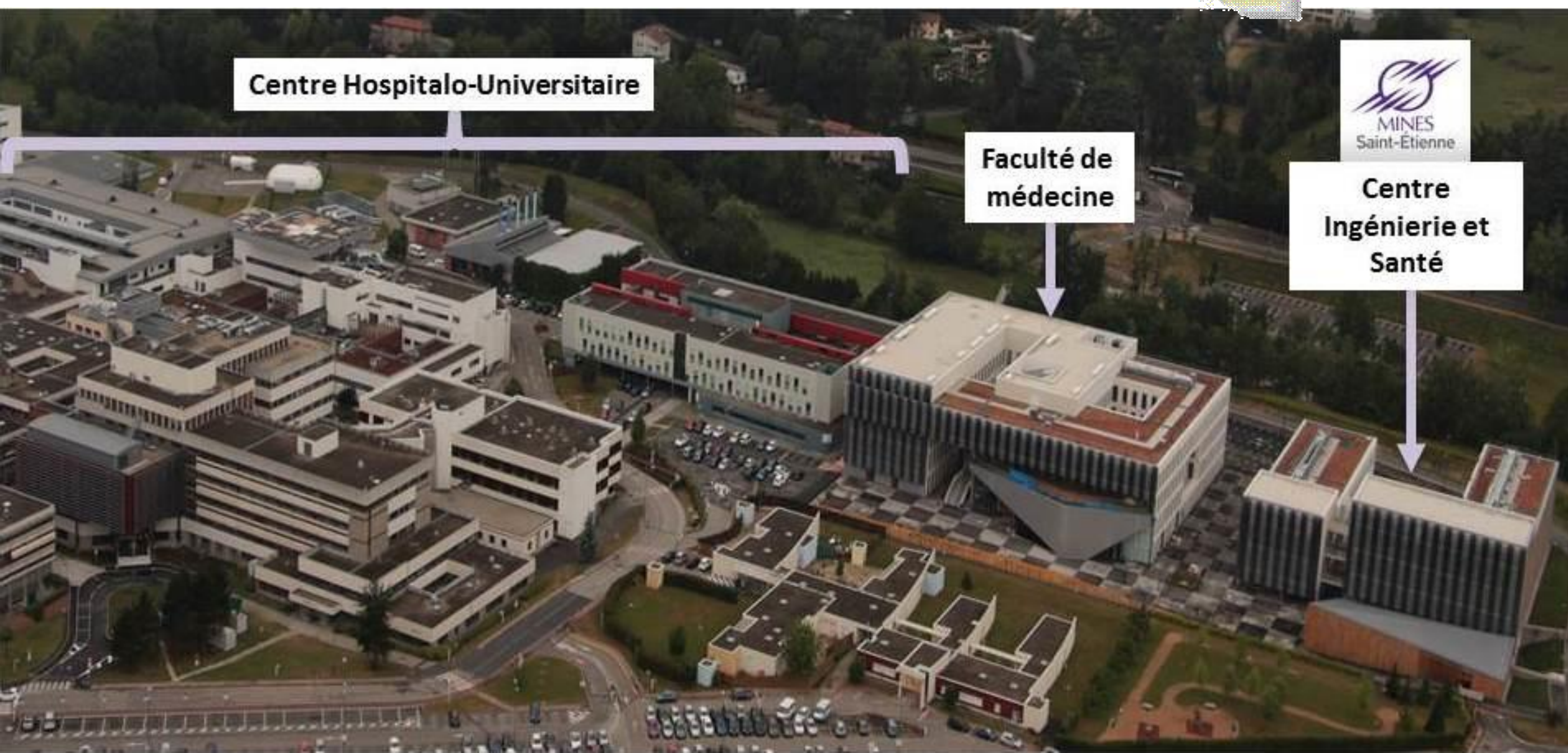
Continuum mechanics in cardiovascular engineering and biology

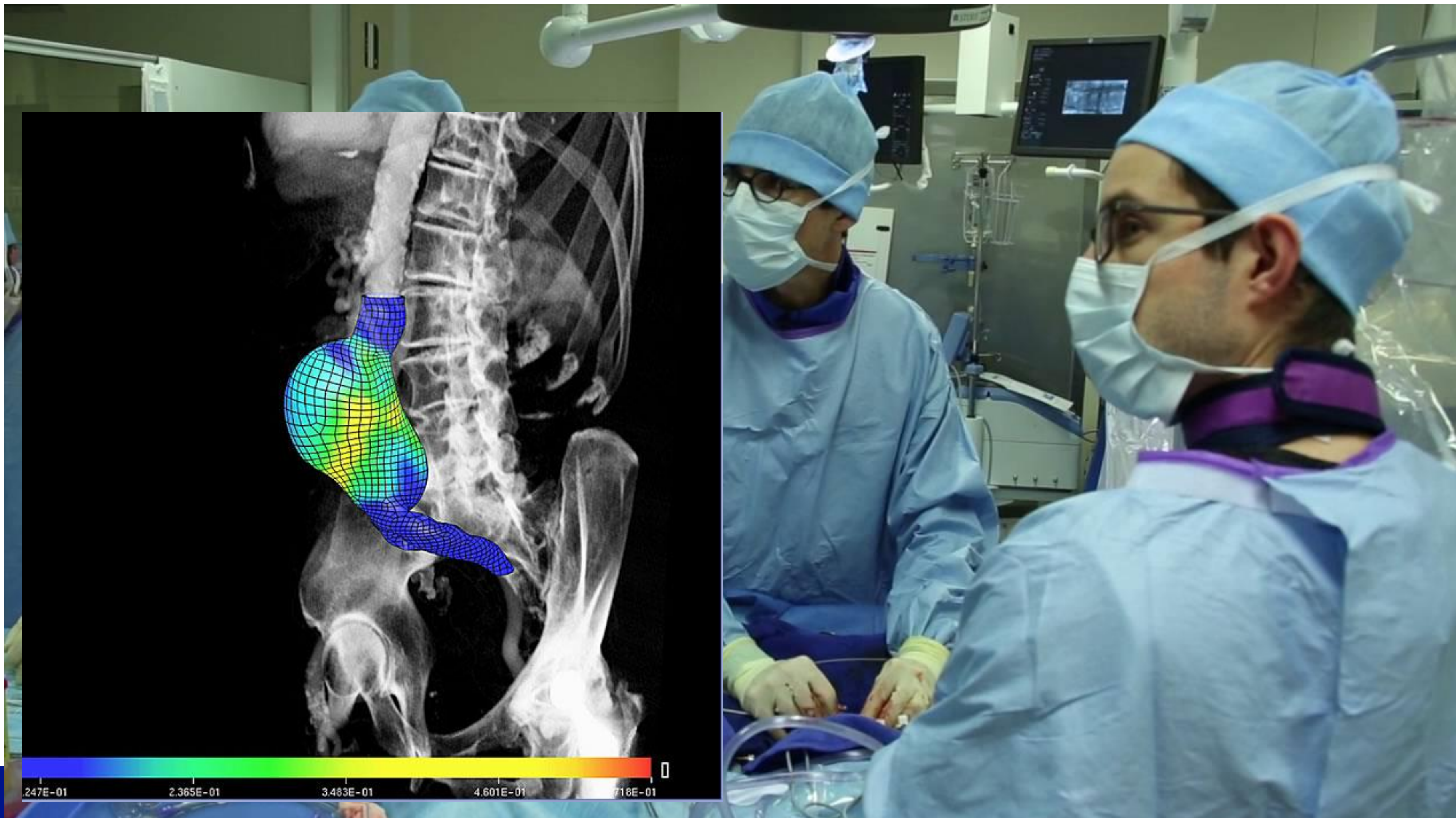
Prof. Stéphane AVRIL

MINES SAINT-ETIENNE
First Grande Ecole
outside Paris
Founded in 1816



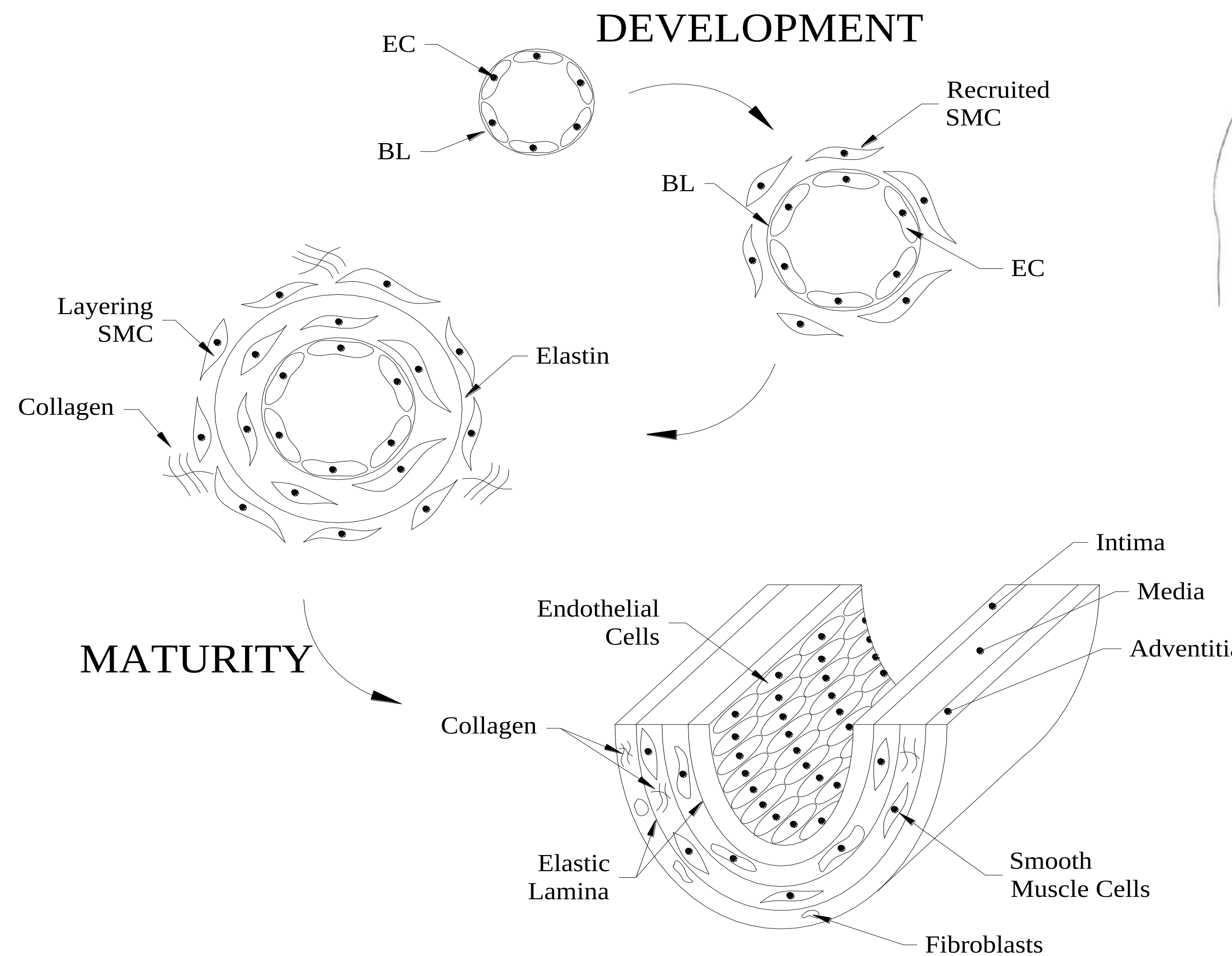
Computational mechanics in the OR for vascular surgery?

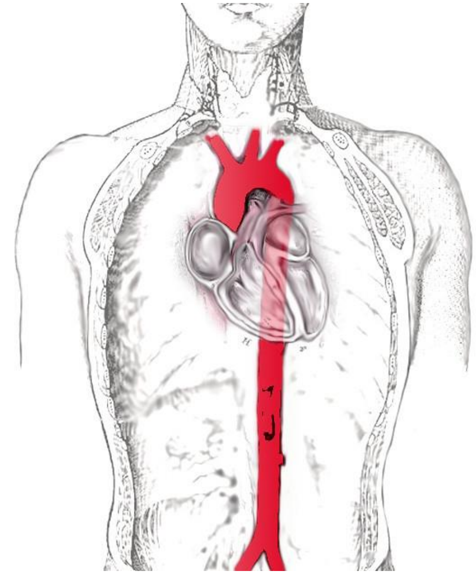
www.predisurge.com

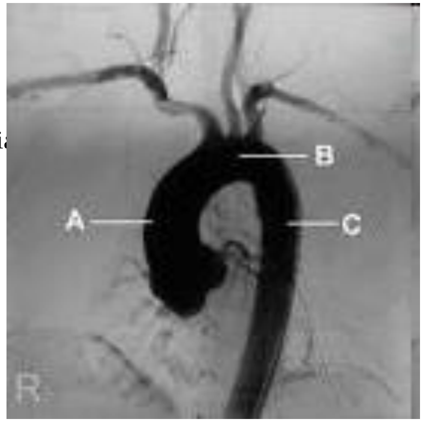


Arterial biomechanics and mechanobiology – Position of photomechanics

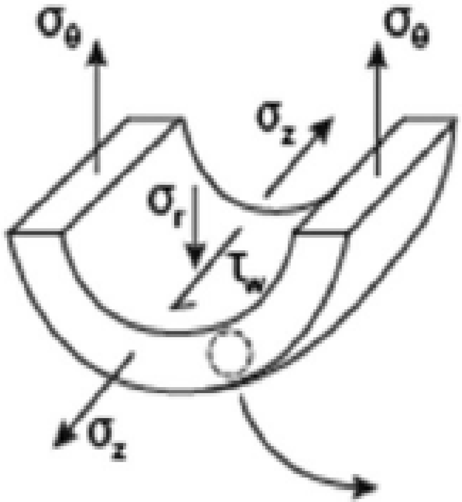
Schematic representation of aortic structure







Basics of aortic mechanics

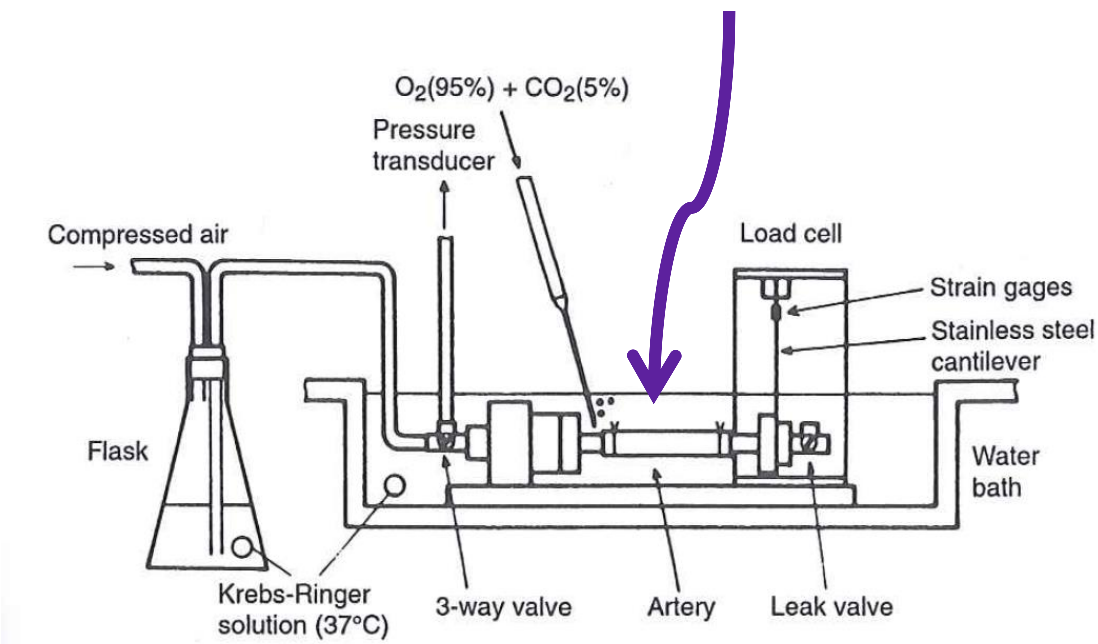


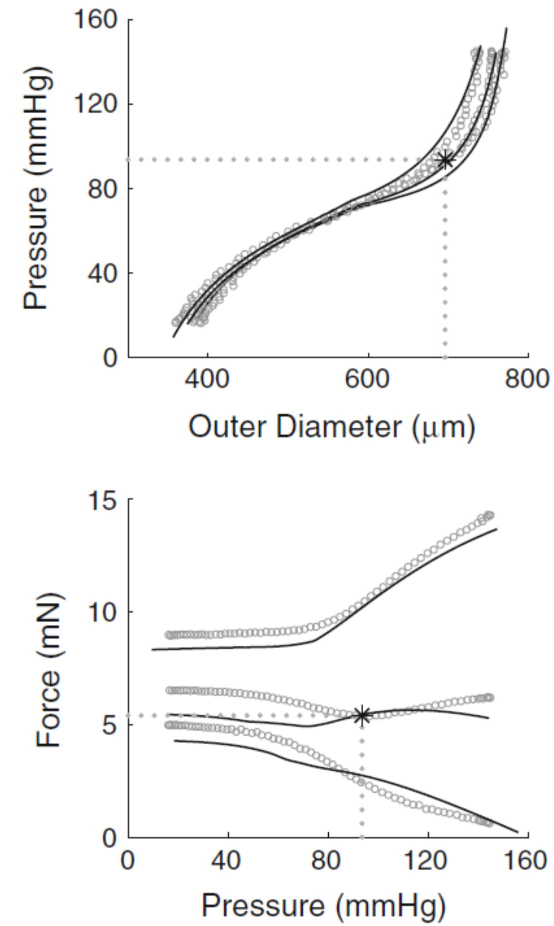
$$\tau_w = \frac{4\mu Q}{\pi a^3}, \qquad \sigma_\theta = \frac{Pa}{h}$$

$$\sigma_z = \frac{f_z}{\pi\left(b^2 - a^2\right)} = \frac{f_z}{\pi h\left(2a - h\right)}$$

Humphrey JD (2002) *Cardiovascular Solid Mechanics: Cells, Tissues, and Organs*, Springer-Verlag, NY

