THE VIRTUAL FIELDS METHOD

PRINCIPLE, RECENT ADVANCES AND APPLICATIONS

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OUTLINE

- Introduction and basic concepts;
- Application 1: anisotropic elasticity in composites;
- Application 2: visco-elasticity in PMMA;
- Application 3: elasto-visco-plasticity in metals;
- Application 4: heterogeneous modulus distribution;
- Conclusion and main prospects.

Introduction and basic concepts

Widespread use of full-field optical techniques in experimental mechanics

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Largely used for qualitative analyses

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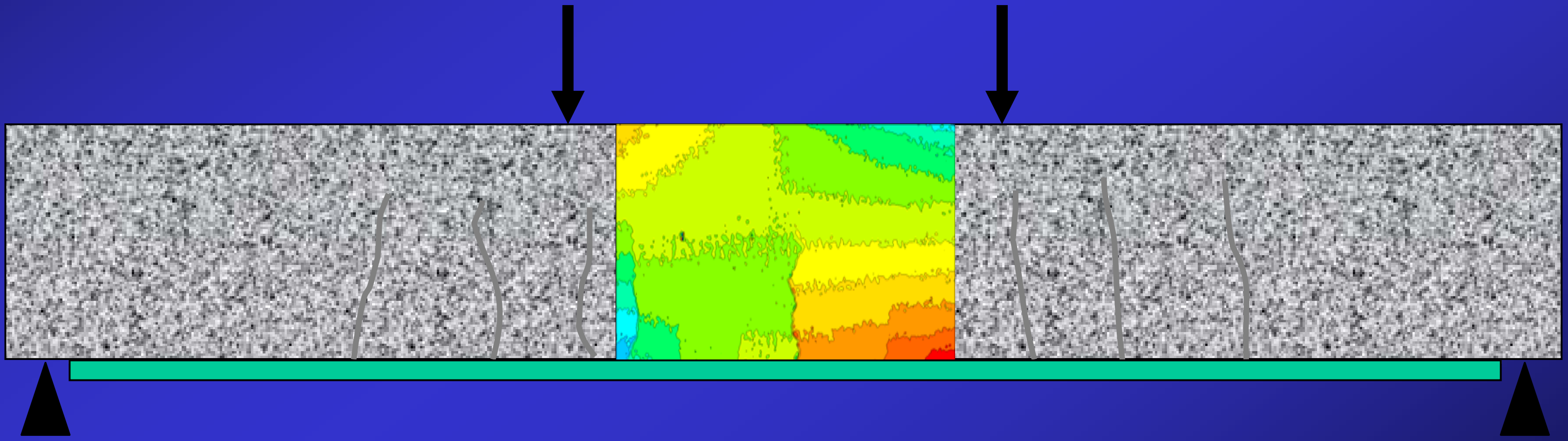
Largely used for qualitative analyses



Largely used for qualitative analyses

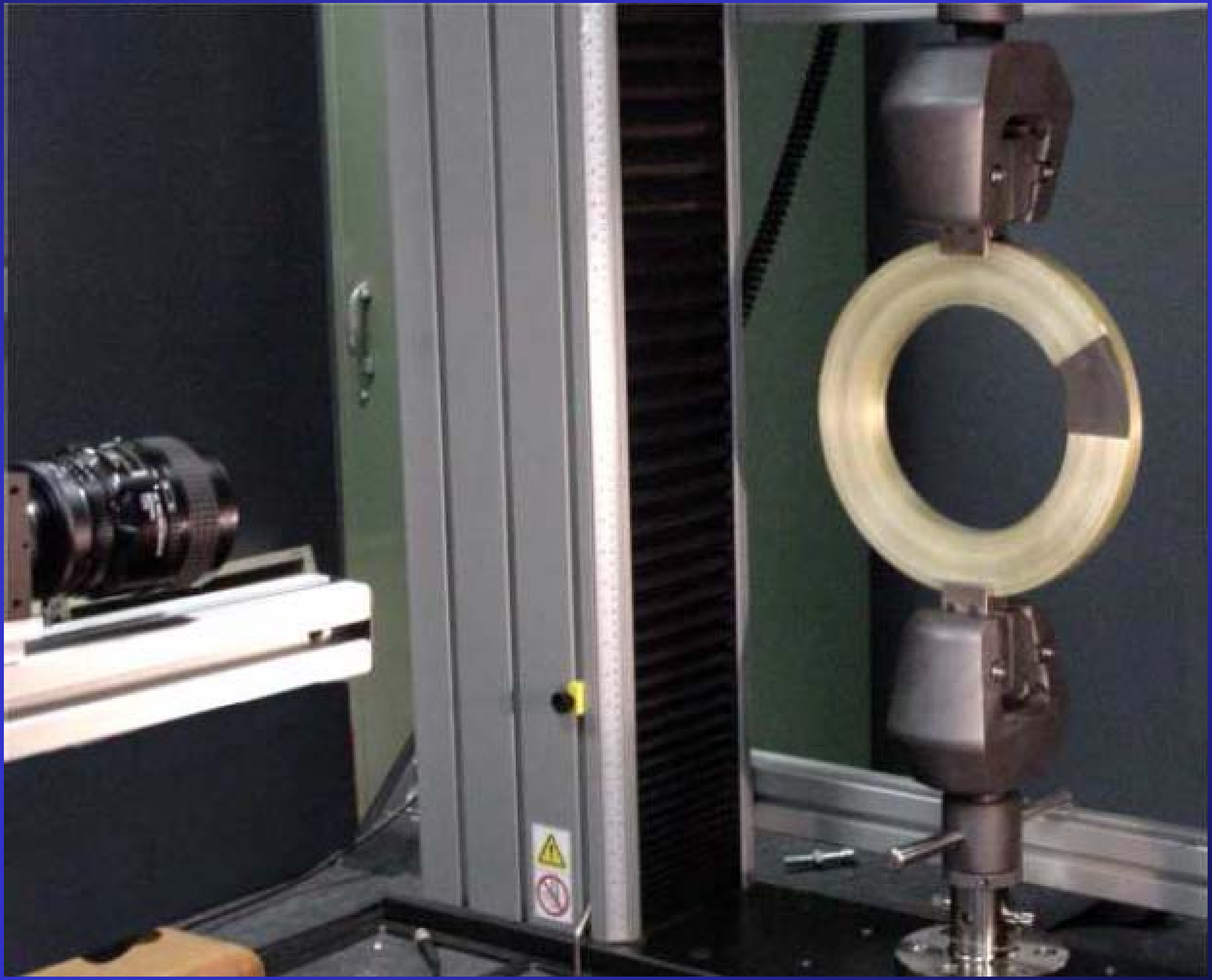
– Detection of shear bands or cracks ;

– Model validation ;

– Verification of boundary conditions.

Interest in quantitative use for model identification

Interest in quantitative use for model identification



Interest in quantitative use for model identification

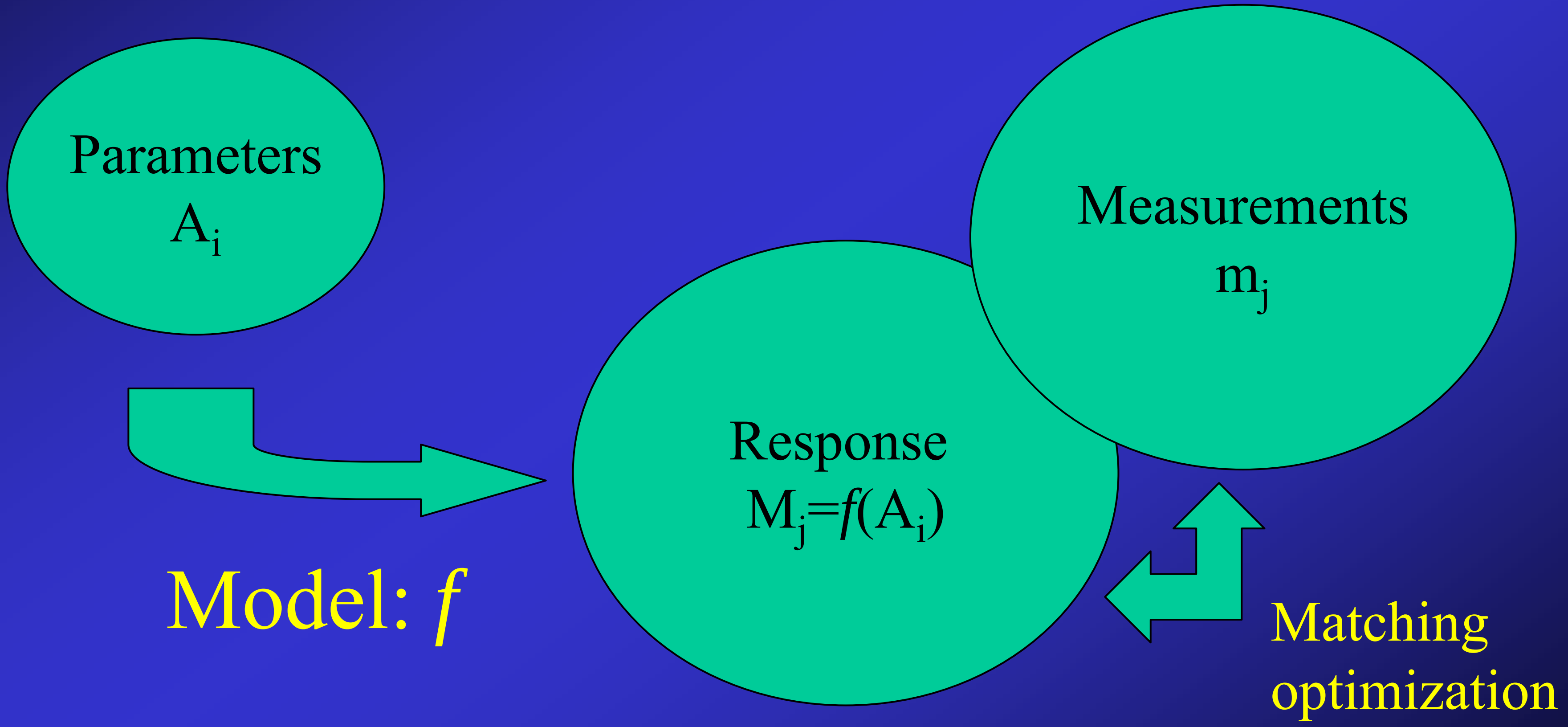
- Identification of sophisticated models in one single test ;
- Specimen geometry is not a constraint ;
- Localization effects can be handled ;
- Identification of graded properties in the same specimen.