Functional biomechanical behavior





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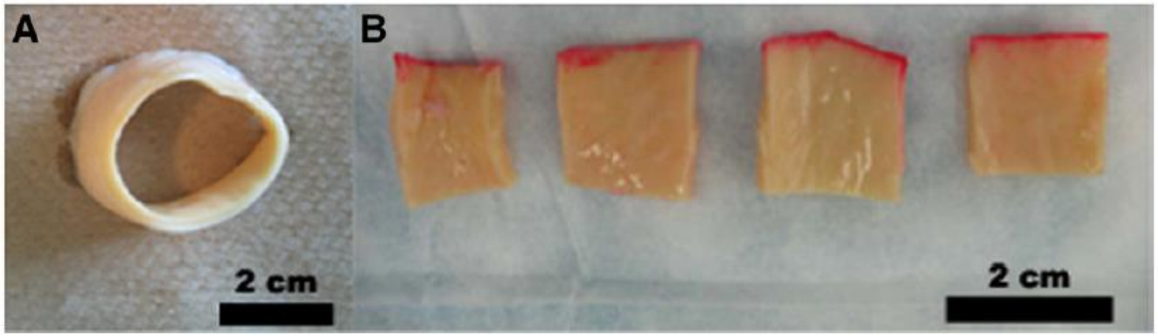
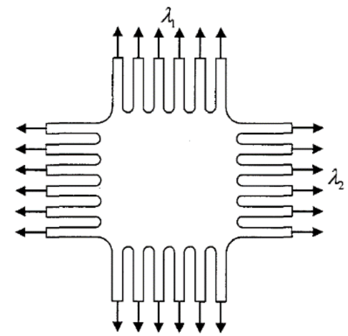
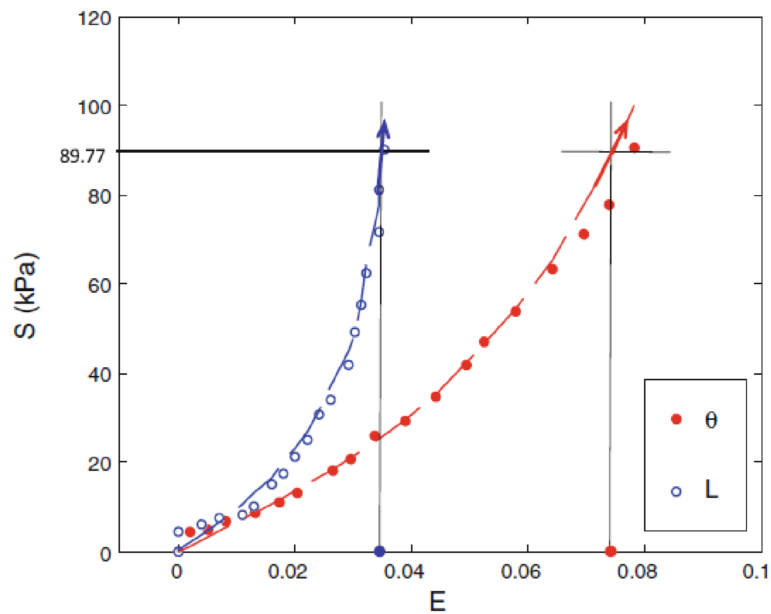
Material characterization and constitutive modeling





$$W = C_{10}\left(\bar{I}_1 - 3\right) + \frac{1}{D}\left(\frac{J^2-1}{2} - \ln J\right) + \frac{k_1}{2k_2}\sum_{\alpha=1}^{N}\left\{\exp\left[k_2\left\langle \bar{E}_\alpha\right\rangle^2\right] - 1\right\}$$



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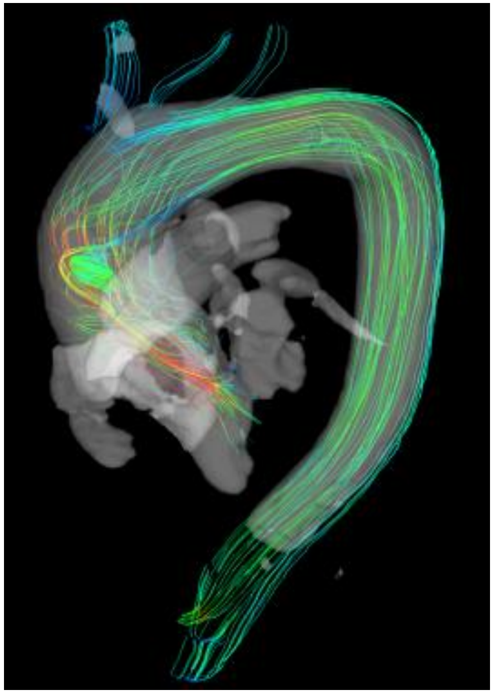
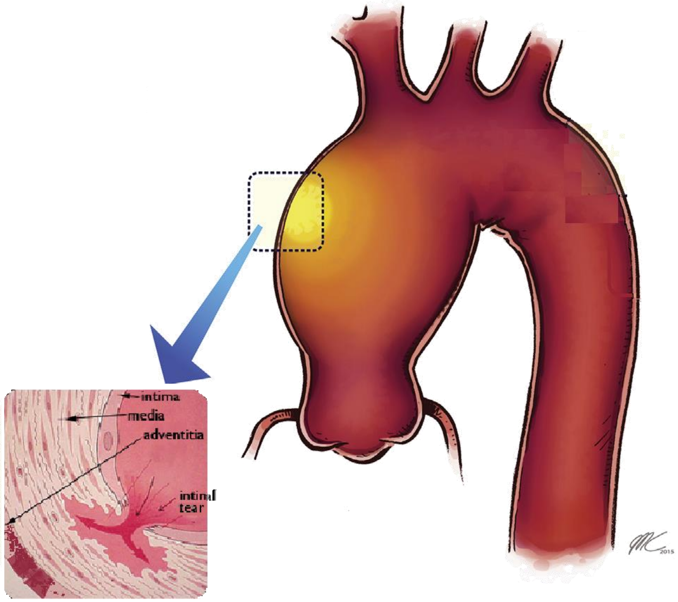
Prediction of risk of rupture and dissection





dissection

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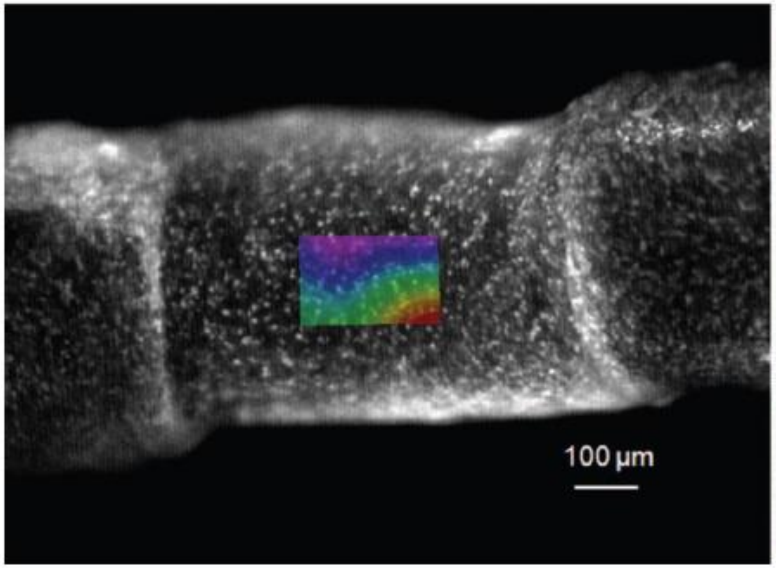
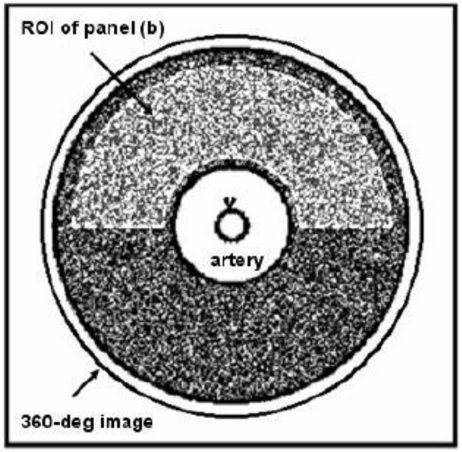
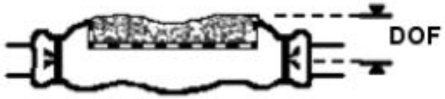
MEASUREMENT OF THE RESPONSE USING DIGITAL IMAGE CORRELATION

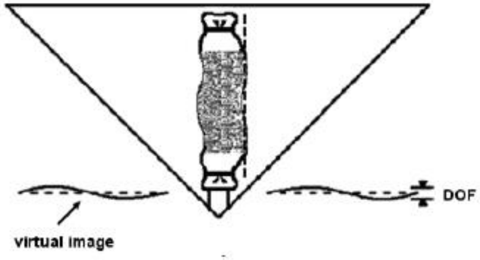
classical

panoramic





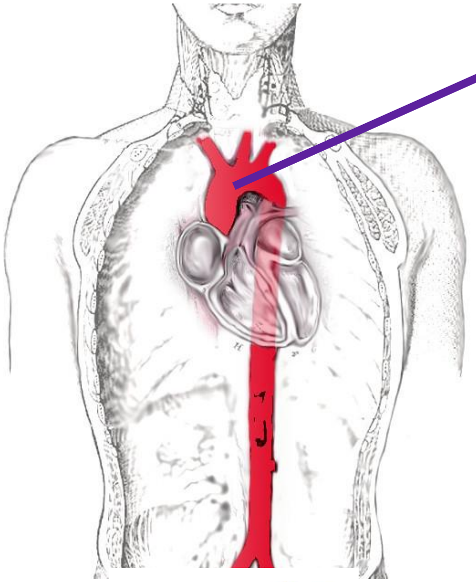


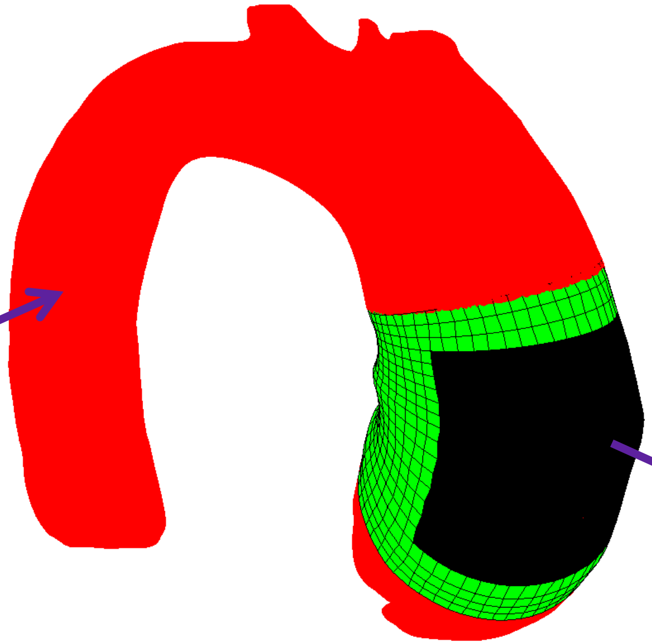


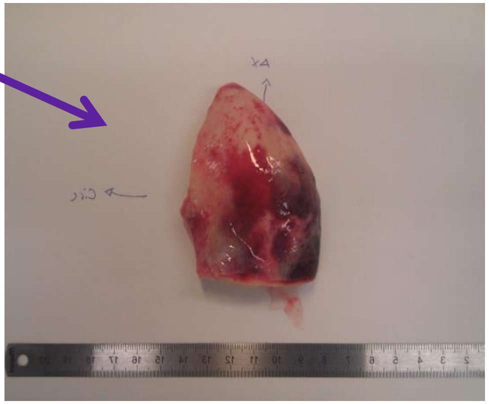
Badel et al. CMBBE, **15**, p 37-48, 2012.

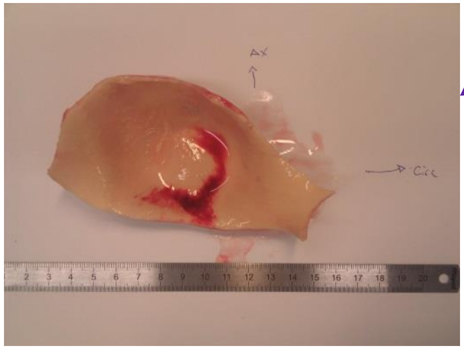
Genovese. Optics Lasers Eng, **47**, p 995-1008, 2009.

Collection of the samples

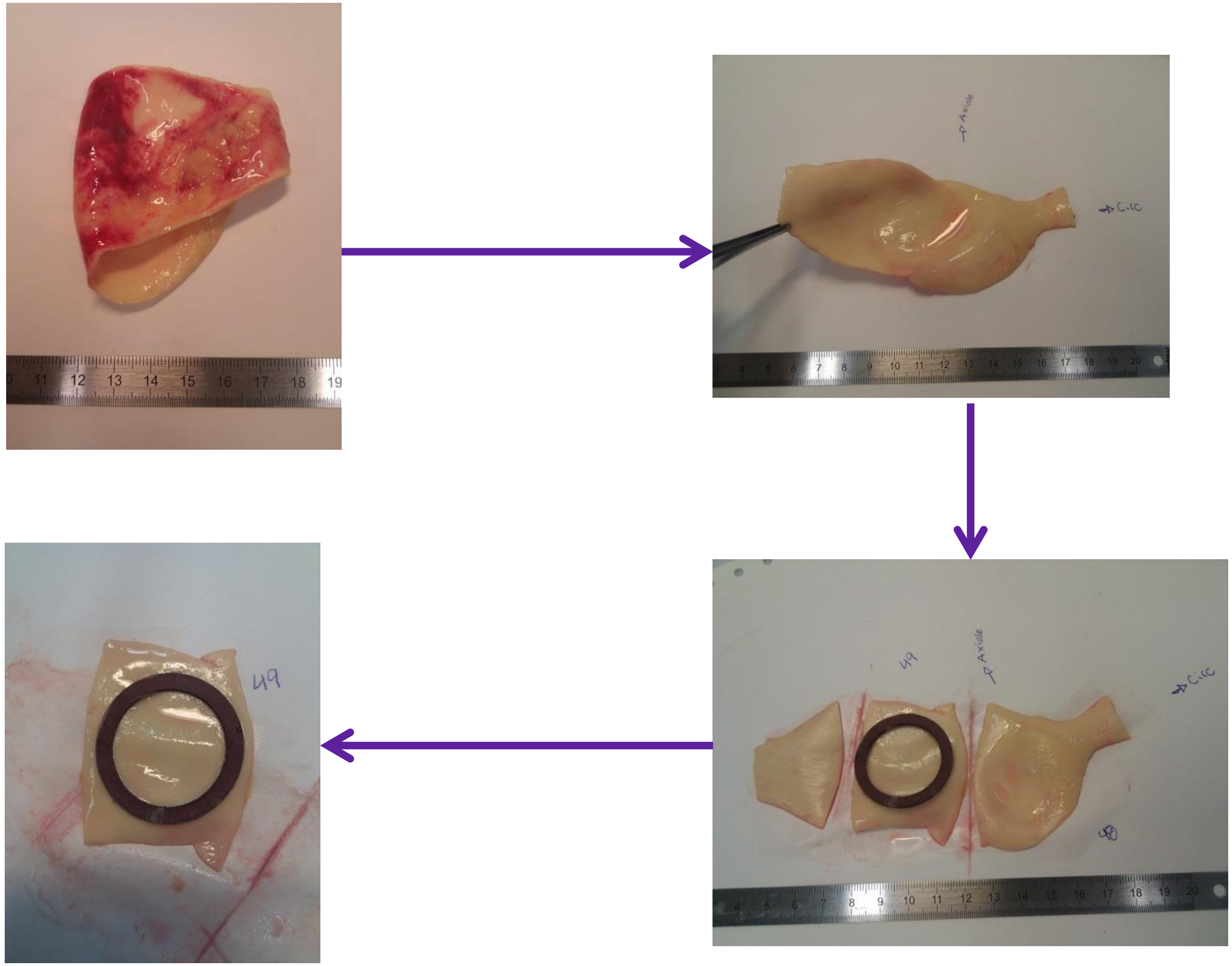








Preparation



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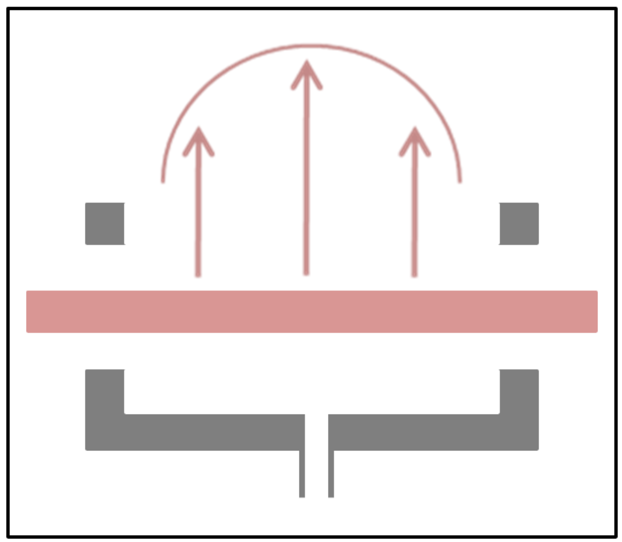
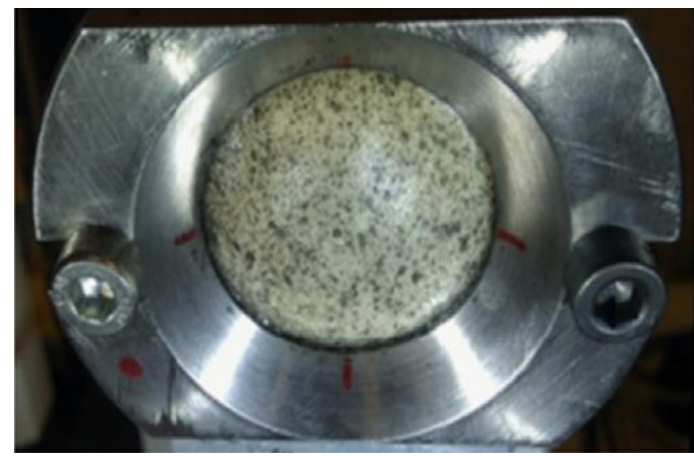
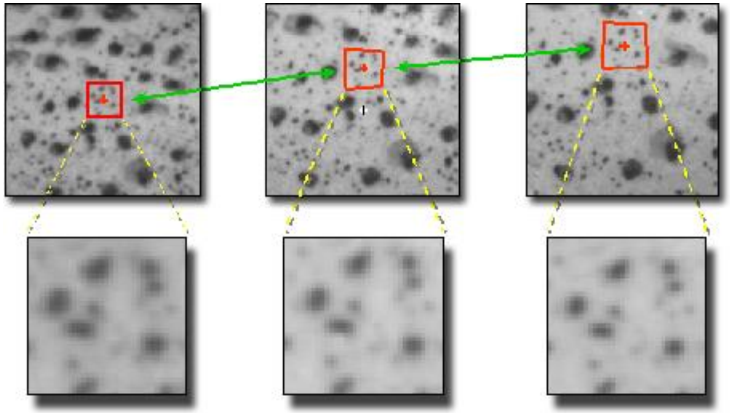
Bulge inflation test

Romo et al. Journal of Biomechanics -2014.