Reason for poor use for model identification

- No standard characterization of uncertainty in full-field optical measurement techniques.

- Large amount of data to process require specific inverse approach.

Resolution of an inverse problem
Basic approach: updating



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Particular case of full-field measurements: alternative approach

Model *f* is derived from:
- the constitutive equations,
- the conservation equations.

$F(A_i,M)=0$

Measurements $m(x,y,z)$

A_i ← $F(A_i,m)=0$ ← Direct computation

Application 1: anisotropic elasticity in composites

The virtual fields method
Example in linear elasticity

I Equilibrium equations

$$\sigma_{ij,j} = 0 \quad + \text{boundary conditions} \qquad \text{strong (local)}$$

or

$$-\int_V \sigma_{ij}\,\varepsilon^*_{ij}\,dV + \int_{\partial V} T_i\,u^*_i\,dS = 0 \qquad \text{weak (global)}$$

II Constitutive equations

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}$$

III Kinematical equations (small strains/displacements)

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$$

The virtual fields method
Example in linear elasticity

Eq. I (weak form, static)

$$-\int_V \sigma_{ij}\varepsilon^*_{ij}dV + \int_{\partial V} T_i u^*_i dS = 0$$

Substitute stress from Eq. II

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$$

$$\Rightarrow \quad -\int_V C_{ijkl}\varepsilon_{kl}\varepsilon^*_{ij}dV + \int_{\partial V} T_i u^*_i dS = 0$$

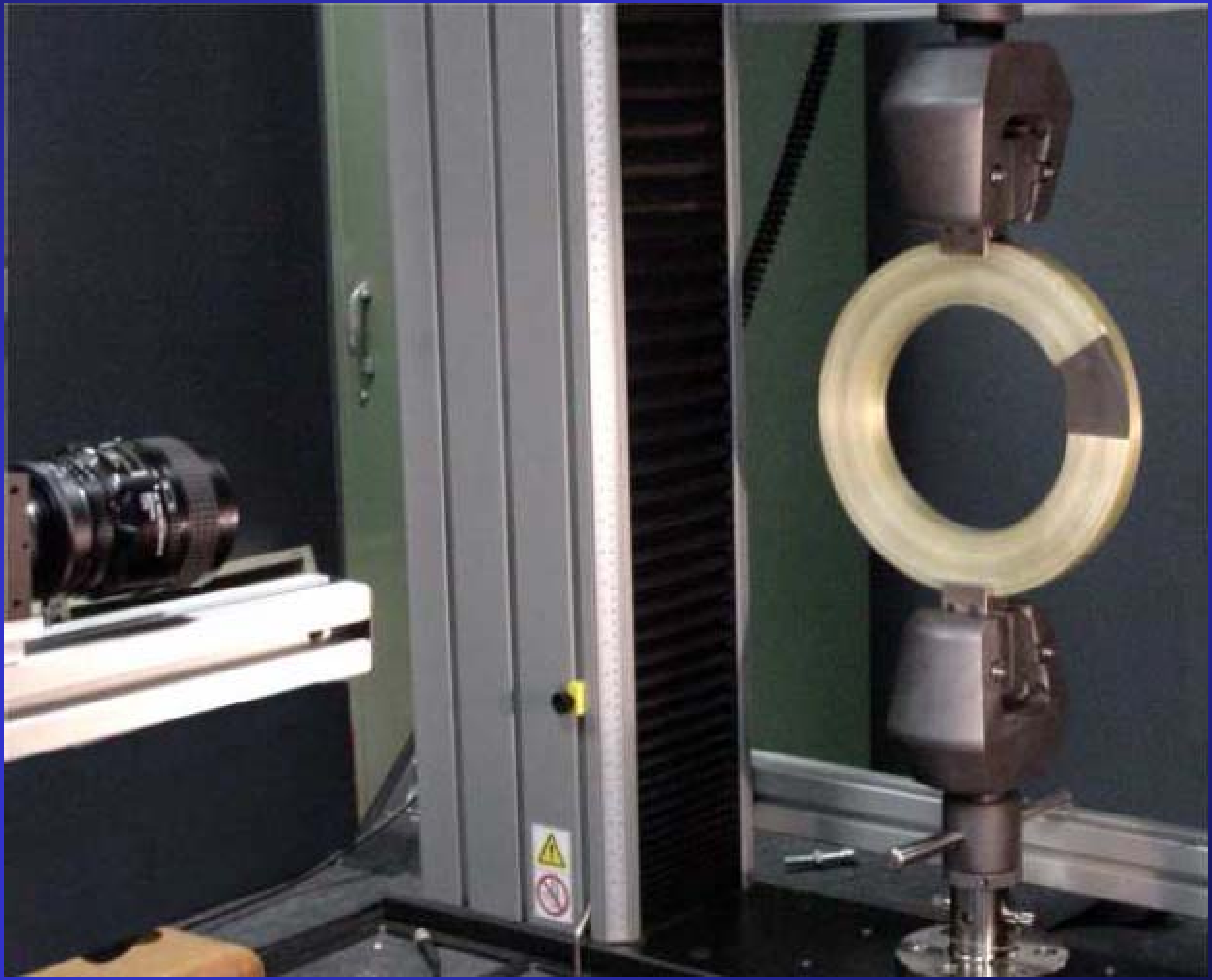
The virtual fields method
Example in linear elasticity

$$-C_{ijkl}\int_V \varepsilon_{kl}\varepsilon_{ij}^{*}dV+\int_{\partial V}T_i u_i^{*}dS=0$$

Equation valid for any KA virtual fields.
For each choice of virtual field: 1 equation.
Choice of as many VF as unknown parameters.

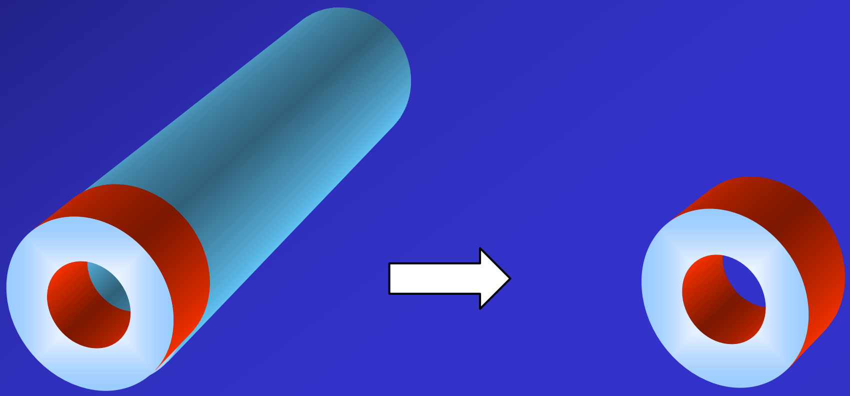
In elasticity:
→ Linear system of equations to be inverted.
→ Optimal choice of virtual fields for minimizing result uncertainty (Avril et al., Comp. Mech., 2004)
→ More robust than updating approach (Avril et al., Int. J. Sol. Struct., 2007)

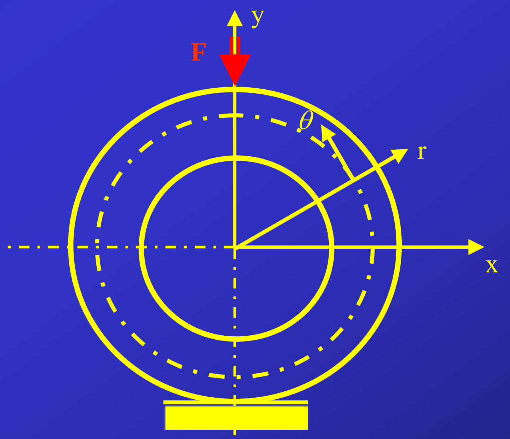
Problem presentation



Example

Glass epoxy ring cut from a tube:



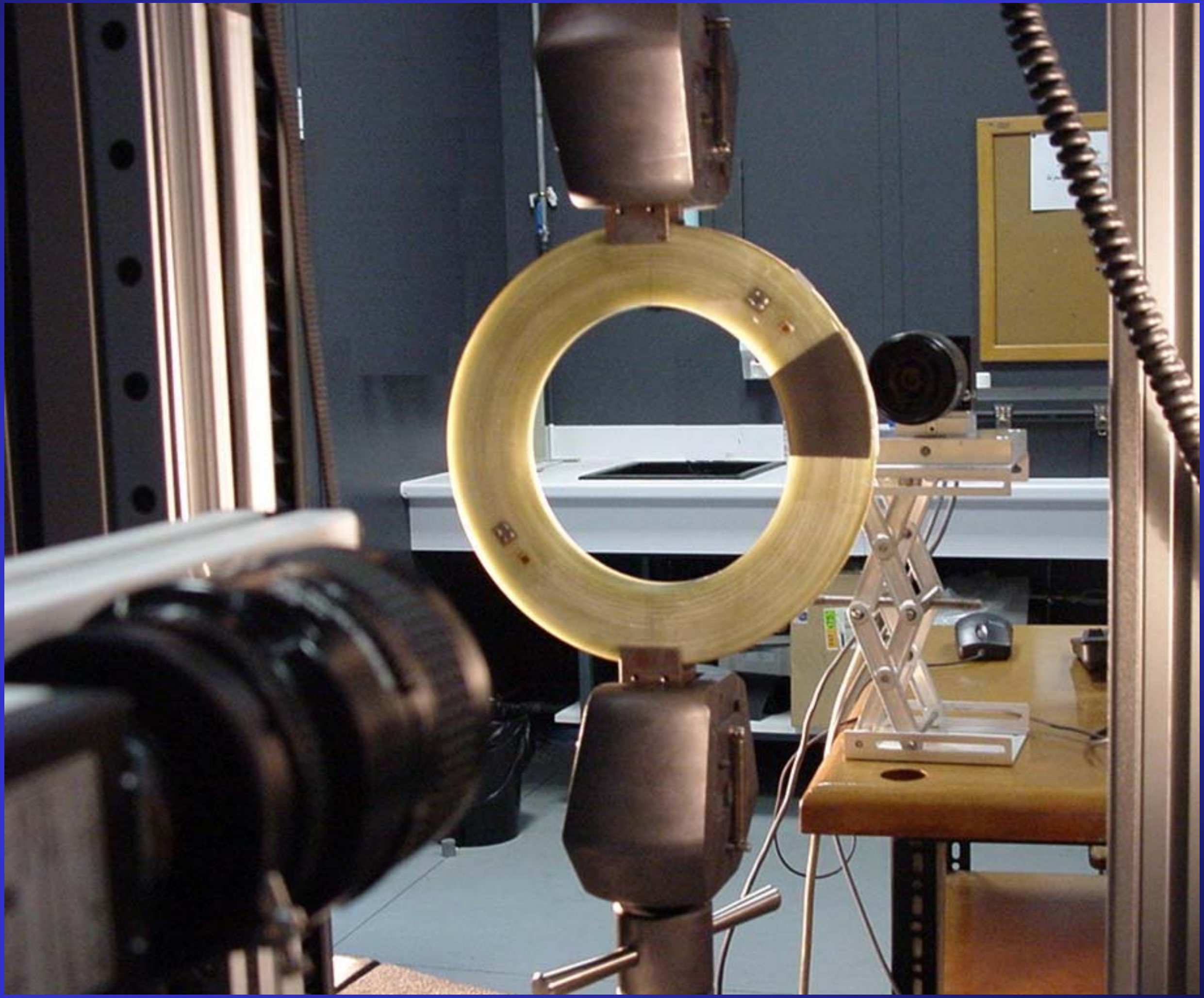


$$\begin{pmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_s \end{pmatrix} = \begin{bmatrix} Q_{rr} & Q_{r\theta} & 0 \\ Q_{r\theta} & Q_{\theta\theta} & 0 \\ 0 & 0 & Q_{ss} \end{bmatrix} \begin{pmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_s \end{pmatrix}$$

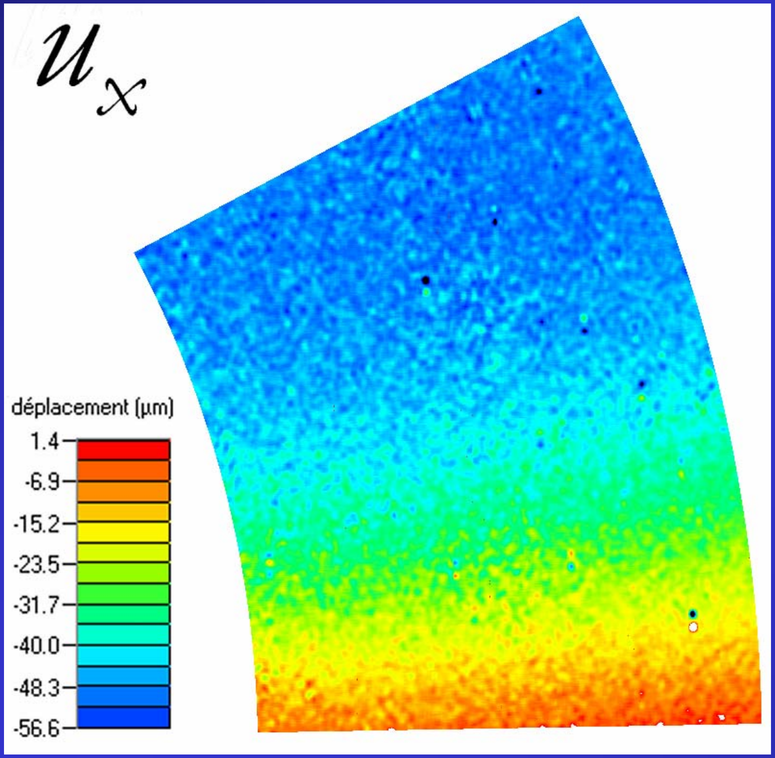
→ 4 parameters to identify

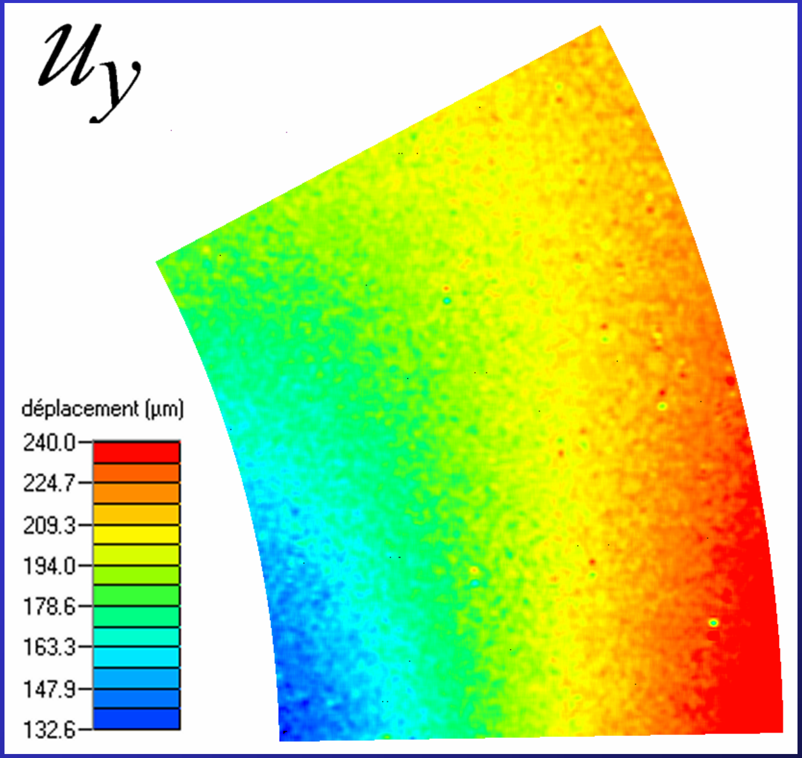
→ No standard test available

Need of two cameras "back to back"



Deformation maps using two cameras

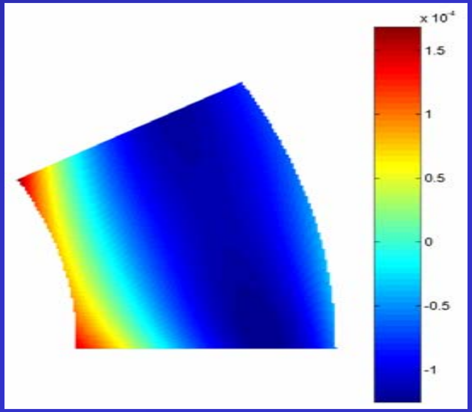




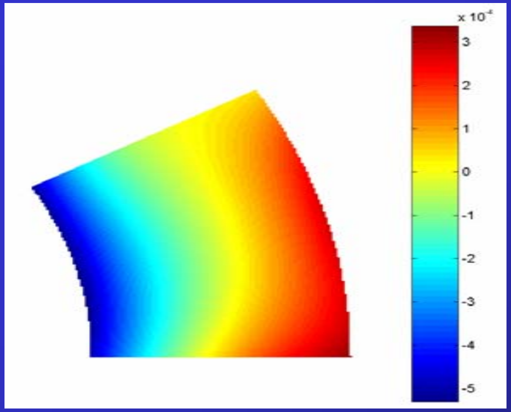
Strain maps

Polynomial fit, degree 3, transform to cylindrical and analytical differentiation:

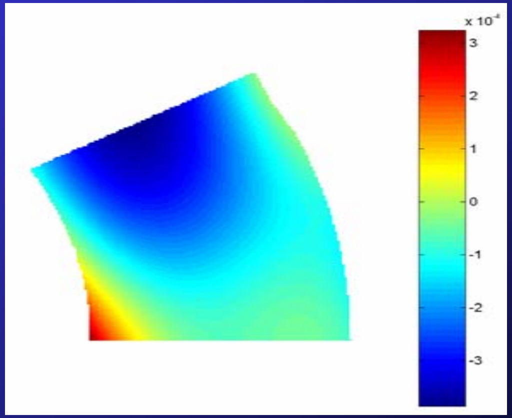
ε_r



ε_θ



ε_s



Identification

$$e\int_S \underline{\underline{\sigma}} : \underline{\underline{\varepsilon}}^* dS = \int_{S_f} \vec{T}(M)\vec{u}^*(M)\,dS$$

&

$$\begin{pmatrix}\sigma_r\\ \sigma_\theta\\ \sigma_s\end{pmatrix} = \begin{bmatrix} Q_{rr} & Q_{r\theta} & 0\\ Q_{r\theta} & Q_{\theta\theta} & 0\\ 0 & 0 & Q_{ss}\end{bmatrix}\begin{pmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \varepsilon_s\end{pmatrix}$$

PVW — **Constitutive equations**

$$Q_{rr}.e\int_S \varepsilon_r\varepsilon_r^* dS + Q_{\theta\theta}.e\int_S \varepsilon_\theta\varepsilon_\theta^* dS + Q_{r\theta}.e\int_S \left(\varepsilon_r\varepsilon_\theta^* + \varepsilon_\theta\varepsilon_r^*\right) dS$$
$$+ Q_{ss}.e\int_S \varepsilon_s\varepsilon_s^* dS = \int_{S_f} \vec{T}(M)\vec{u}^*(M)\,dS$$

4 virtual fields

$$[A]\times(Q) = (B)$$

Results

	Q_{rr}	$Q_{\theta\theta}$	$Q_{r\theta}$	Q_{ss}
Reference* (GPa)	10	40	3	4
Identified (GPa)	11.4	45.4	2.62	6.78
Coeff. var (%) – 9 tests	29	10	29	4

Moulart R., Avril S., Pierron F., Identification of the through-thickness rigidities of a thick laminated composite tube, ***Composites Part A: Applied Science and Manufacturing***, vol. 37, n° 2, pp. 326-336, 2006.