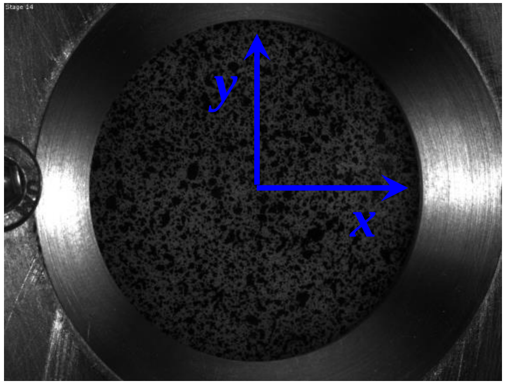


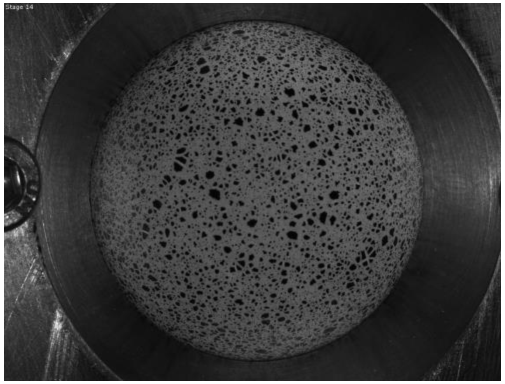
Full-field measurements using sDIC



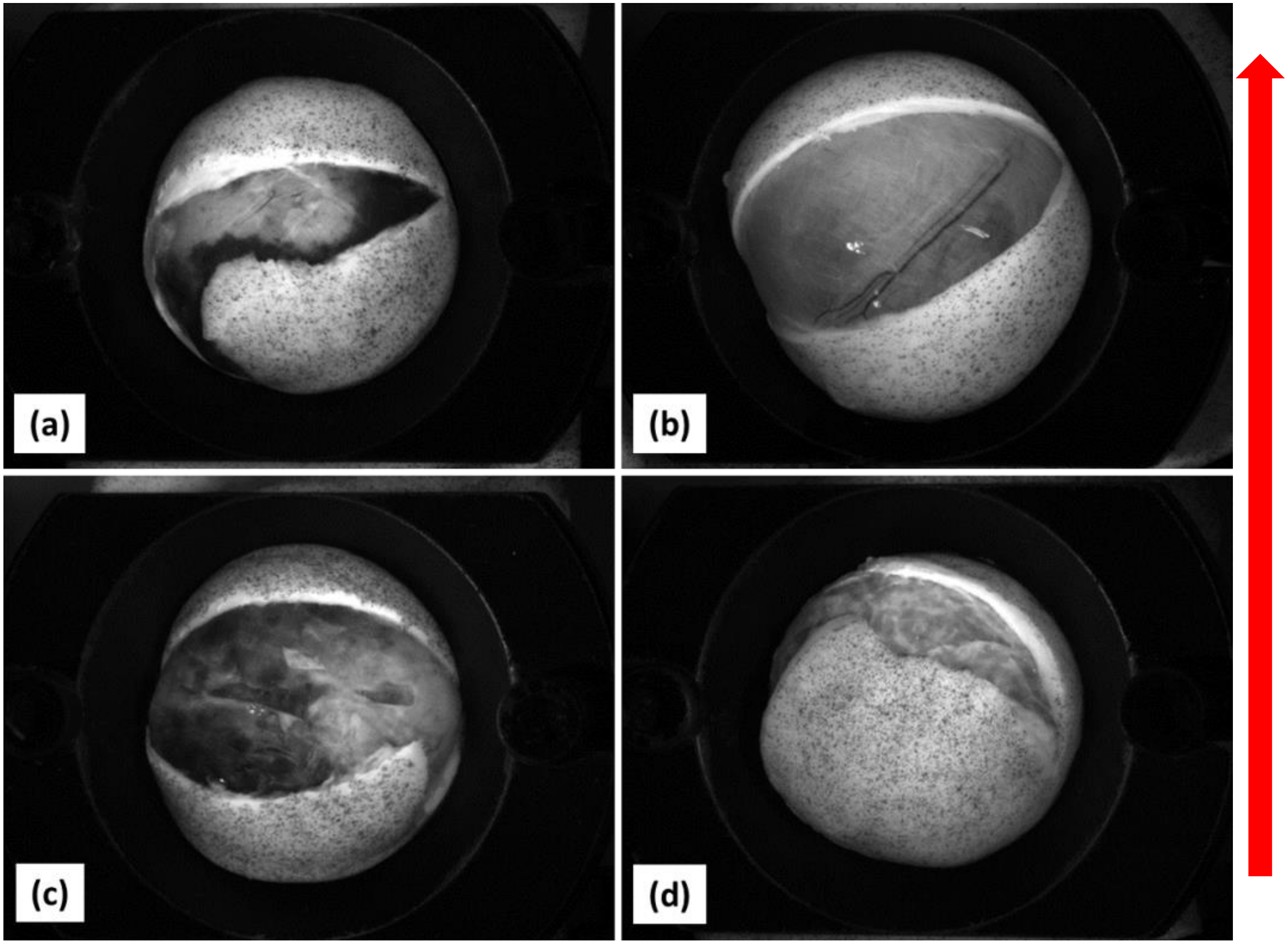
Undeformed

Deformed





Rupture profiles



50% of aortas ruptured with an angle θ equal to 90 °

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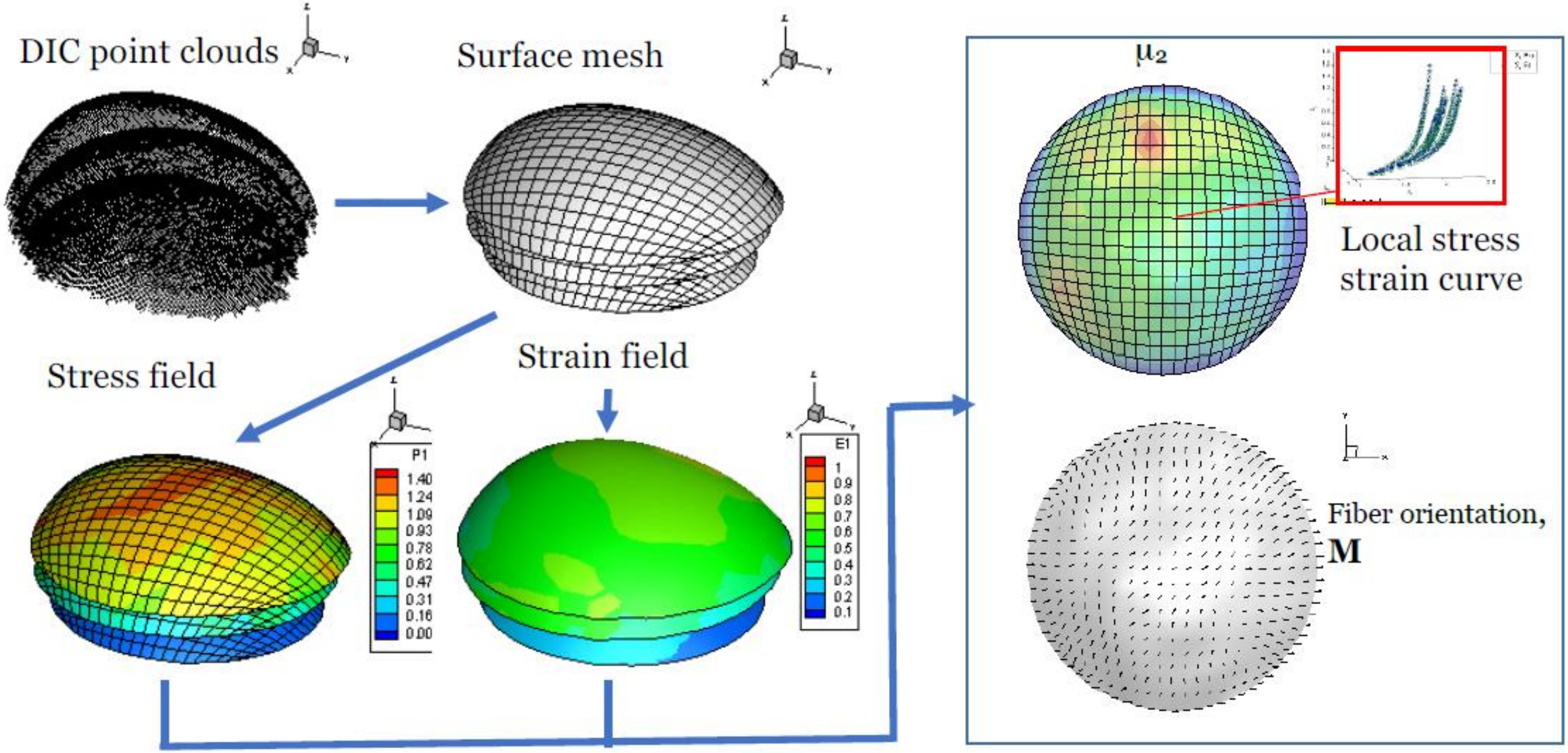
Identification of local material properties

Davis et al. BMMB – 2015.
Davis et al. JMBBM – 2016
Zhao et al. Acta Biomaterialia - 2016



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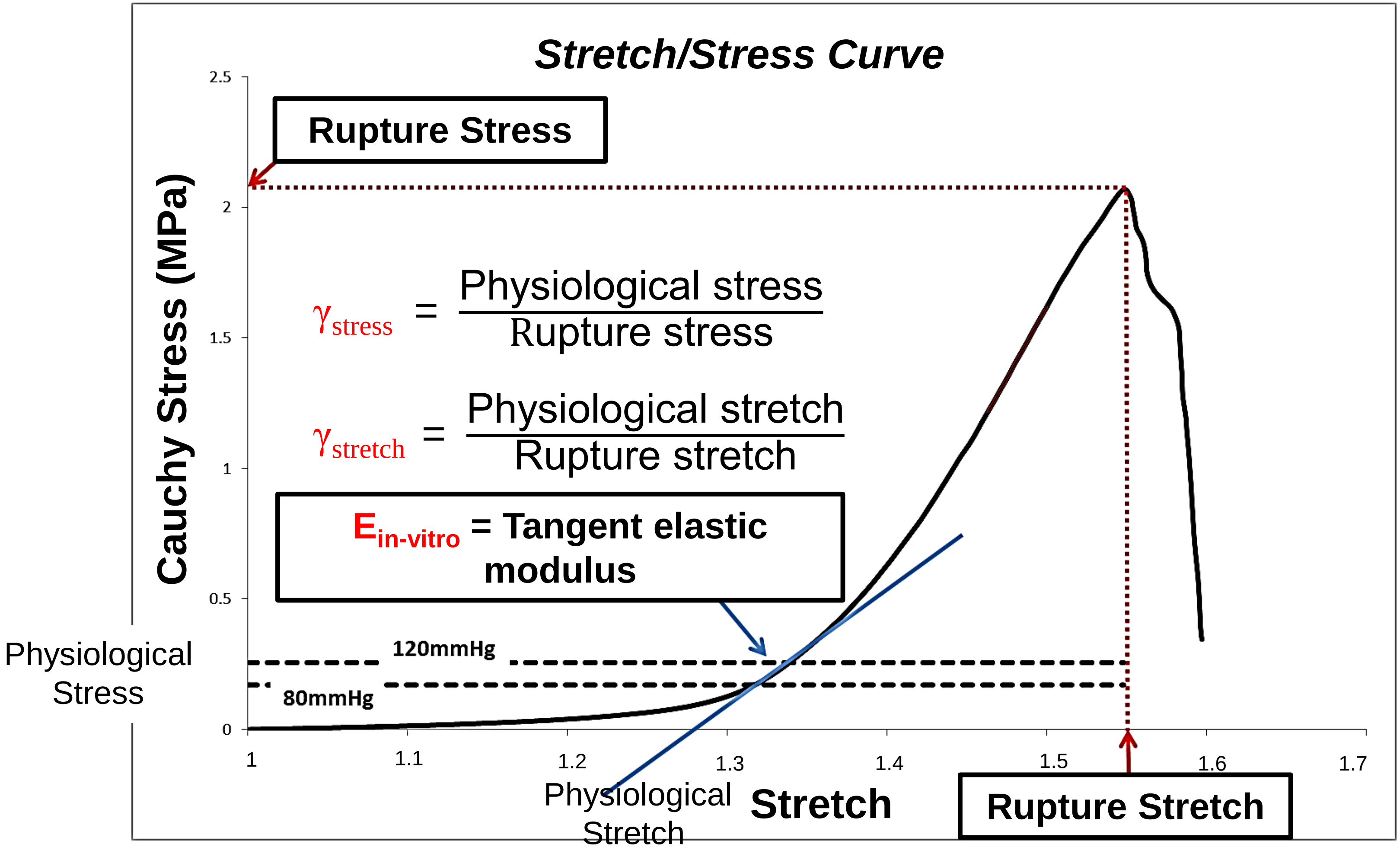
Rupture risk estimation



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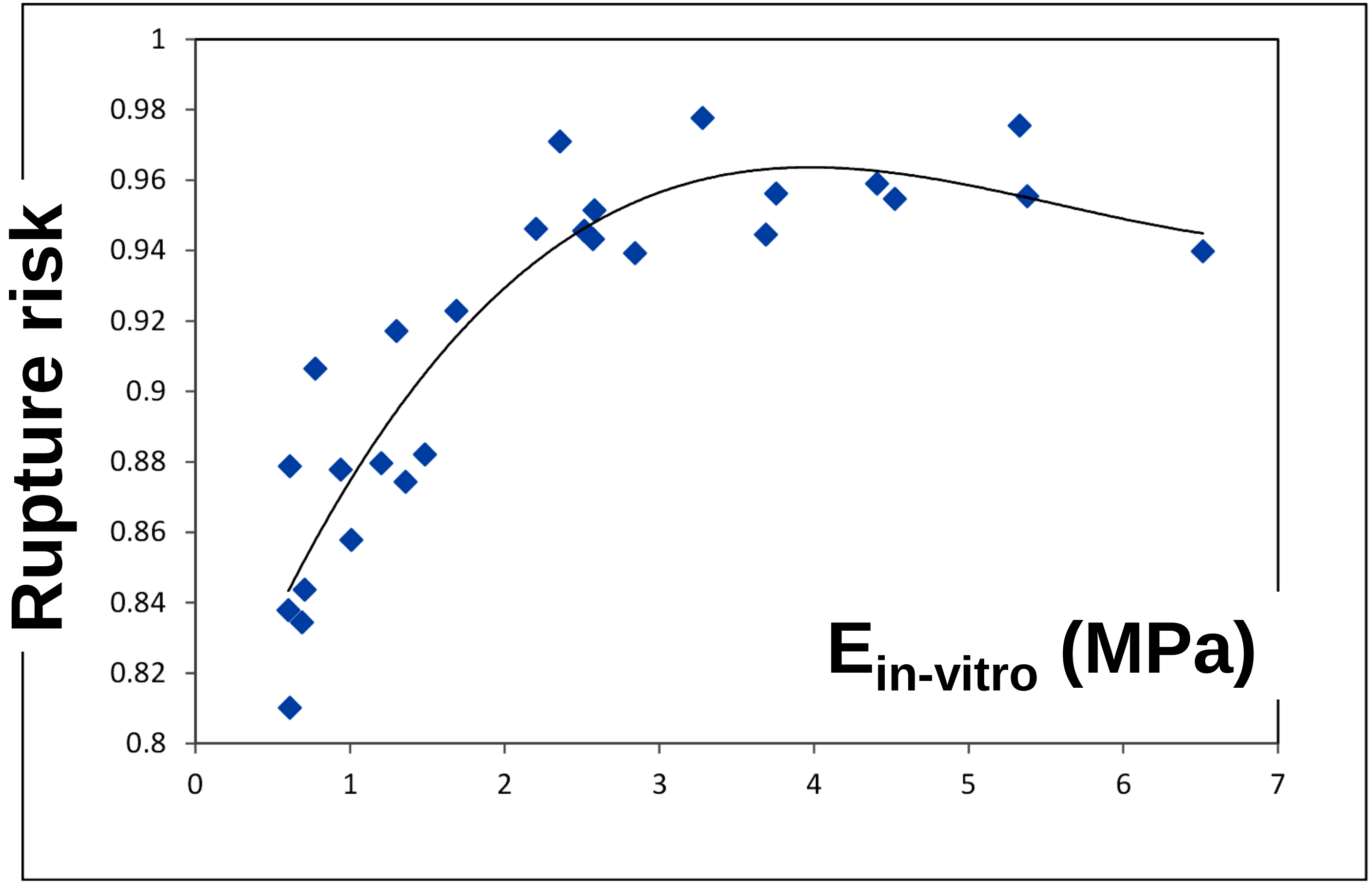
Correlation between the stretch-based rupture risk and the tangent elastic modulus