Application 2: visco-elasticity in polycarbonate

Constitutive equations and assumptions

Inertial excitation with steady state and linear response.

$$\sigma(x,y) = \bar{\sigma}(x,y)\cos(\omega t)$$
$$\varepsilon(x,y) = \bar{\varepsilon}(x,y)\cos[\omega t + \varphi(x,y)]$$

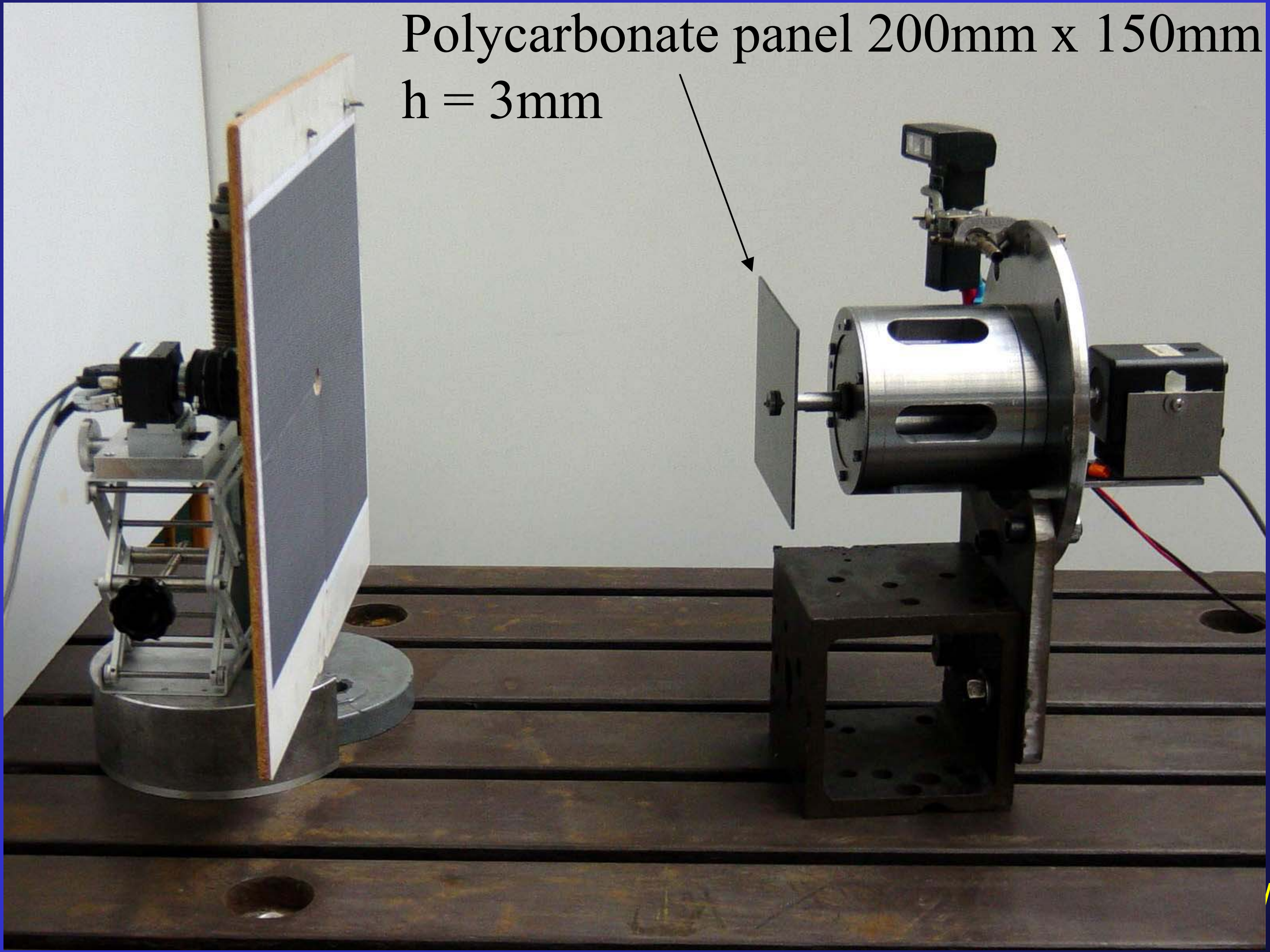
⇒

$$\sigma(x,y) = \bar{\sigma}(x,y)e^{i\omega t}$$
$$\varepsilon(x,y) = \bar{\varepsilon}(x,y)e^{i[\omega t + \varphi(x,y)]}$$

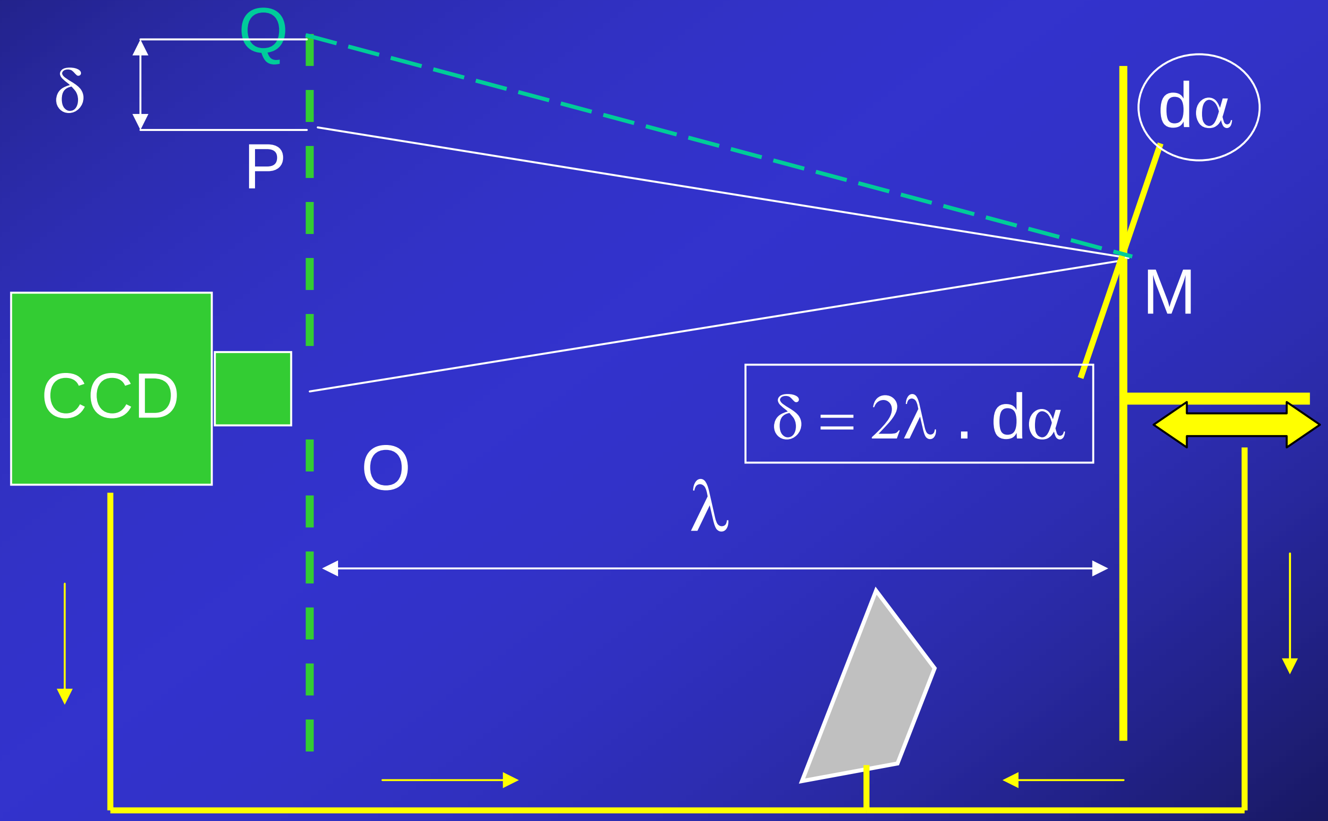
$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \underbrace{\begin{bmatrix} Q^r_{xx} & Q^r_{xy} & 0 \\ Q^r_{xy} & Q^r_{xx} & 0 \\ 0 & 0 & Q^r_{xx} - Q^r_{xy} \end{bmatrix}}_{\text{STIFFNESS}} \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{pmatrix} + i\omega \underbrace{\begin{bmatrix} Q^i_{xx} & Q^i_{xy} & 0 \\ Q^i_{xy} & Q^i_{xx} & 0 \\ 0 & 0 & Q^i_{xx} - Q^i_{xy} \end{bmatrix}}_{\text{DAMPING}} \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{pmatrix}$$