Duprey A, et al. Biaxial rupture properties of ascending thoracic aortic aneurysms. ***Acta Biomaterialia 2016.***

Understanding aneurysm growth using mechanobiology and photomechanics

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Altered mechanics induce biological responses, including gene expression, protein activation and cell phenotype

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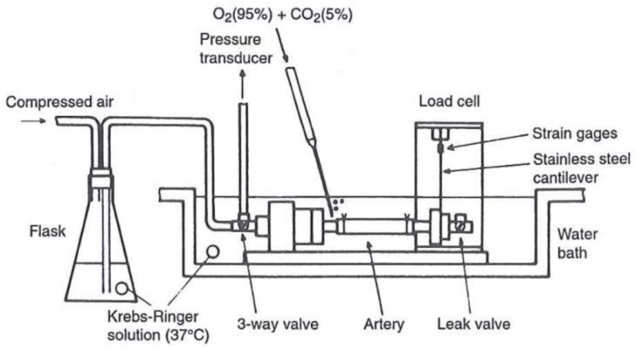
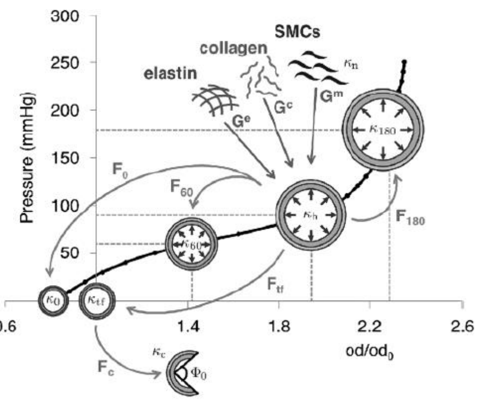
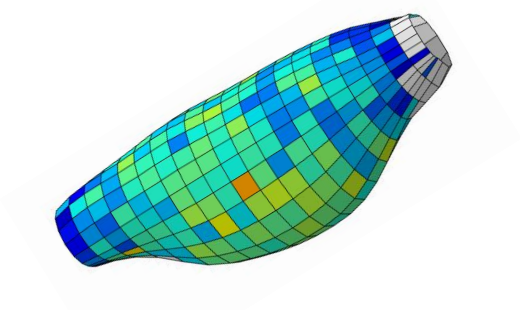
APPROACH

1. Experiments
2. Material model
3. Inverse method







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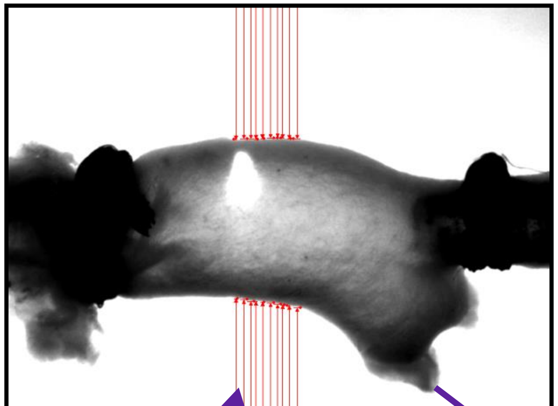
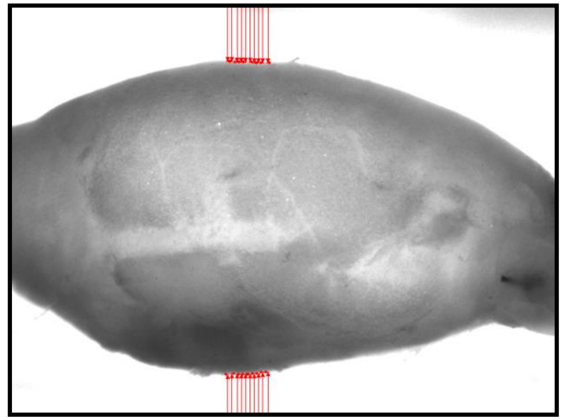
Study Design

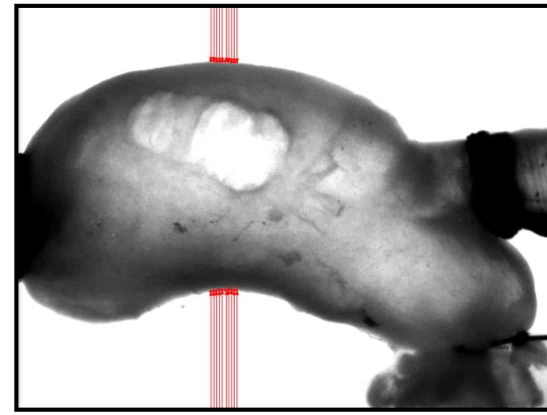
Control

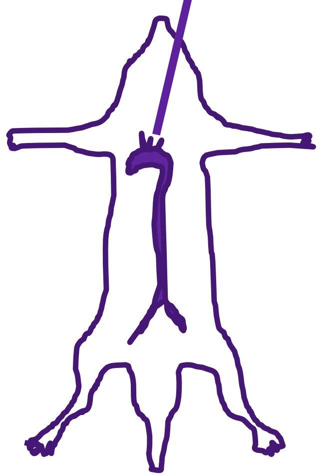


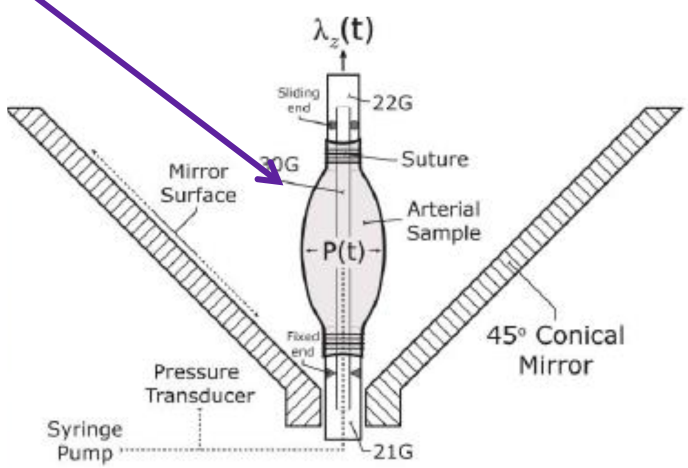
Fibulin 4 SMC KO

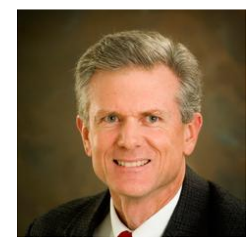


Fibrillin 1 *mgR/mgR*

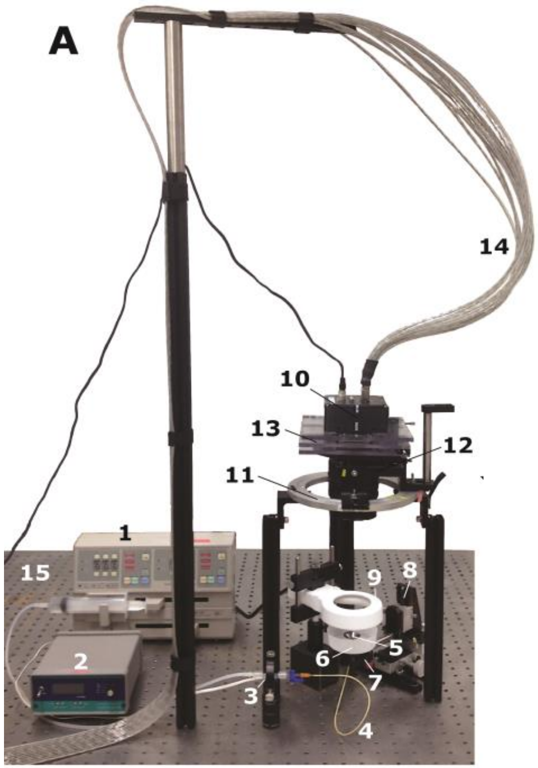


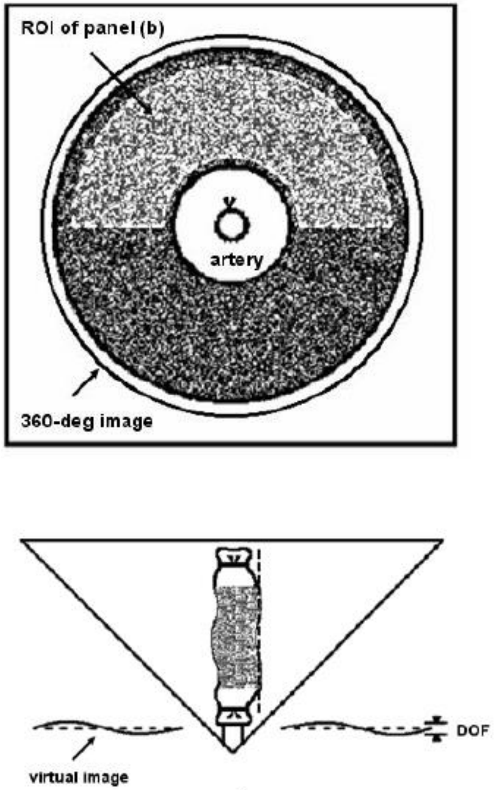


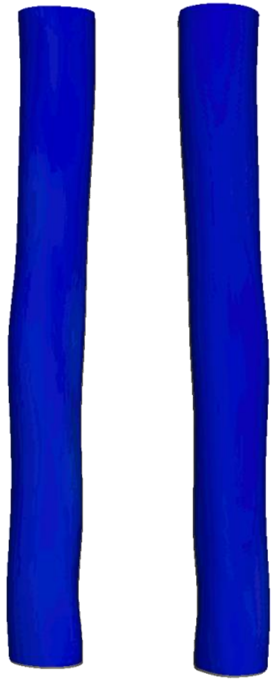




The pDIC technique







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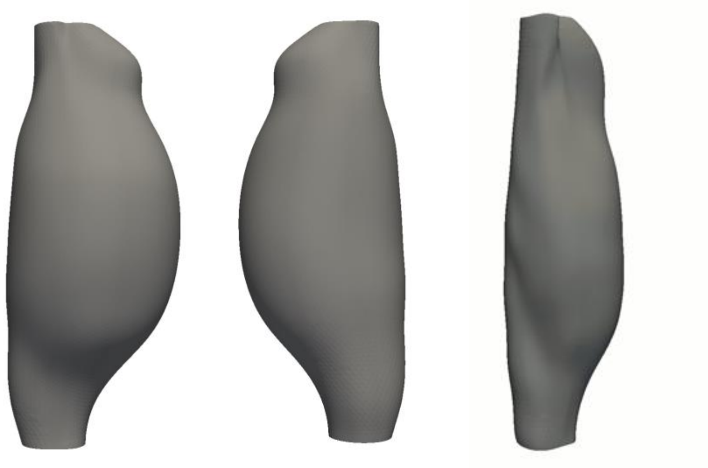
pDIC measurements

Fibulin 4 SMC KO

ventral *dorsal* *inflation*

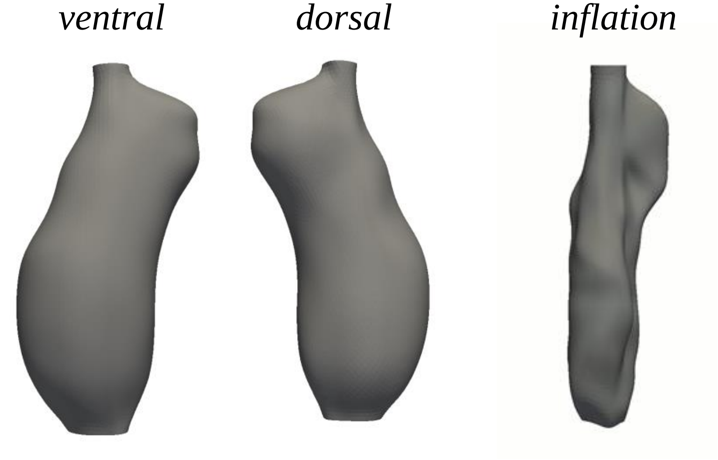
6852



Fibrillin 1 *mgR/mgR*

ventral *dorsal* *inflation*

CS38



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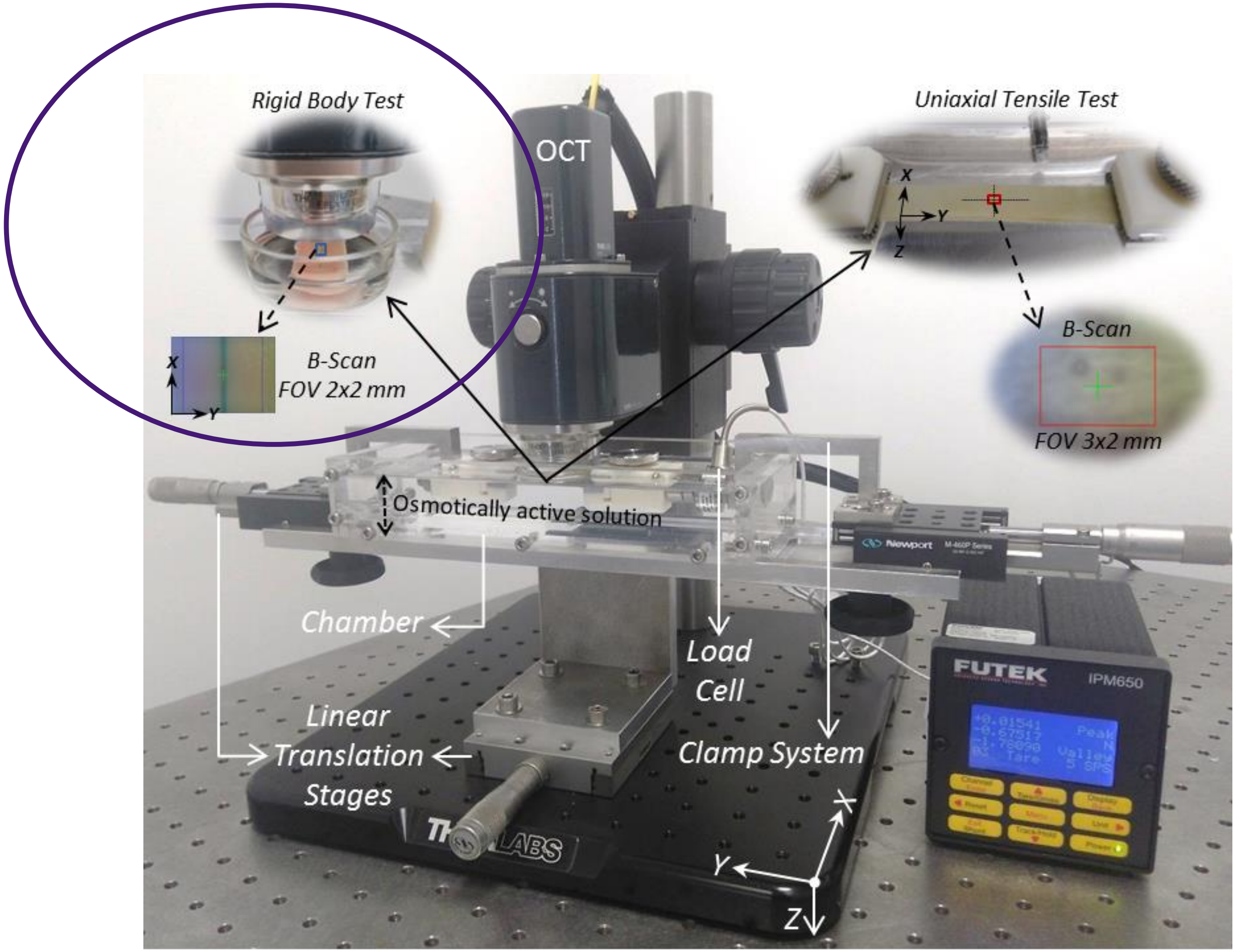
OCT-DVC applied to arterial mechanics



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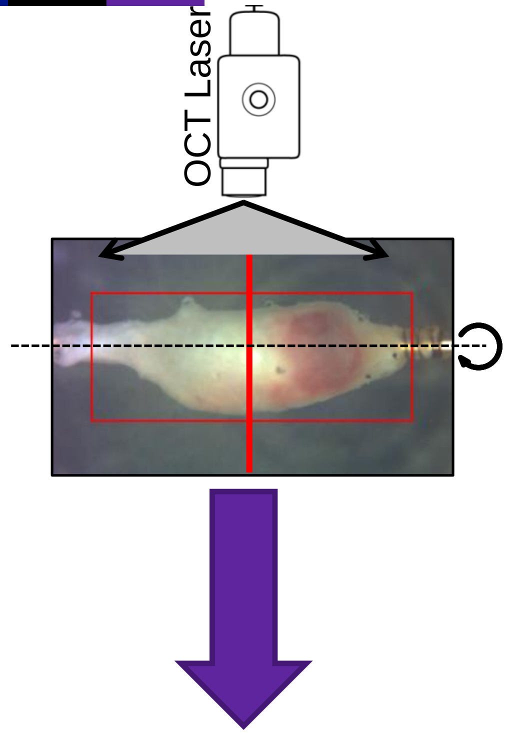
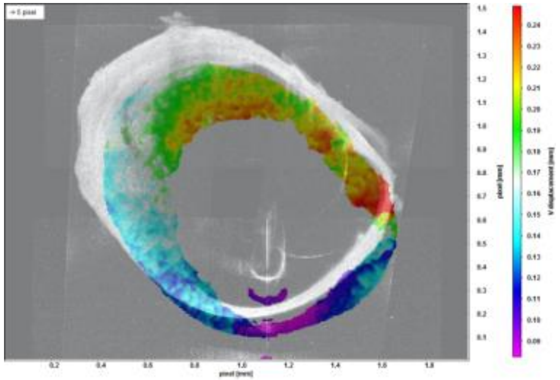
Measurement of bulk deformation fields by Digital Volume Correlation on OCT images





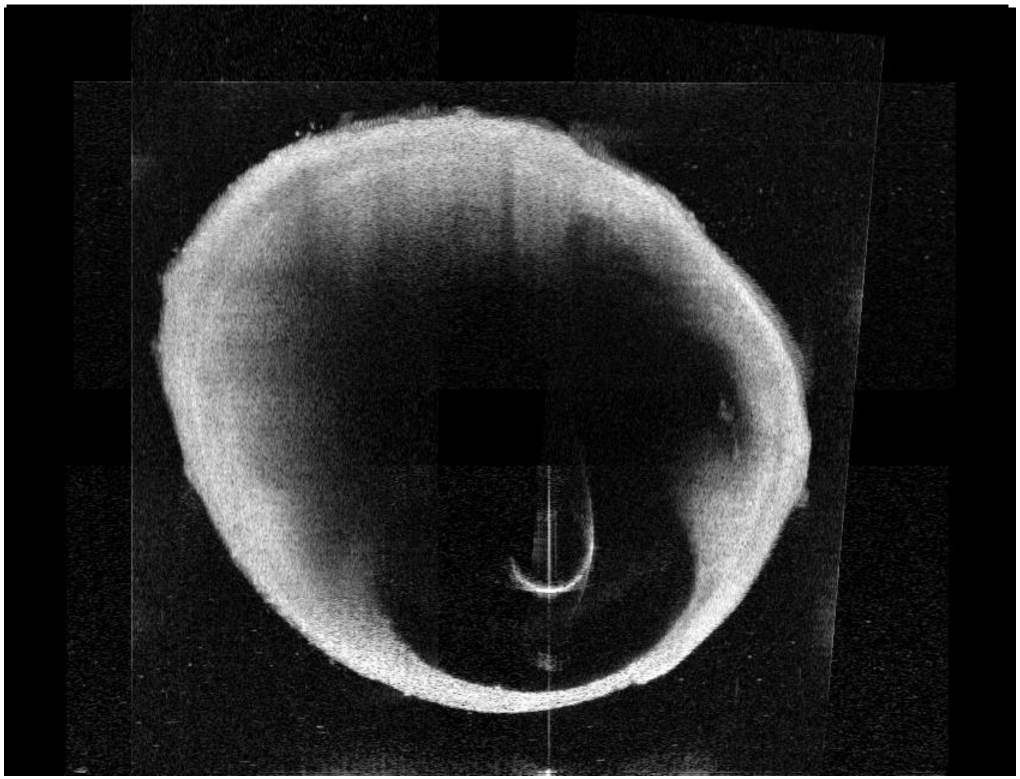


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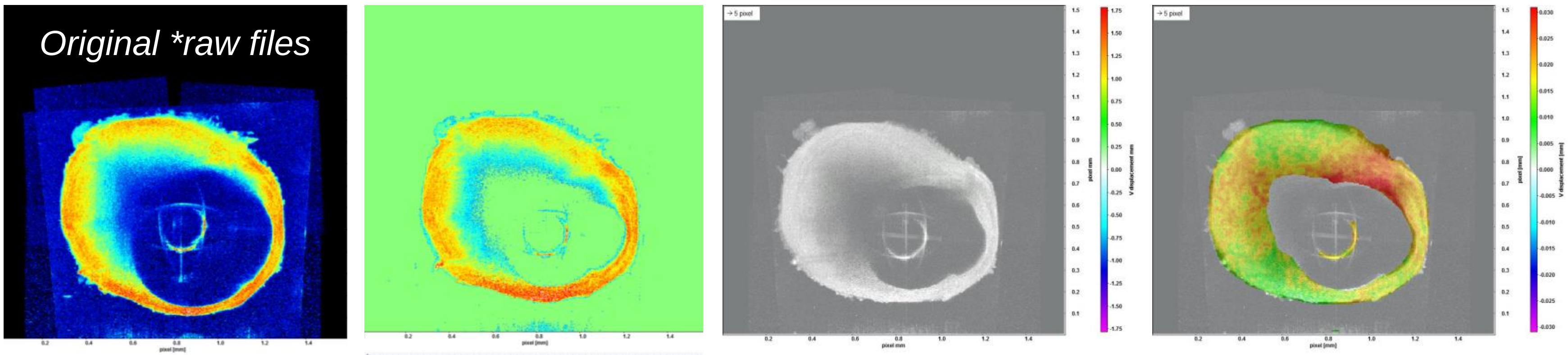
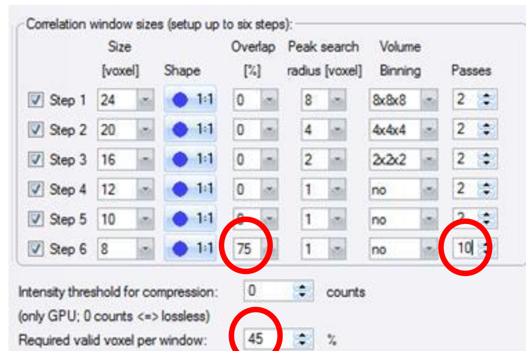
Image Processing Methodology – Inflation Test



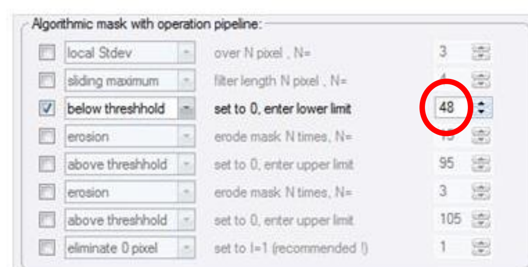
Title: REF
Width: 667 pixels
Height: 640 pixels
Depth: 100 pixels
Voxel size: 1x1x1 pixel^3 (2.38um)

Original Image Parameters



Pressures: 20, 40, 60, 80, 100, 120, 140
Lambda (λ) 1 – 2 – 3

- ***Algorithmic Mask – Threshhold: 48***
- ***Correlation window size [voxel]: 24 (8x8x8)***
- ***Overlap : 75%***
- ***Passes : 10***
- ***Required valid voxel per window: 44%***



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APPROACH

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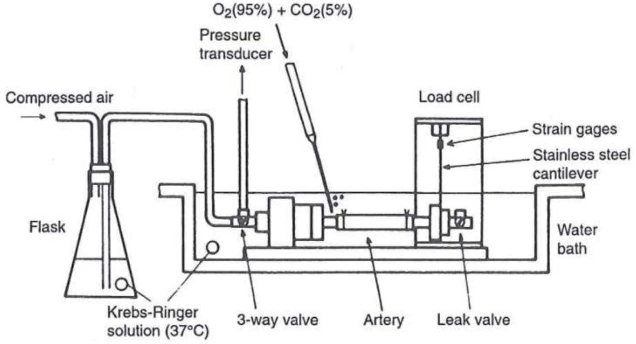

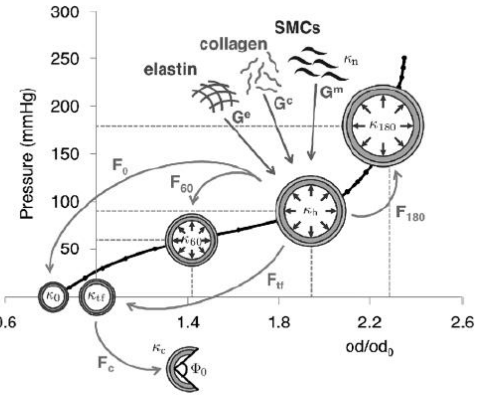

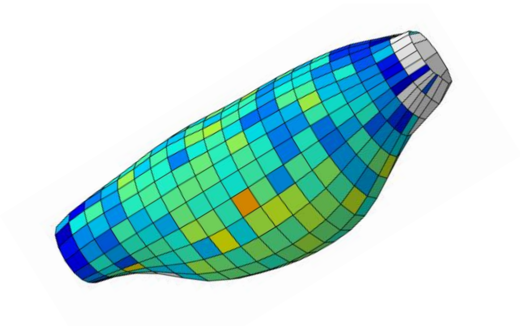

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CONSTITUTIVE MODEL

Bellini, et al., Ann. Biomed. Eng., **42**(3), pp. 488–502, 2014

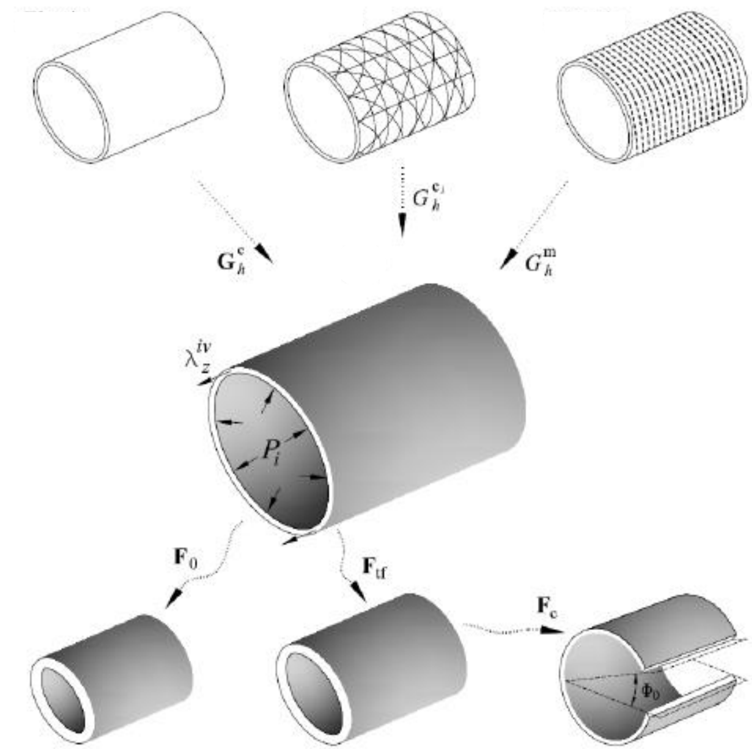
Strain energy functions:

$$W = \phi^{\mathrm{e}} W^{\mathrm{e}}(\mathbf{F}^{\mathrm{e}}) + \phi^{\mathrm{m}} W^{\mathrm{m}}(\lambda^{\mathrm{m}}) + \sum_{j=1}^{4} \phi^{\mathrm{c}_j} W^{\mathrm{c}_j}(\lambda^{\mathrm{c}_j})$$

$$W^{\mathrm{e}}(\mathbf{F}^{\mathrm{e}}) = \frac{c^{\mathrm{e}}}{2}\left[\mathrm{tr}\left((\mathbf{F}^{\mathrm{e}})^{\mathrm{T}}\mathbf{F}^{\mathrm{e}}\right) - 3\right]$$

$$W^{\mathrm{m}}(\lambda^{\mathrm{m}}) = \frac{c_2^{\mathrm{m}}}{4c_3^{\mathrm{m}}}\left[e^{c_3^{\mathrm{m}}\left((\lambda^{\mathrm{m}})^2-1\right)^2} - 1\right]$$

$$W^{\mathrm{c}}(\lambda^{\mathrm{c}_j}) = \frac{c_2^{\mathrm{c}}}{4c_3^{\mathrm{c}}}\left[e^{c_3^{\mathrm{c}}\left((\lambda^{\mathrm{c}_j})^2-1\right)^2} - 1\right]$$



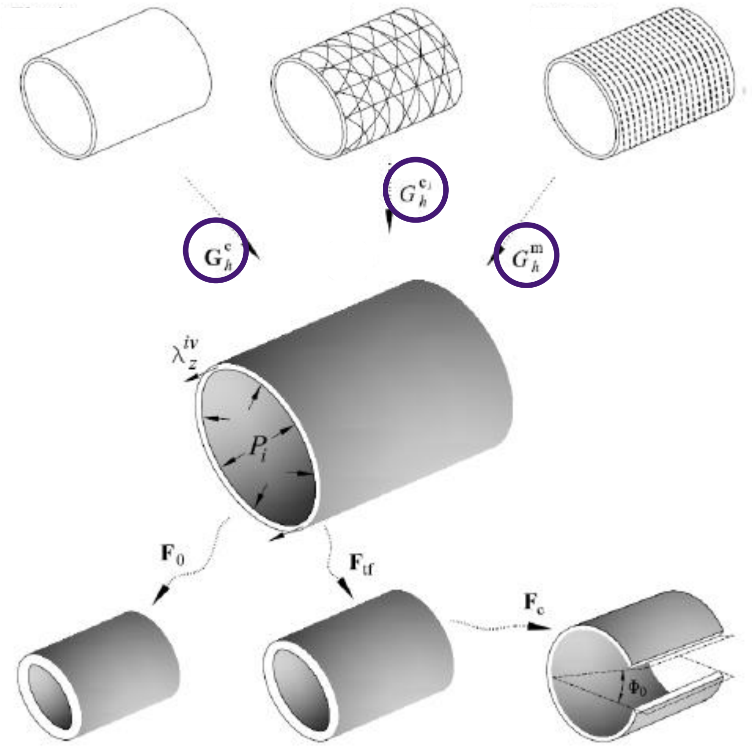
PARAMETERS TO BE IDENTIFIED

$$W = \phi^{\mathrm{e}} W^{\mathrm{e}}(\mathbf{F}^{\mathrm{e}}) + \phi^{\mathrm{m}} W^{\mathrm{m}}(\lambda^{\mathrm{m}}) + \sum_{j=1}^{4} \phi^{\mathrm{c}_j} W^{\mathrm{c}_j}(\lambda^{\mathrm{c}_j})$$

$$W^{\mathrm{e}}(\mathbf{F}^{\mathrm{e}}) = \frac{c^{\mathrm{e}}}{2}\left[\mathrm{tr}\left((\mathbf{F}^{\mathrm{e}})^{\mathrm{T}}\mathbf{F}^{\mathrm{e}}\right) - 3\right]$$

$$W^{\mathrm{m}}(\lambda^{\mathrm{m}}) = \frac{c_2^{\mathrm{m}}}{4c_3^{\mathrm{m}}}\left[e^{c_3^{\mathrm{m}}((\lambda^{\mathrm{m}})^2-1)^2} - 1\right]$$

$$W^{\mathrm{c}}(\lambda^{\mathrm{c}_j}) = \frac{c_2^{\mathrm{c}}}{4c_3^{\mathrm{c}}}\left[e^{c_3^{\mathrm{c}}((\lambda^{\mathrm{c}_j})^2-1)^2} - 1\right]$$



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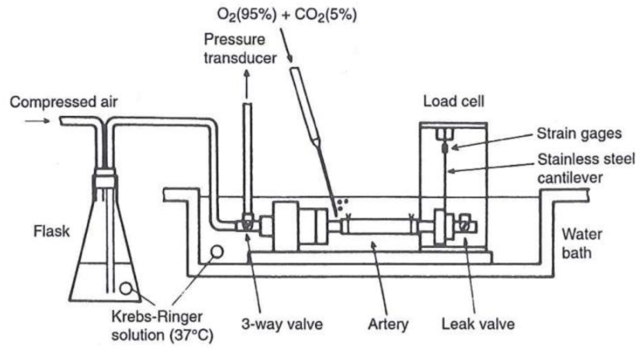
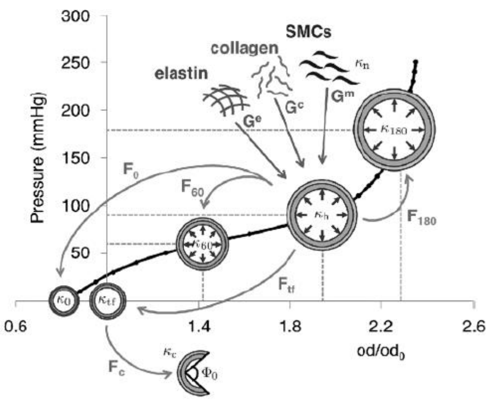
APPROACH

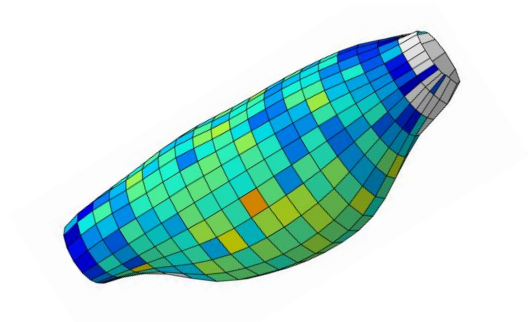


1. Experiments



2. Material model

3. Inverse method



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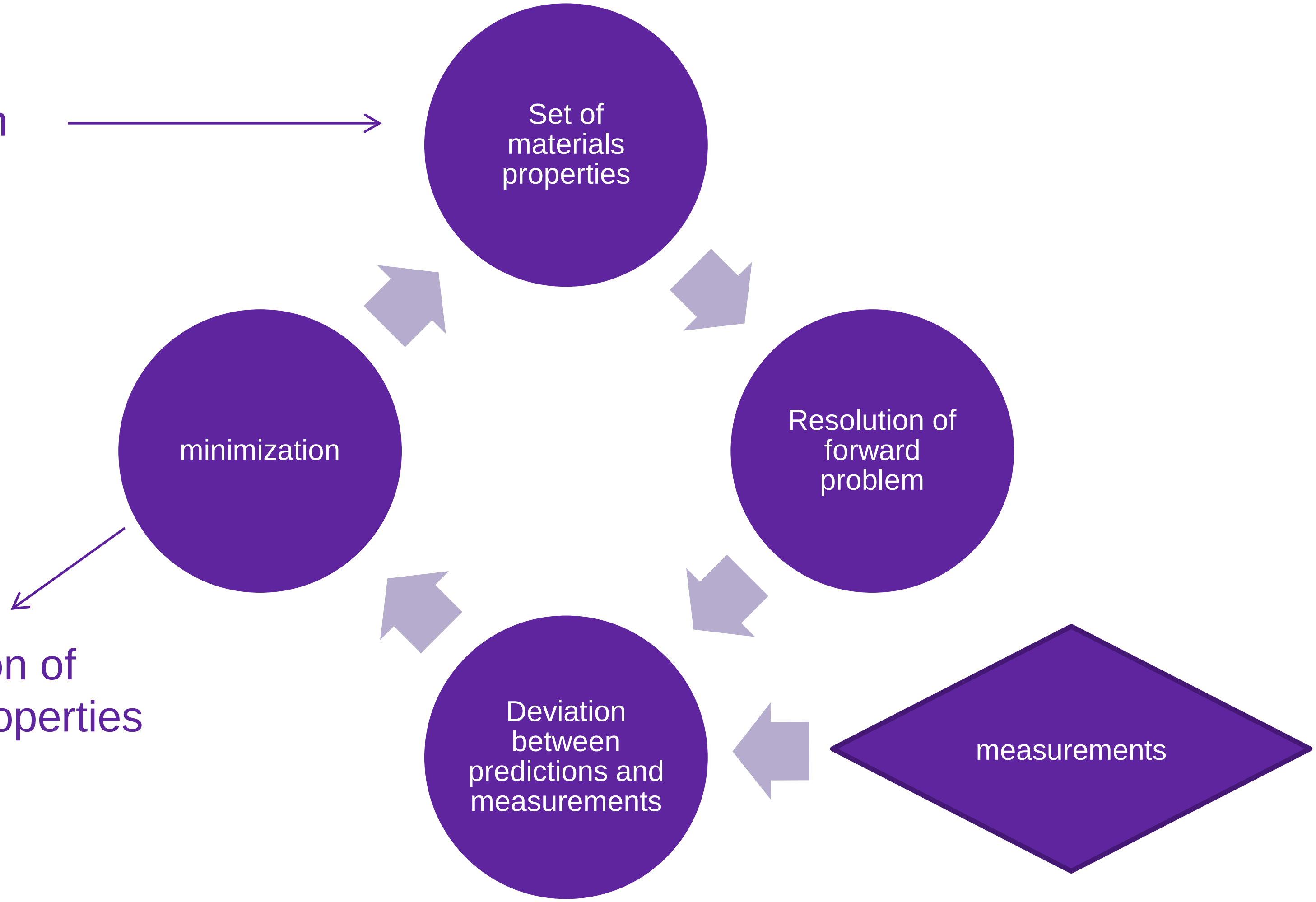
Inverse approach – traditional approach