STIFFNESS **DAMPING**

Experimental arrangements

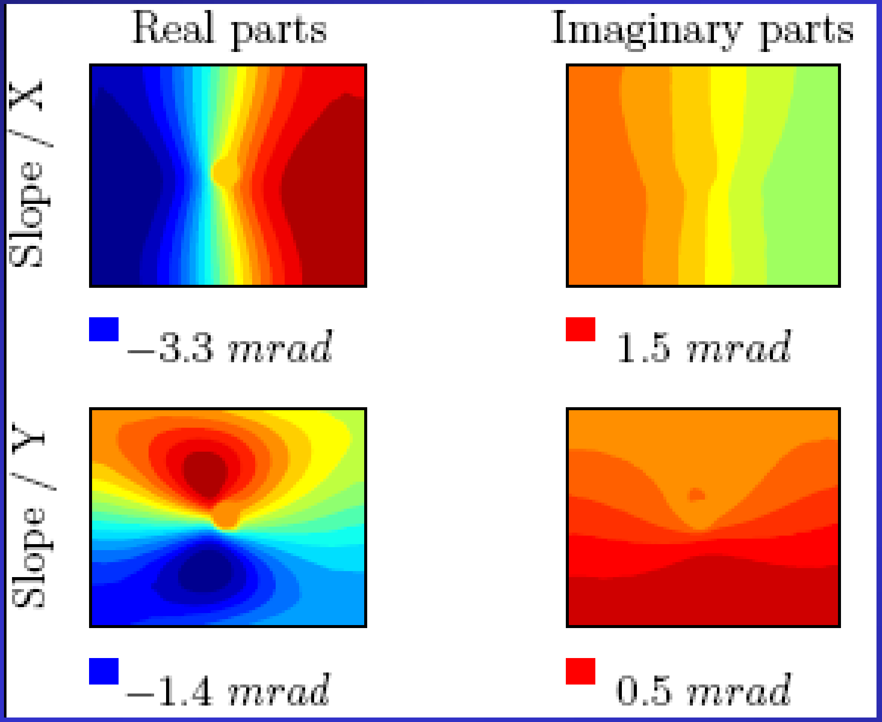


The deflectometry technique

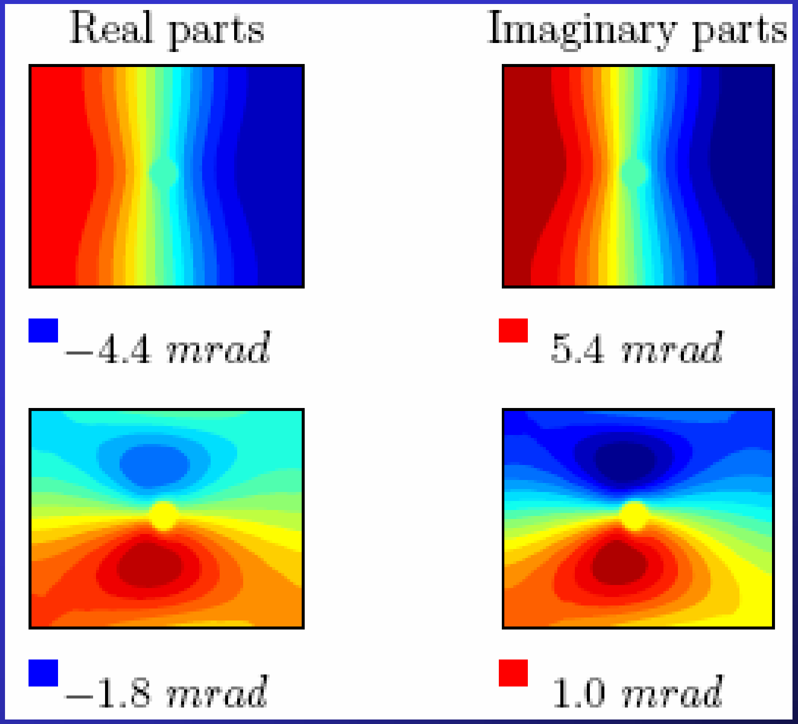


Slope measurements

Out of resonance 80 Hz

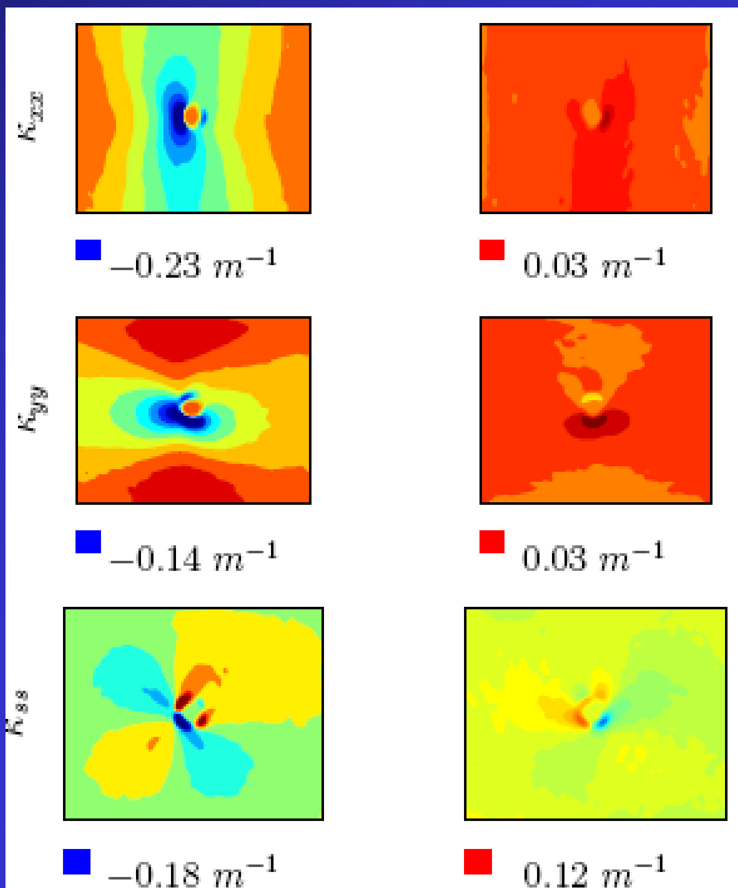


Near resonance 100 Hz

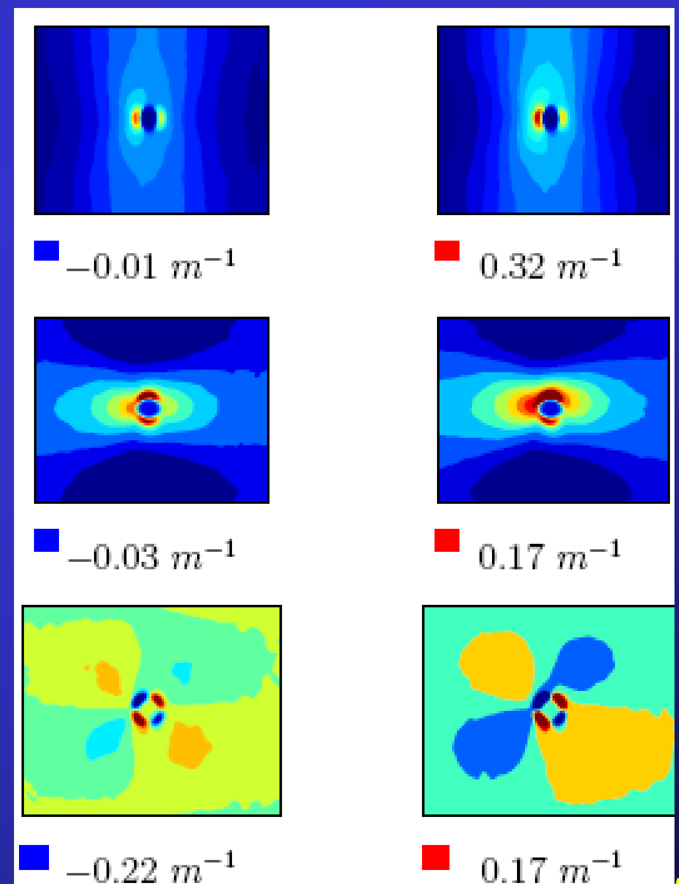


Deduced curvatures

Out of resonance 80 Hz



Near resonance 100 Hz



Identification

$$\begin{Bmatrix} M_x \\ M_y \\ M_s \end{Bmatrix} = \left(\begin{bmatrix} D_{xx} & D_{xy} & 0 \\ D_{xy} & D_{yy} & 0 \\ 0 & 0 & \dfrac{D_{xx}-D_{xy}}{2} \end{bmatrix} + i\omega \begin{bmatrix} B_{xx} & B_{xy} & 0 \\ B_{xy} & B_{yy} & 0 \\ 0 & 0 & \dfrac{B_{xx}-B_{xy}}{2} \end{bmatrix} \right) \begin{Bmatrix} k_x \\ k_y \\ k_s \end{Bmatrix}$$