initialization



Identification of material properties

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Inverse approach – FEMU approach

initialization →

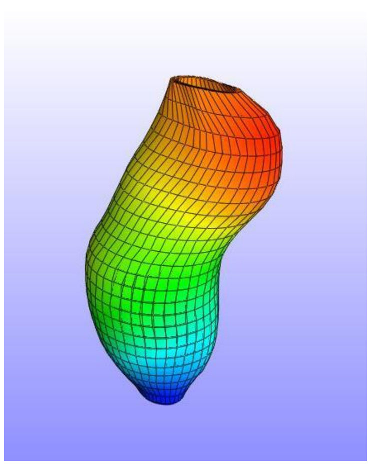
Set of materials properties

Oberai et al., Inverse problems, **19**, pp. 297-313, 2003

Resolution of forward problem

Deviation between predictions and measurements

measurements



$$J(\mu) = \|T(u) - T(u^{exp})\|^2 + \frac{\alpha}{2} B(\mu)$$

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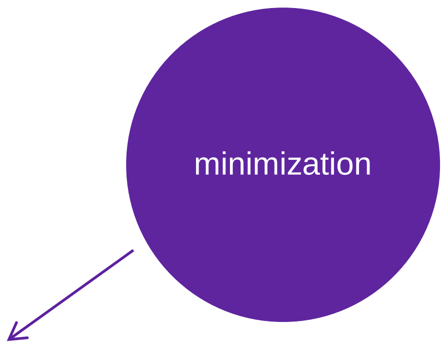
Inverse approach – FEMU approach

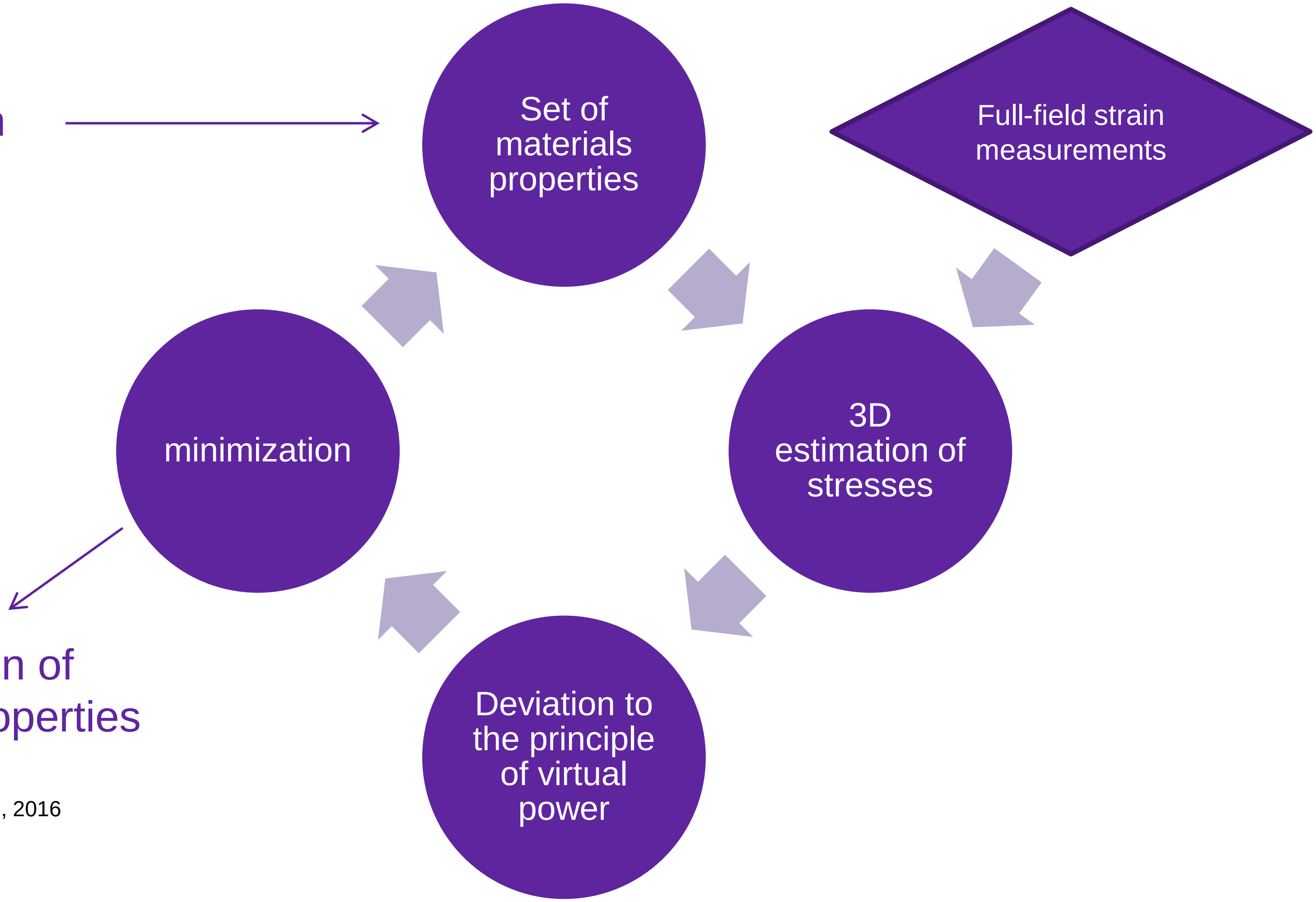


Identification of material properties

1. Use a gradient based method (steepest descent or BFGS)
2. Need to derive the gradient of J with respect to μ at each iteration. With the adjoint method, this requires the resolution of 2 forward problems
3. Very unstable with hyperelastic models: **many risks that the forward problems have a poor convergence**

Alternative inverse approach: the virtual fields method

initialization

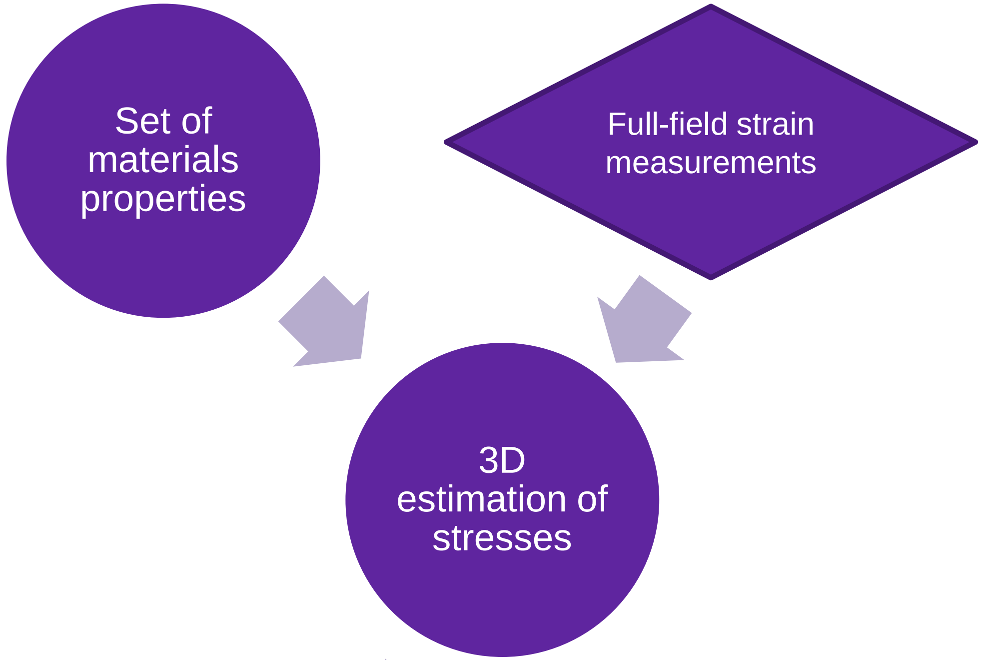


Identification of material properties

Bersi et al., J Biomech Eng, 2016

Full-field stress reconstruction

initialization →



$$W = \phi^{\mathrm{e}} W^{\mathrm{e}}(\mathbf{F}^{\mathrm{e}}) + \phi^{\mathrm{m}} W^{\mathrm{m}}(\lambda^{\mathrm{m}}) + \sum_{j=1}^{4} \phi^{\mathrm{c}_j} W^{\mathrm{c}_j}(\lambda^{\mathrm{c}_j})$$

$$W^{\mathrm{e}}(\mathbf{F}^{\mathrm{e}}) = \frac{c^{\mathrm{e}}}{2}\left[\mathrm{tr}\left((\mathbf{F}^{\mathrm{e}})^{\mathrm{T}}\mathbf{F}^{\mathrm{e}}\right) - 3\right]$$

$$W^{\mathrm{m}}(\lambda^{\mathrm{m}}) = \frac{c_2^{\mathrm{m}}}{4c_3^{\mathrm{m}}}\left[e^{c_3^{\mathrm{m}}\left((\lambda^{\mathrm{m}})^2-1\right)^2} - 1\right]$$

$$W^{\mathrm{c}}(\lambda^{\mathrm{c}_j}) = \frac{c_2^{\mathrm{c}}}{4c_3^{\mathrm{c}}}\left[e^{c_3^{\mathrm{c}}\left((\lambda^{\mathrm{c}_j})^2-1\right)^2} - 1\right]$$

Simple application of the constitutive model for each element

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Minimization of the equilibrium gap using the principle of virtual power

minimization

$$J = \sum_p \sum_\lambda \left(\underbrace{- \int_{\omega(t)} \underline{\underline{\sigma}} : \left(\underline{\nabla} \otimes \underline{\xi}^* \right) d\omega}_{P^*_{int}} + \underbrace{\oint_{\partial\omega(t)} \underline{T} : \underline{\xi}^* \, ds}_{P^*_{ext}} \right)^2$$

Bersi et al., J Biomech Eng, 2016

Resolution:

$$\min_{c_3^1, c_3^{2,3}, c_3^4, \alpha, \beta} \left[\min_{c^e, c_2^1, c_2^{2,3}, c_2^4} \left[\frac{J(u)}{A} + \frac{J(v)}{B} \right] \right]$$

Linear least-squares (inner minimization)

Genetic algorithm (outer minimization)

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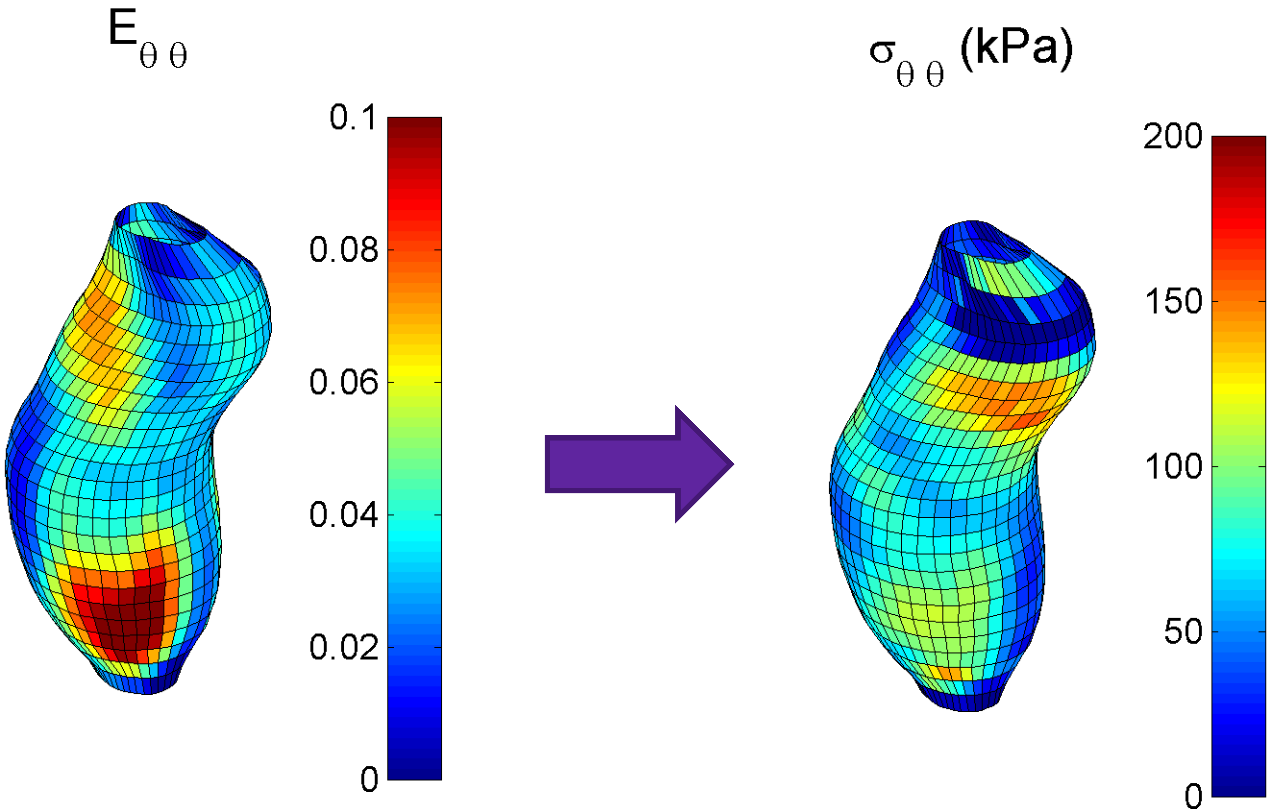
Derivation of stress tensor using layer specific constitutive behavior

3D estimation of strains

↓

3D estimation of stress



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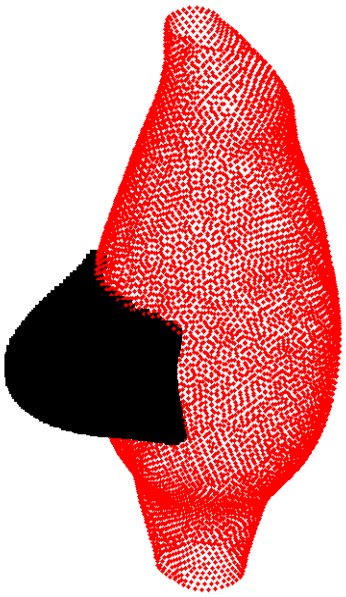
Virtual power 1

1. A local virtual radial “bulge”: $\mathbf{u}(\mathbf{x}) = [f(\mathbf{x}-\mathbf{x}_0)/r^2]\,\mathbf{e}_r$



$$\underbrace{-\int_{\omega(t)} \underline{\underline{\sigma}} : \left(\underline{\nabla} \otimes \underline{\xi}^*\right) d\omega}_{P^*_{int}} + \underbrace{\oint_{\partial\omega(t)} \underline{T} : \underline{\xi}^* \, ds}_{P^*_{ext}} = 0$$

After deriving virtual strains (infinitesimal) local internal virtual work is derived at every Gauss point and integrated across the volume

The virtual field is normalized such as the external virtual work equals P.

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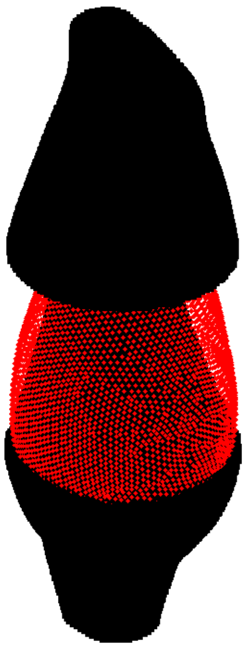
Virtual power 2

2. Local virtual axial extension: **v**(**x**) = f(**x**-**x**$_0$) (z **e**$_z$ –x/2 **e**$_x$ –y/2 **e**$_y$)



$$\underbrace{-\int_{\omega(t)} \underline{\underline{\sigma}} : \left(\underline{\nabla} \otimes \underline{\xi}^*\right) d\omega}_{P^*_{int}} + \underbrace{\oint_{\partial\omega(t)} \underline{T} : \underline{\xi}^* \, ds}_{P^*_{ext}} = 0$$

After deriving virtual strains (infinitesimal) local internal virtual work is derived at every Gauss point and integrated across the volume

The virtual field is normalized such as the external virtual work equals F.

Minimizing the equilibrium gap

minimization

$$J = \sum_{p} \sum_{\lambda} \Bigg(\underbrace{- \int_{\omega(t)} \underline{\underline{\sigma}} : \left(\underline{\nabla} \otimes \underline{\xi}^{*} \right) d\omega}_{P^{*}_{int}} + \underbrace{\oint_{\partial\omega(t)} \underline{T} : \underline{\xi}^{*} \, ds}_{P^{*}_{ext}} \Bigg)^{2}$$

Bersi et al., J Biomech Eng, 2016

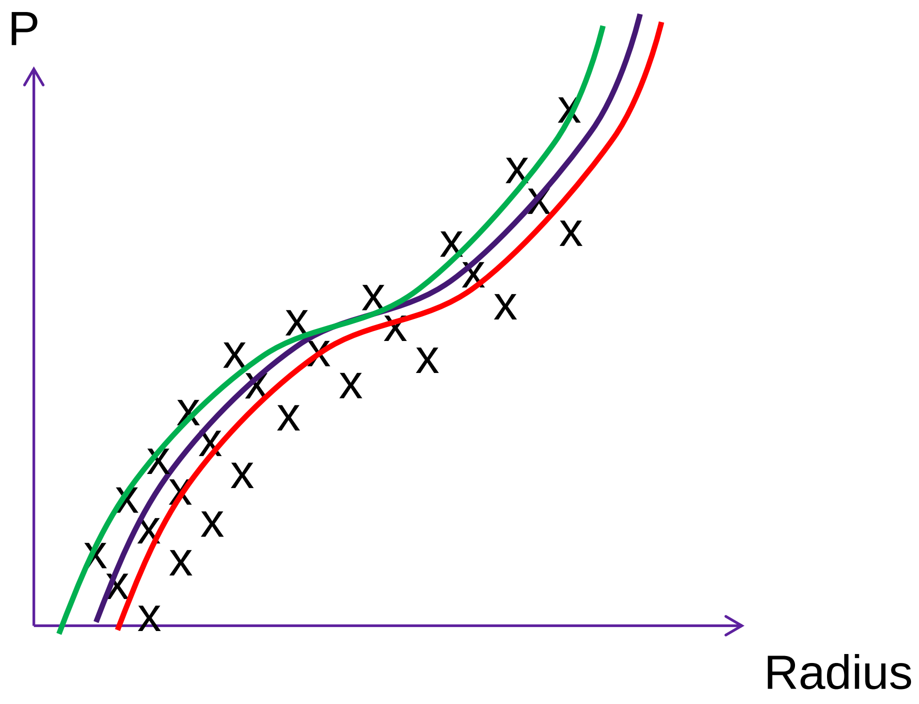
Resolution:

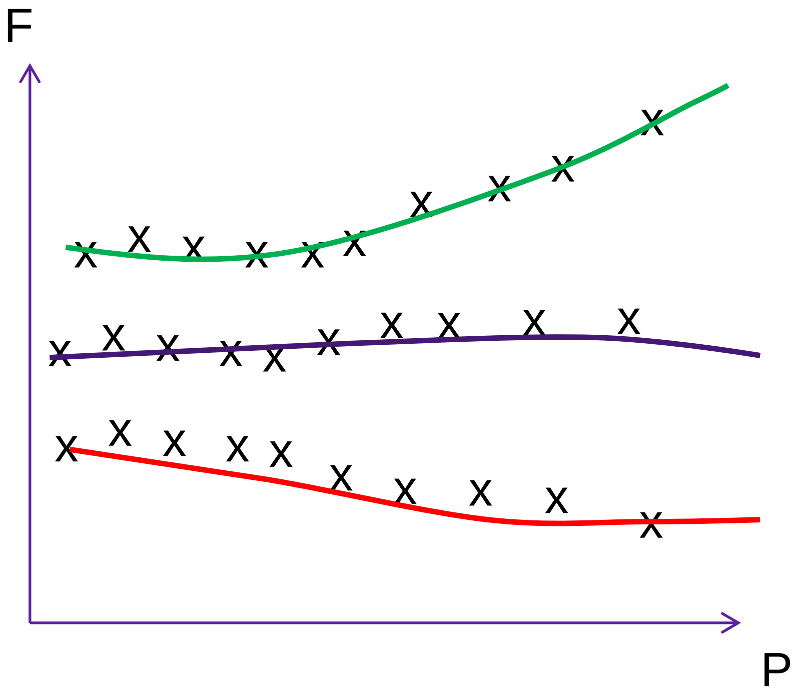
$$\min_{c_3^1, c_3^{2,3}, c_3^4, \alpha, \beta} \left[\min_{c^e, c_2^1, c_2^{2,3}, c_2^4} \left[\frac{J(u)}{A} + \frac{J(v)}{B} \right] \right]$$

Linear least-squares (inner minimization)

Genetic algorithm (outer minimization)

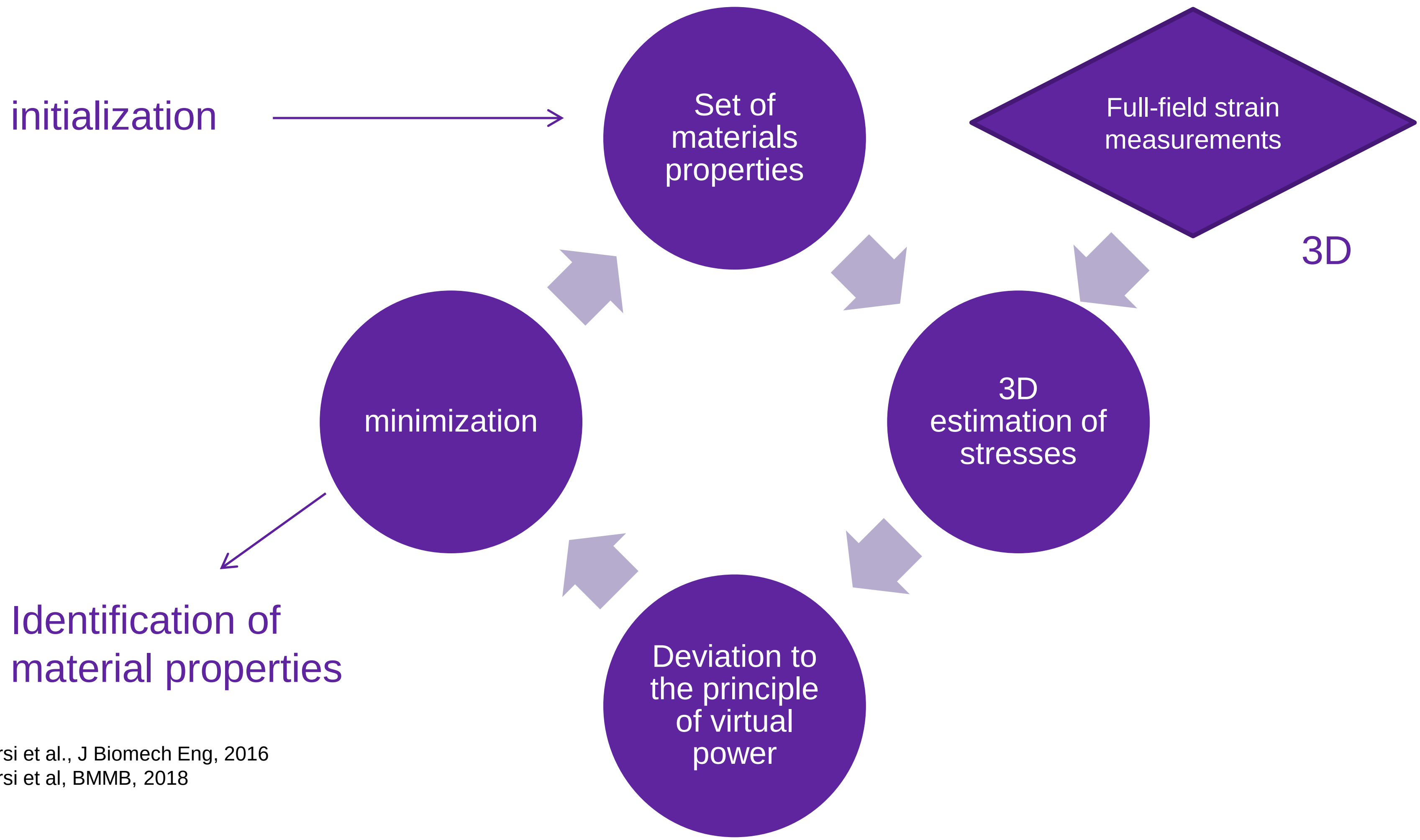
Similar to material fitting at every position





Crosses represent external virtual work for every pressure and axial stretch
Solid lines represent internal virtual work
The goodness of fit is evaluated with the R^2 value

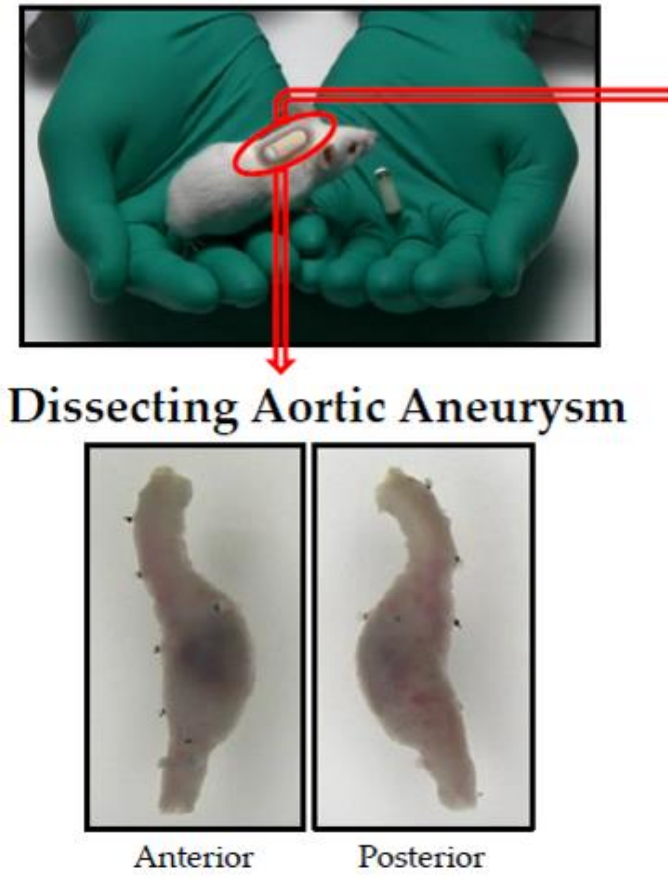
Summary of the inverse approach

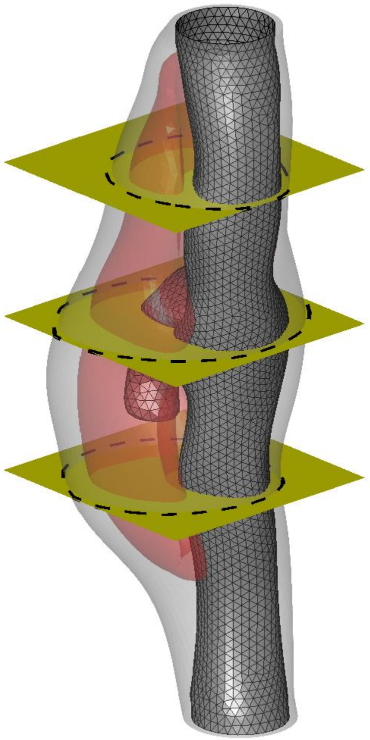


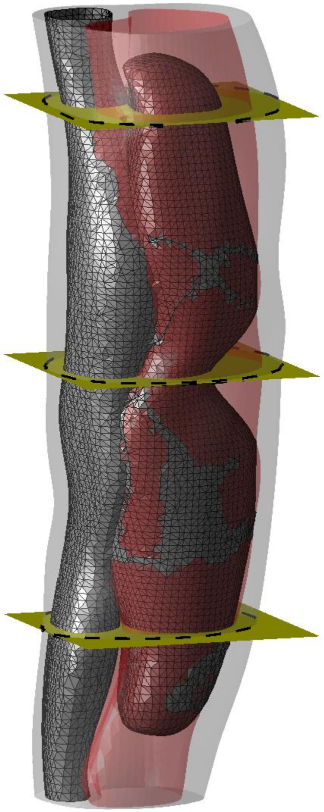
Bersi et al., J Biomech Eng, 2016
Bersi et al, BMMB, 2018

Results - Highlights

DISSECTED ANEURYSMS







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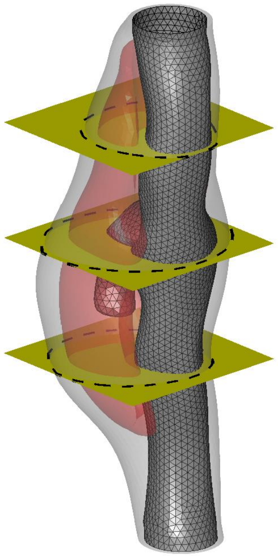
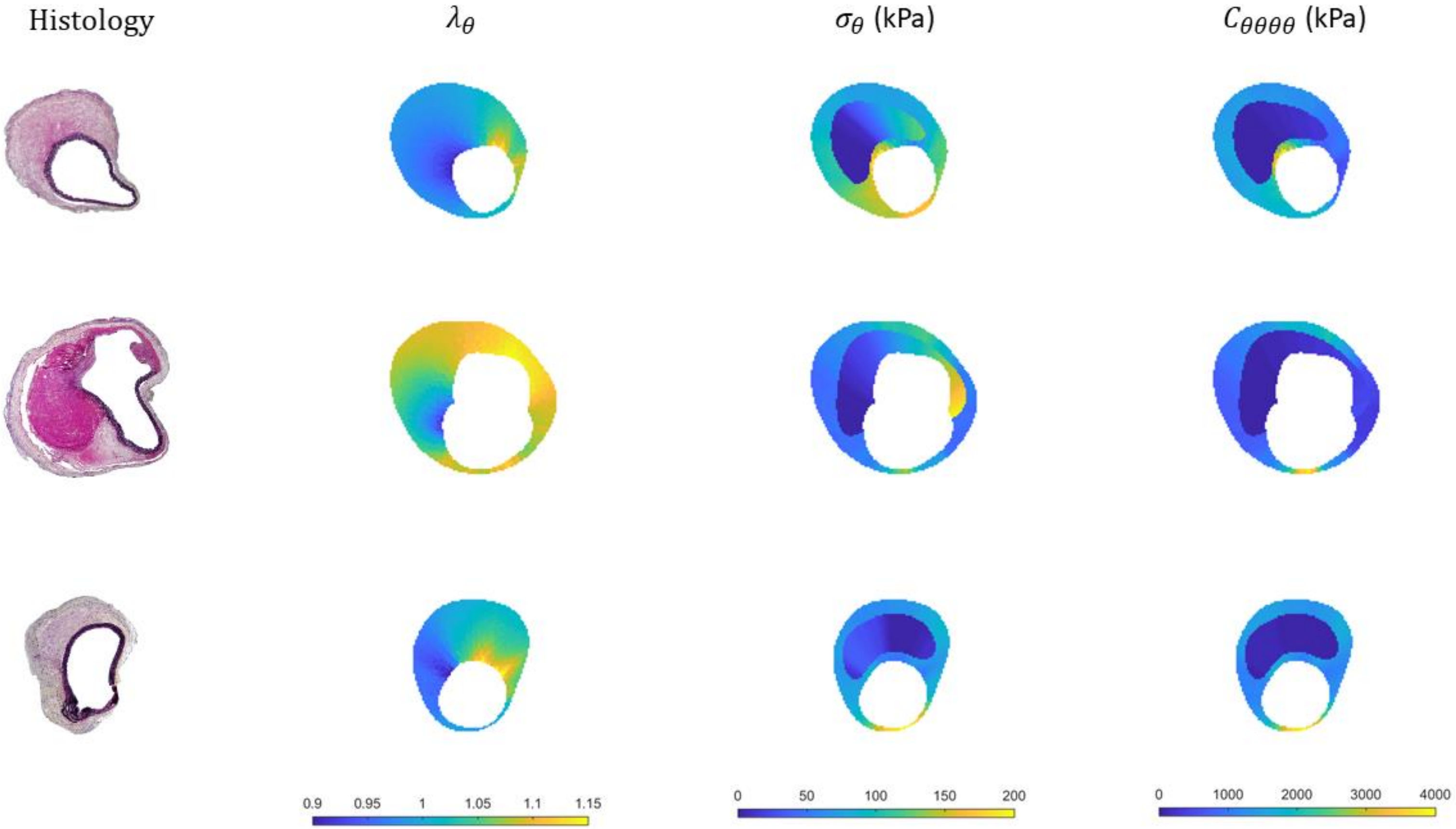
Cross sectional results





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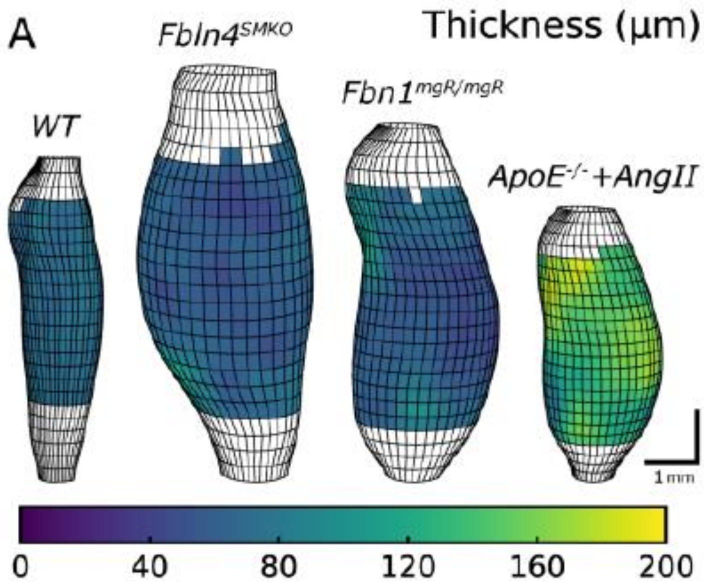
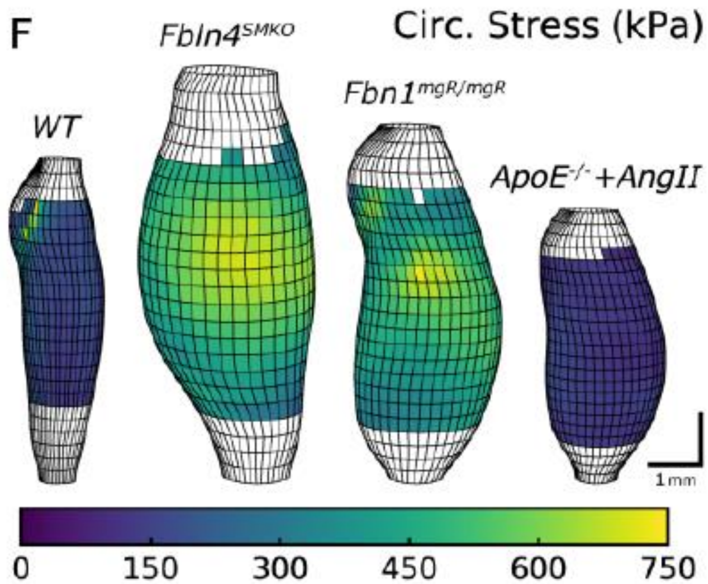
Full-Field Material Parameter Estimation vs thickness distribution