Four parameters to identify

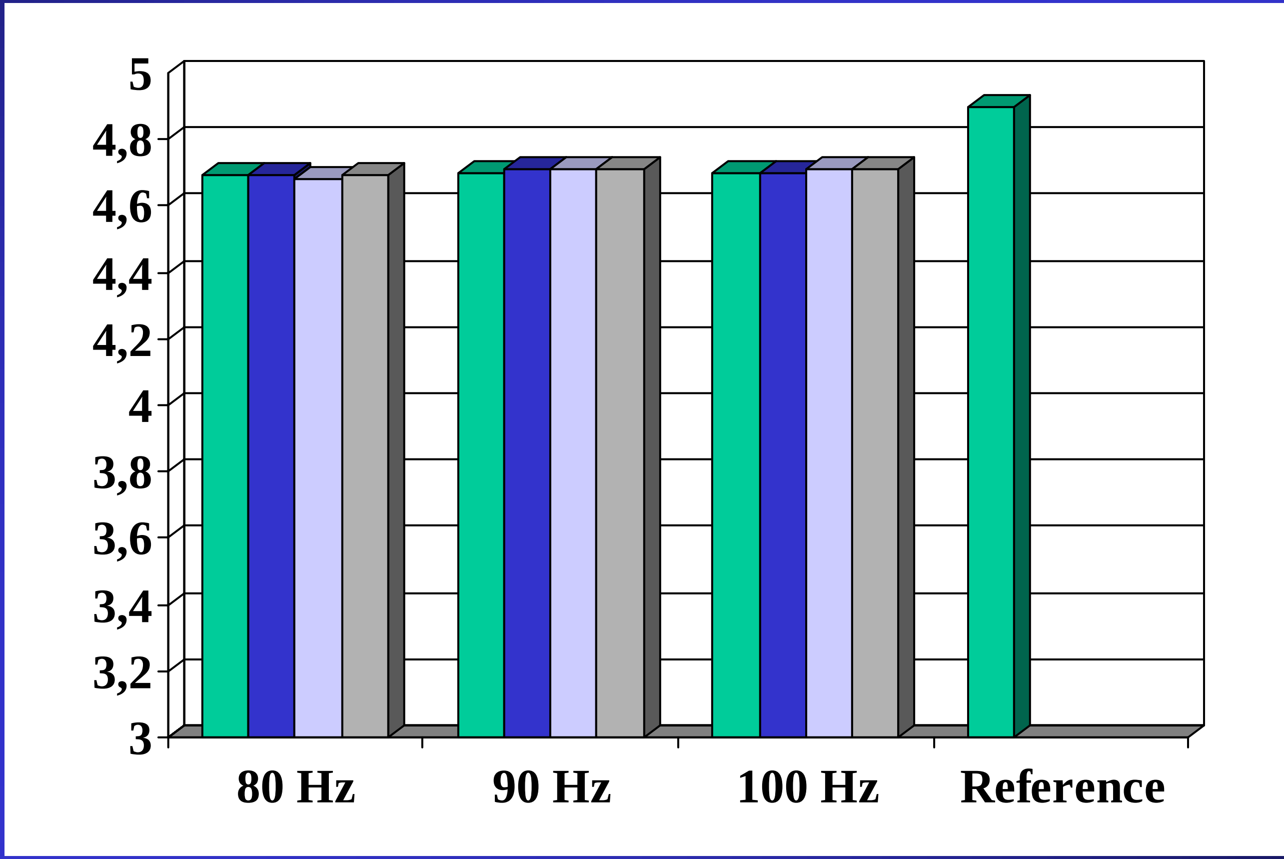
$$\mathrm{E} = \frac{\mathrm{D}_{xx}}{(1-\nu^2)} ; \nu = \frac{\mathrm{D}_{xy}}{\mathrm{D}_{xx}} \qquad \text{Stiffness}$$

$$\beta_{xx} = \frac{\mathrm{B}_{xx}}{\mathrm{D}_{xx}} ; \beta_{xy} = \frac{\mathrm{B}_{xy}}{\mathrm{D}_{xy}} \qquad \text{Damping}$$

$$VF_1 = x^2(1+j), \quad VF_2 = y^2(1+j)$$

Results

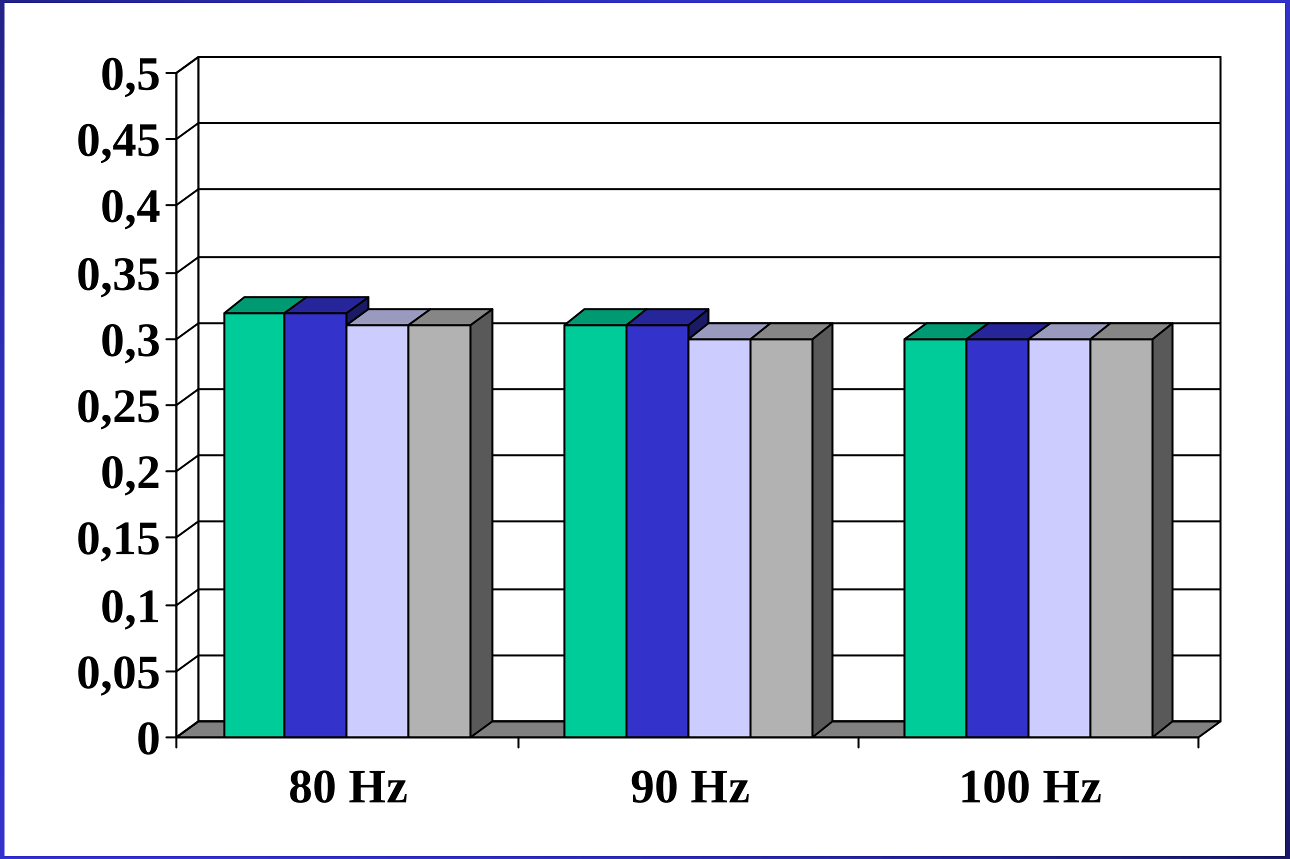
E (GPa)