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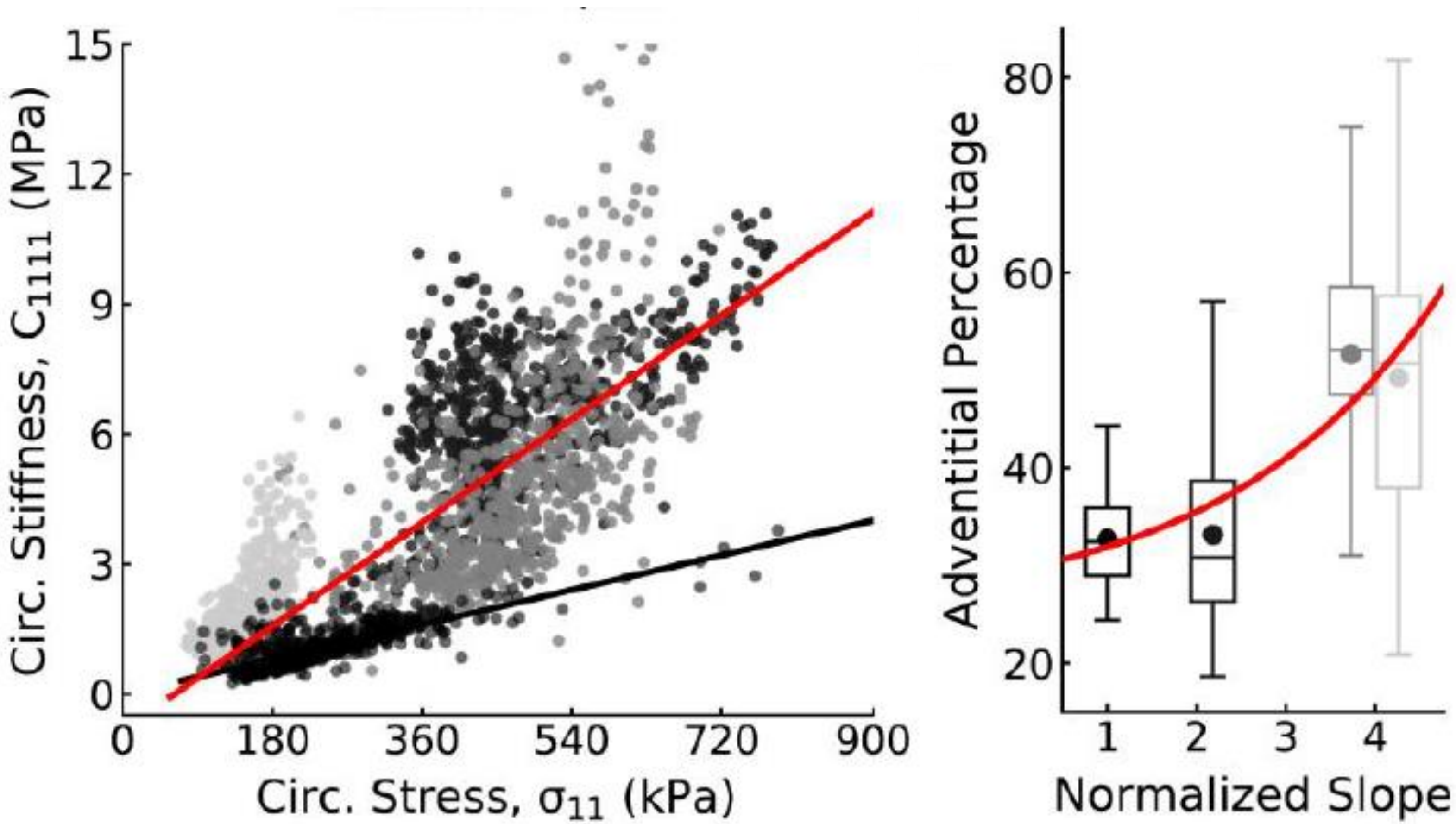
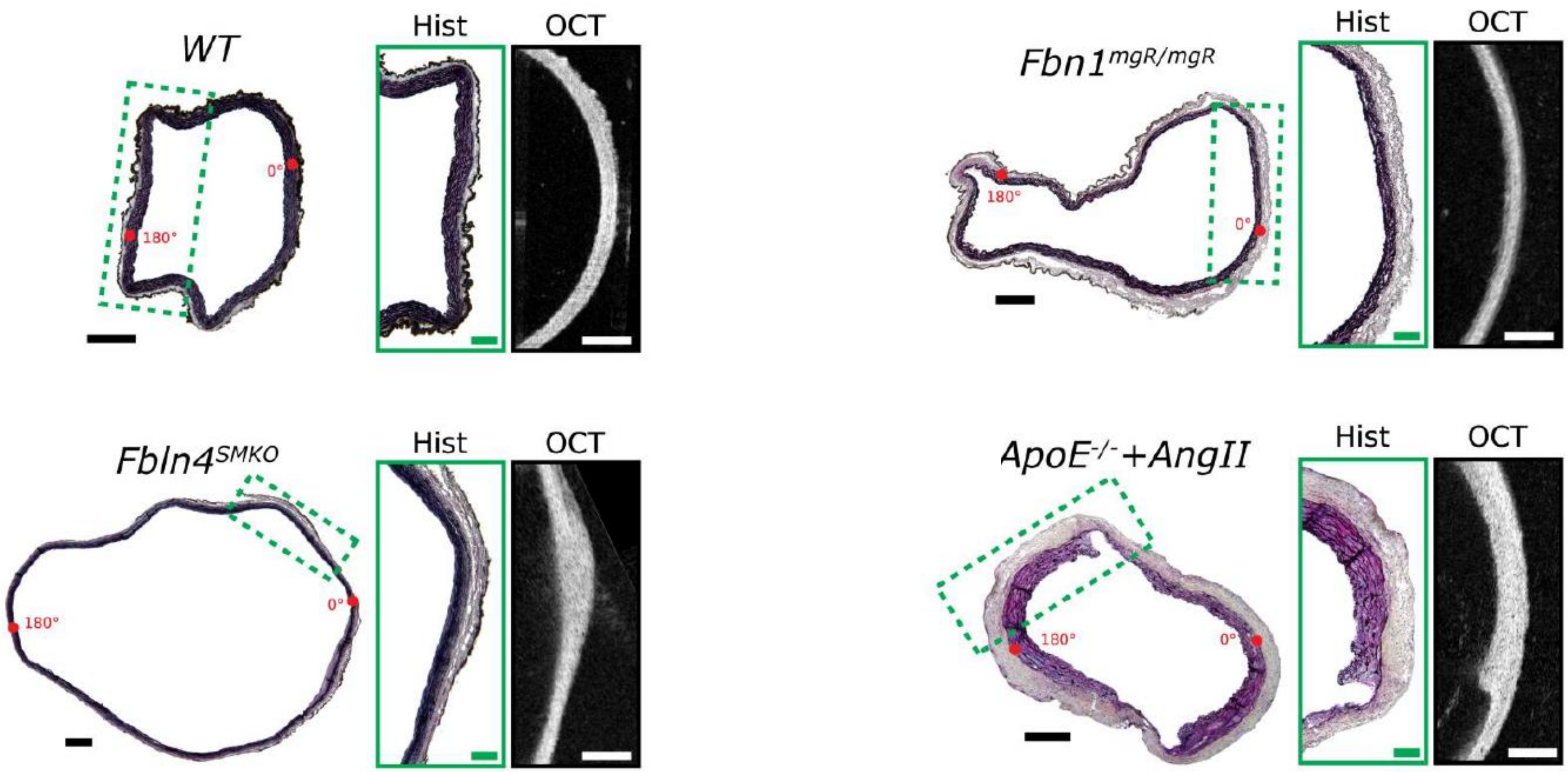
Full-Field Material Parameter Estimation vs local stress

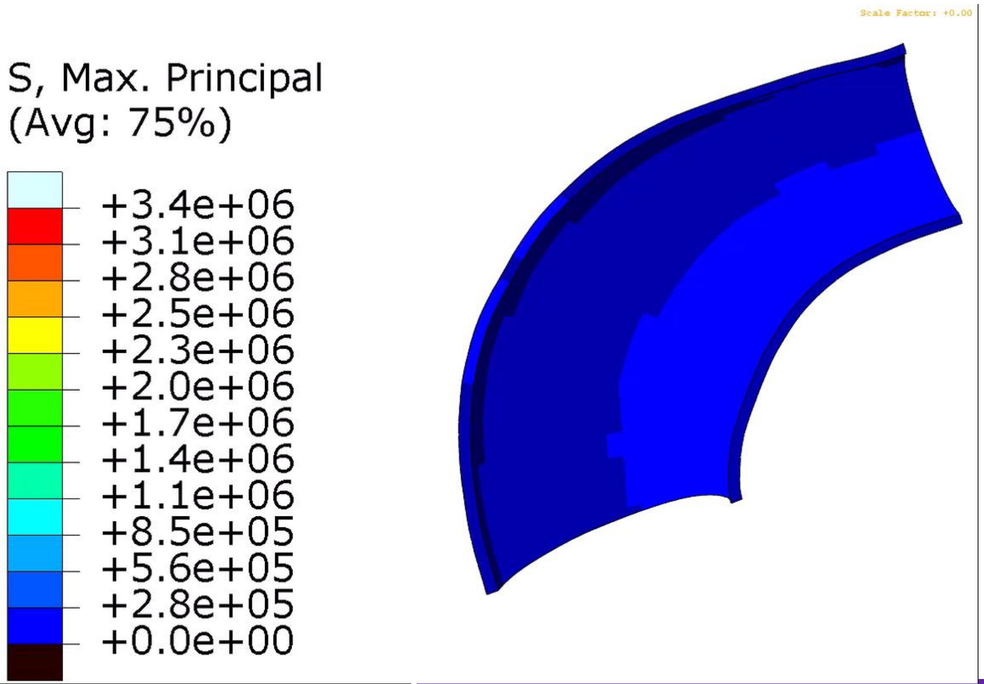


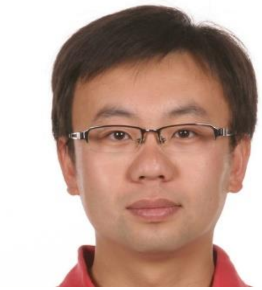
Correlation with tissue µstructure



Vision

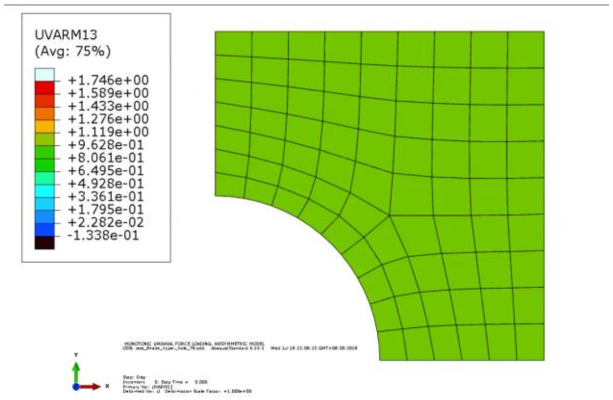
- **Our vision is that the evolution of the strength and of the wall stress of the aorta during the growth of an aneurysm can be predicted on a patient-specific basis by a computational model.**



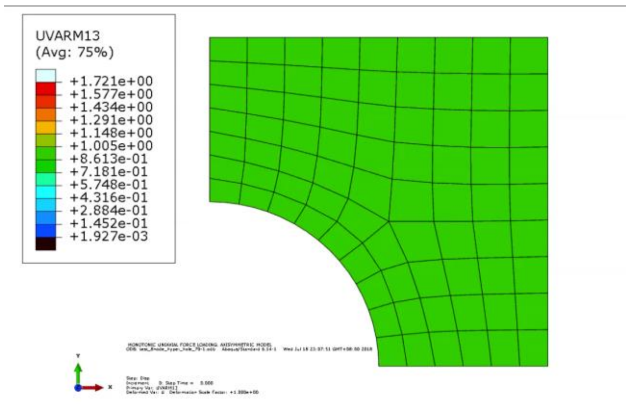


Gradient-enhanced G&R healing model

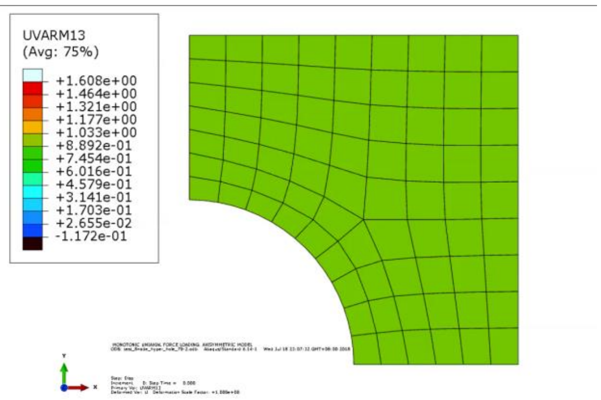
k=0



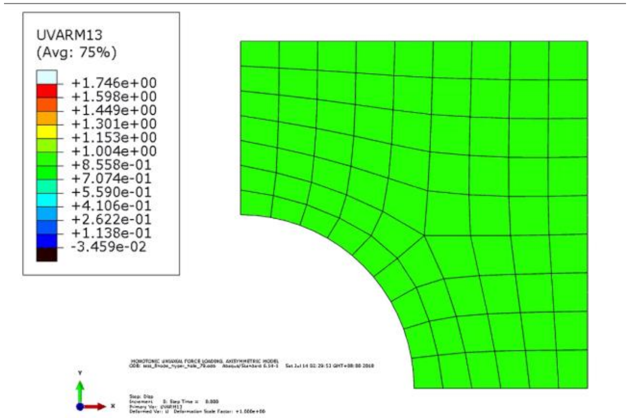
k=0.5



k=1.0



k=5.0



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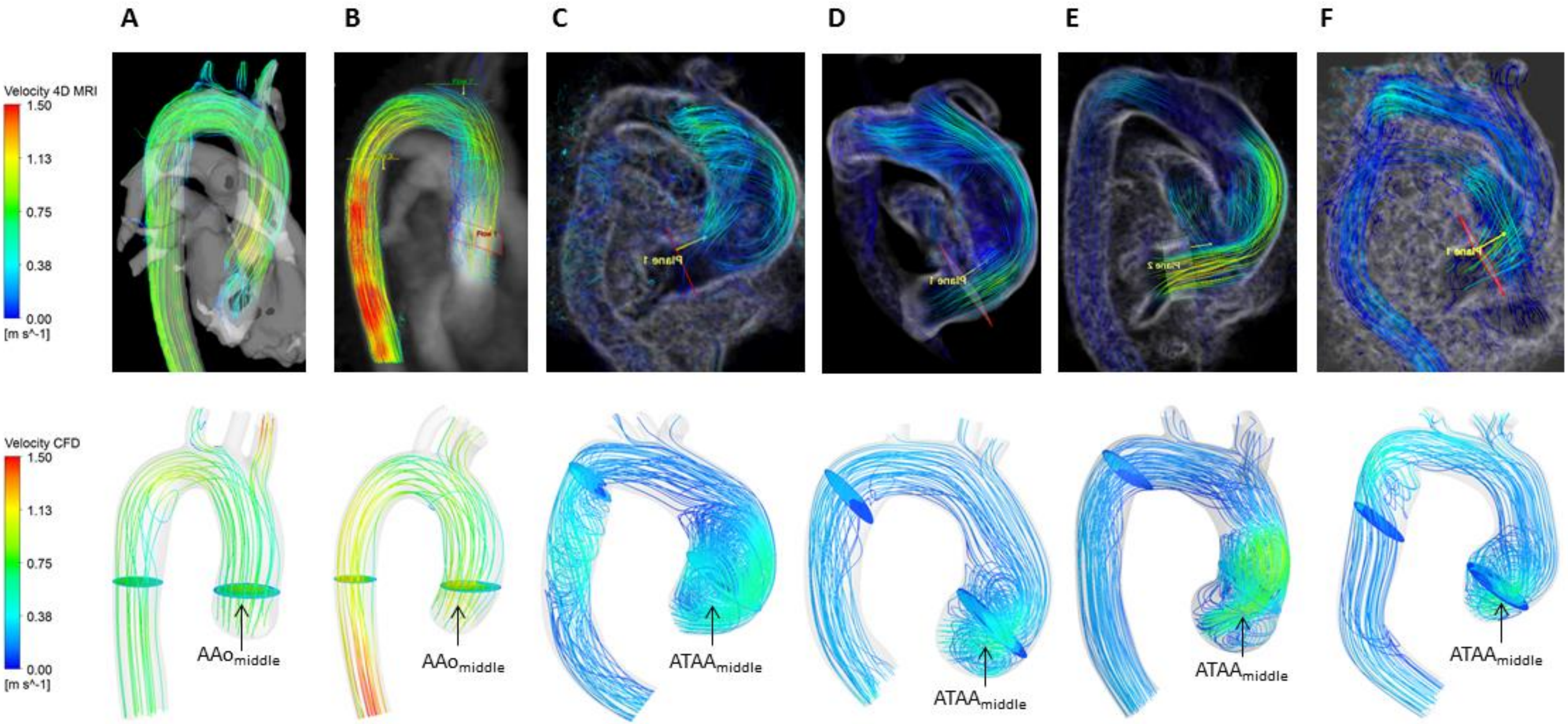
Correlation with flow descriptors



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