Reference: clamped beam

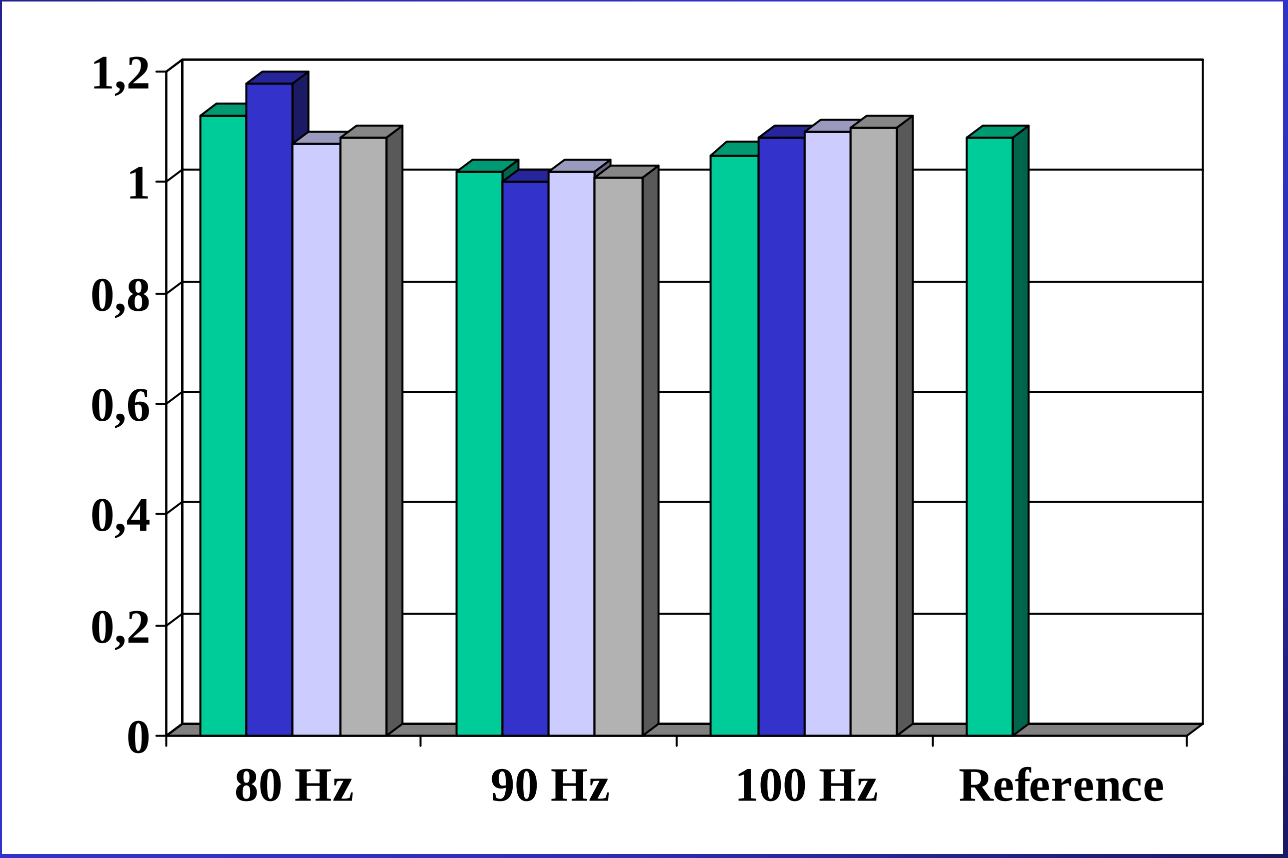
Results

ν



Results

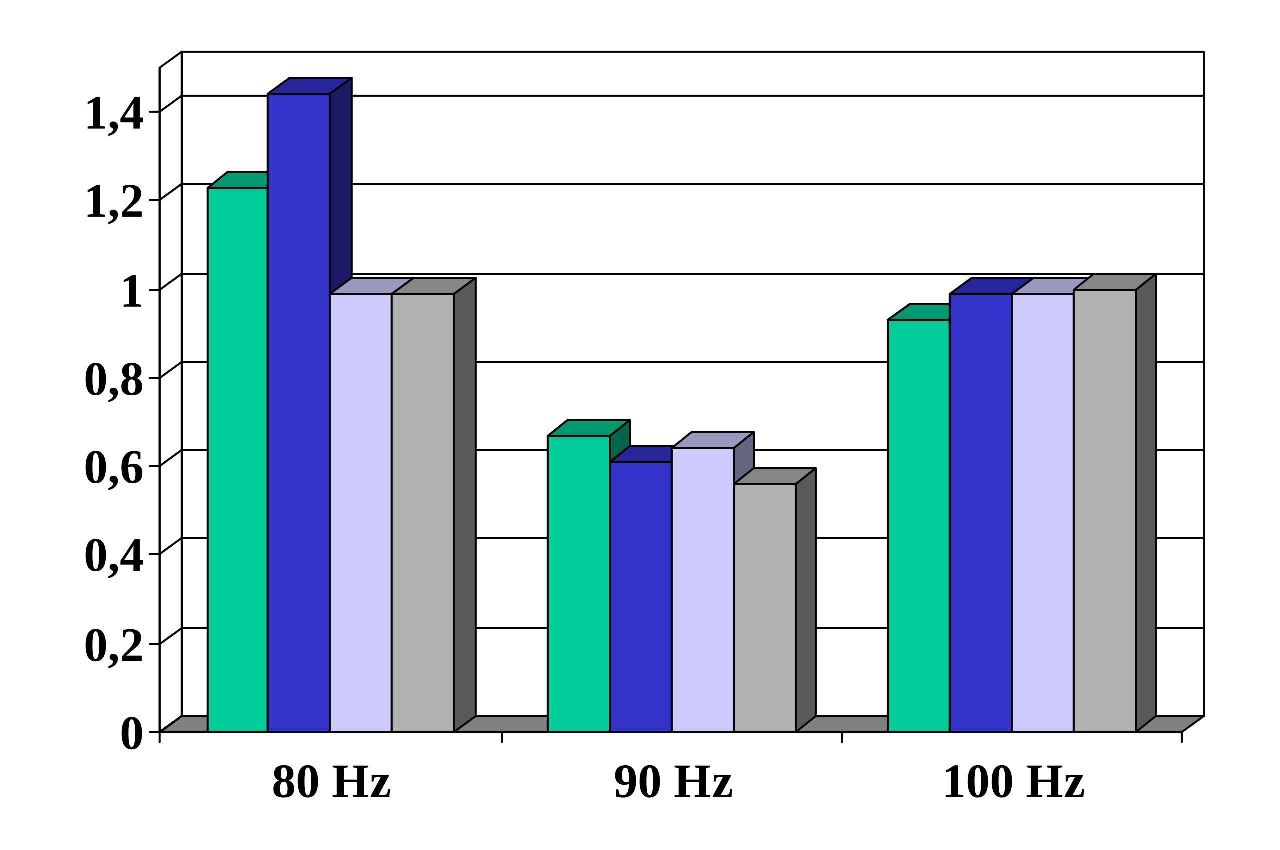
β_{xx} (10^{-4} s)



Reference: clamped beam

Results

β_{xy} (10^{-4} s)



Prospects

- Improvement of the measurement temporal accuracy.

- Extension to anisotropic plates underway

Giraudeau A., Guo B., Pierron F., Stiffness and Damping Identification from Full Field Measurements on Vibrating Plates, ***Experimental Mechanics***, Vol. 46, N°6, pp. 777-787, 2006.

Application 3: elasto-visco-plasticity of metals

Constitutive equations and assumptions

$$\dot{\sigma} = g(\sigma, \dot{\varepsilon}, X)$$

Constitutive parameters to identify

Elasto-plasticity with plane stress and Von Mises criterion

$$\dot{\sigma} = Q(\varepsilon - \varepsilon^{p})$$

$$\begin{Bmatrix} \dot{\sigma}_x \\ \dot{\sigma}_y \\ \dot{\sigma}_s \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & \nu & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \dot{\varepsilon}_x - \dfrac{3}{2}\dot{p}\dfrac{s_{xx}}{\sigma_s} \\ \dot{\varepsilon}_y - \dfrac{3}{2}\dot{p}\dfrac{s_{xy}}{\sigma_s} \\ \dot{\varepsilon}_s - 3\dot{p}\dfrac{s_{xy}}{\sigma_s} \end{Bmatrix}$$

Perzyna's model for viscoplasticity

$$\begin{Bmatrix} \dot{\sigma}_x \\ \dot{\sigma}_y \\ \dot{\sigma}_s \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & \nu & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \dot{\varepsilon}_x - \dfrac{3}{2}\gamma \left\langle \dfrac{\sigma_s - H(p)}{\sigma_0} - 1 \right\rangle^n \dfrac{s_{xx}}{\sigma_s} \\ \dot{\varepsilon}_y - \dfrac{3}{2}\gamma \left\langle \dfrac{\sigma_s - H(p)}{\sigma_0} - 1 \right\rangle^n \dfrac{s_{xy}}{\sigma_s} \\ \dot{\varepsilon}_s - 3\gamma \left\langle \dfrac{\sigma_s - H(p)}{\sigma_0} - 1 \right\rangle^n \dfrac{s_{xy}}{\sigma_s} \end{Bmatrix}$$

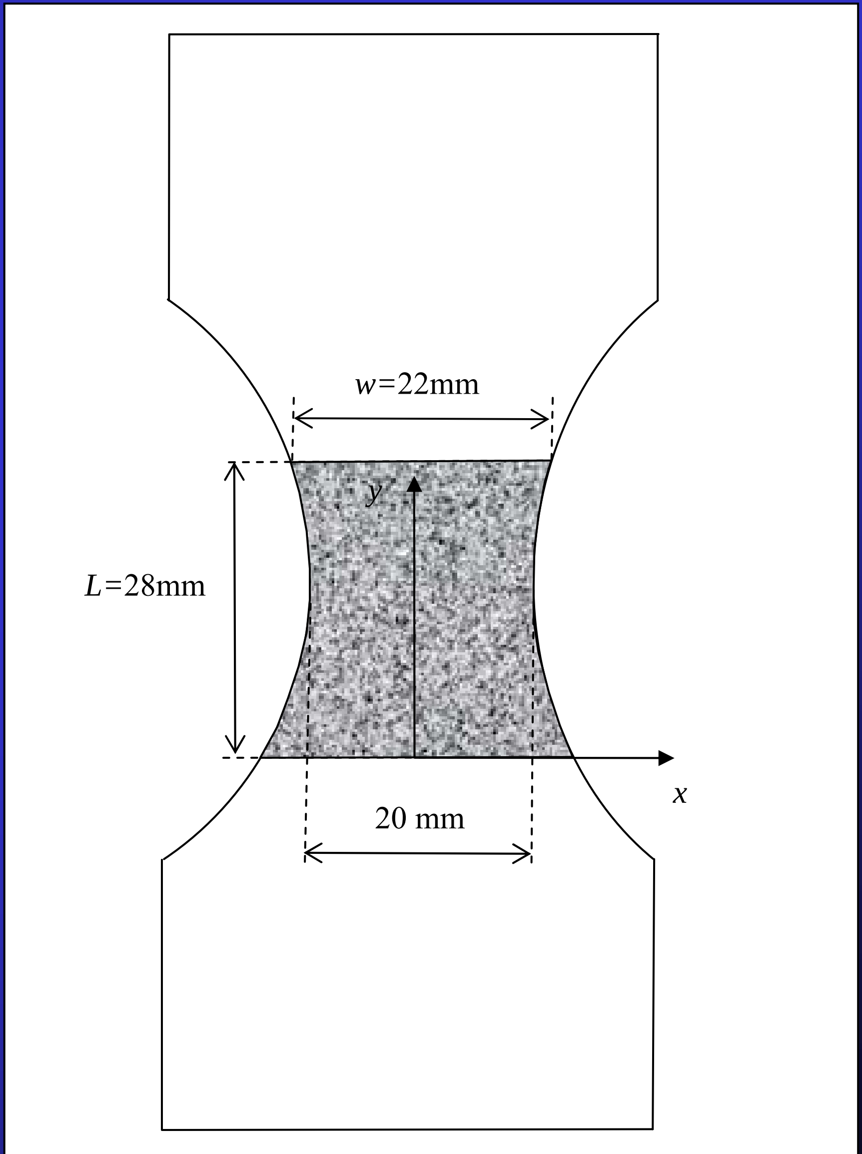
Constitutive parameters to identify: E, ν, γ, σ_0, n, dH/dp.

Mechanical arrangement

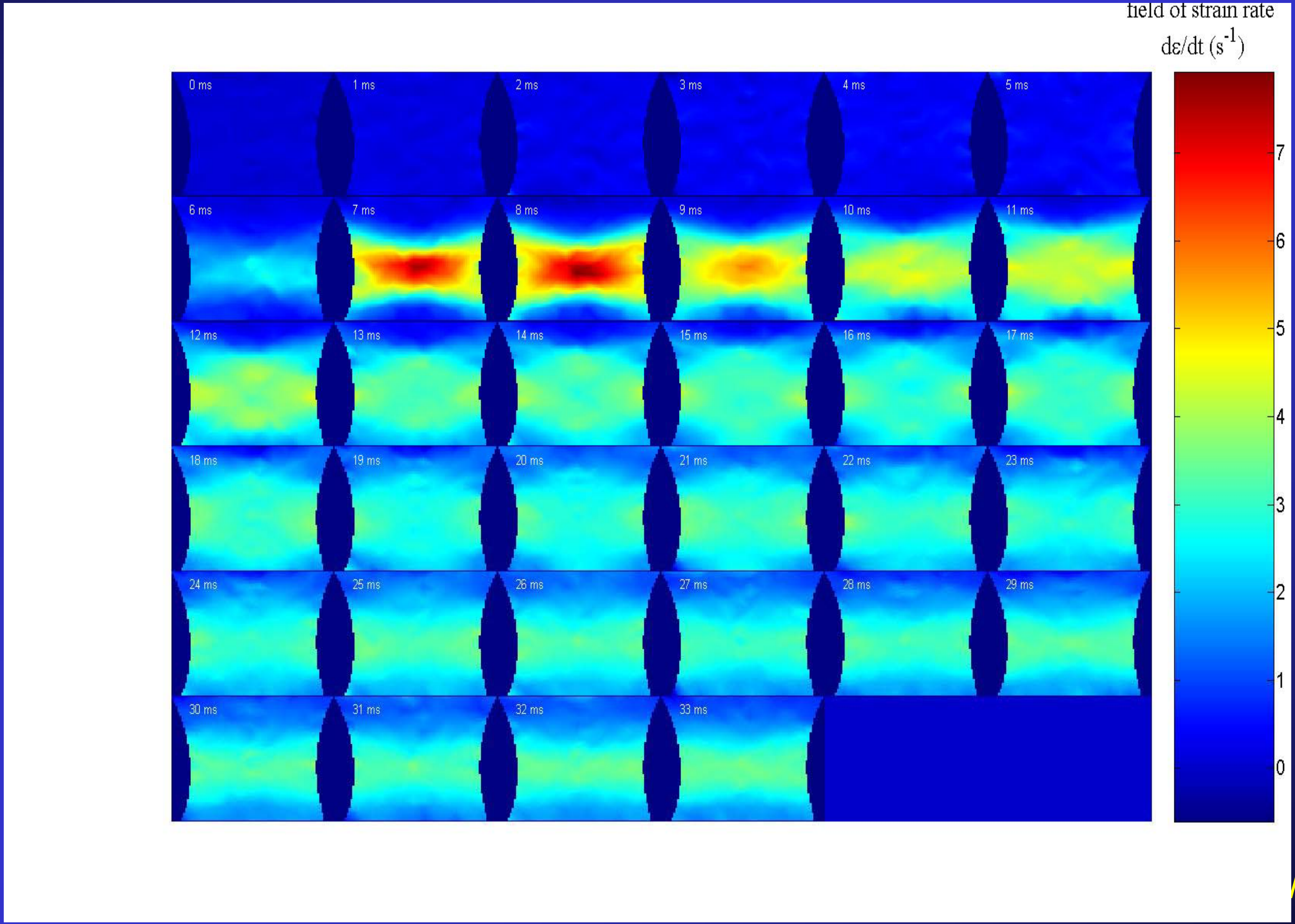
Average strain rate: $1s^{-1}$

Images of the specimen recorded with high speed cameras

Deformation deduced by digital image correlation.



Fields of strain rate



Virtual field

Across the measurement area:

$$u_x^* = 0; u_y^* = -y$$

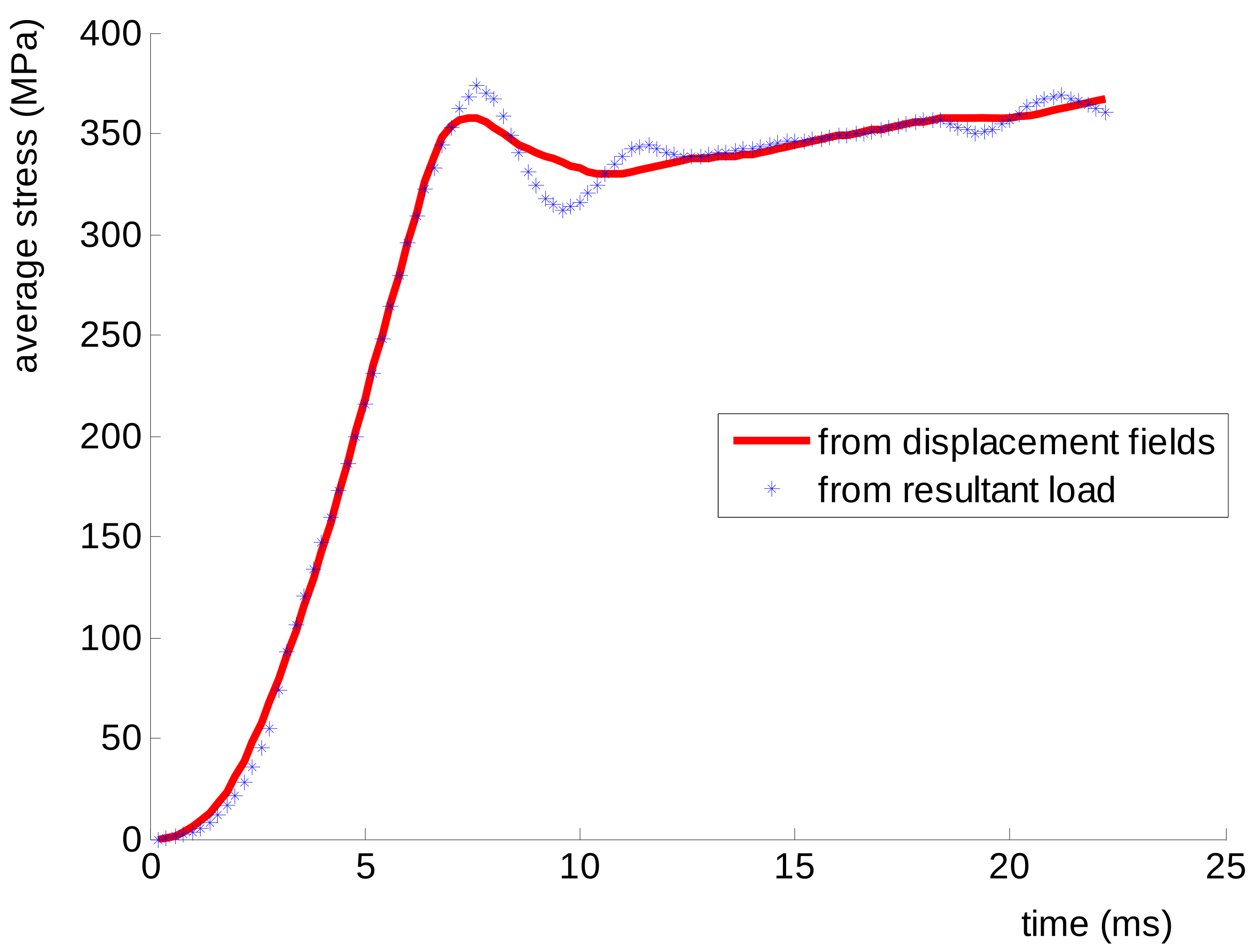
$$\varepsilon_{xx}^* = 0; \varepsilon_{yy}^* = 1; \varepsilon_{xy}^* = 0$$

Identification

→ minimization of a cost function defined from the principle of virtual work:

$$F(\sigma_0, n, \gamma, E_t) = \sum_{l=1}^{N} \left(\frac{1}{S} \int_S \int_0^{t_l} \dot{\sigma}_y(\sigma_0, n, \gamma, E_t, x, y, t)\,dt\,dS - \frac{P(t_l)L}{Sh} \right)^2$$

Identification



Prospects

- Numerical issues

- Larger strain rates → Hopkinson

- Heterogeneities: microscopic scale, welds...

Pannier Y., Avril S., Rotinat R., Pierron F., Identification of elasto-plastic constitutive parameters from statically undetermined tests using the virtual fields method, ***Experimental Mechanics***, Vol. 46, N°6, pp. 735-755, 2006.

Avril S., Pierron F., Yan J., Sutton M., Identification of viscoplastic parameters using DIC and the virtual fields method. In Proceedings of the SEM annual conference, Springfield (USA), 2007.

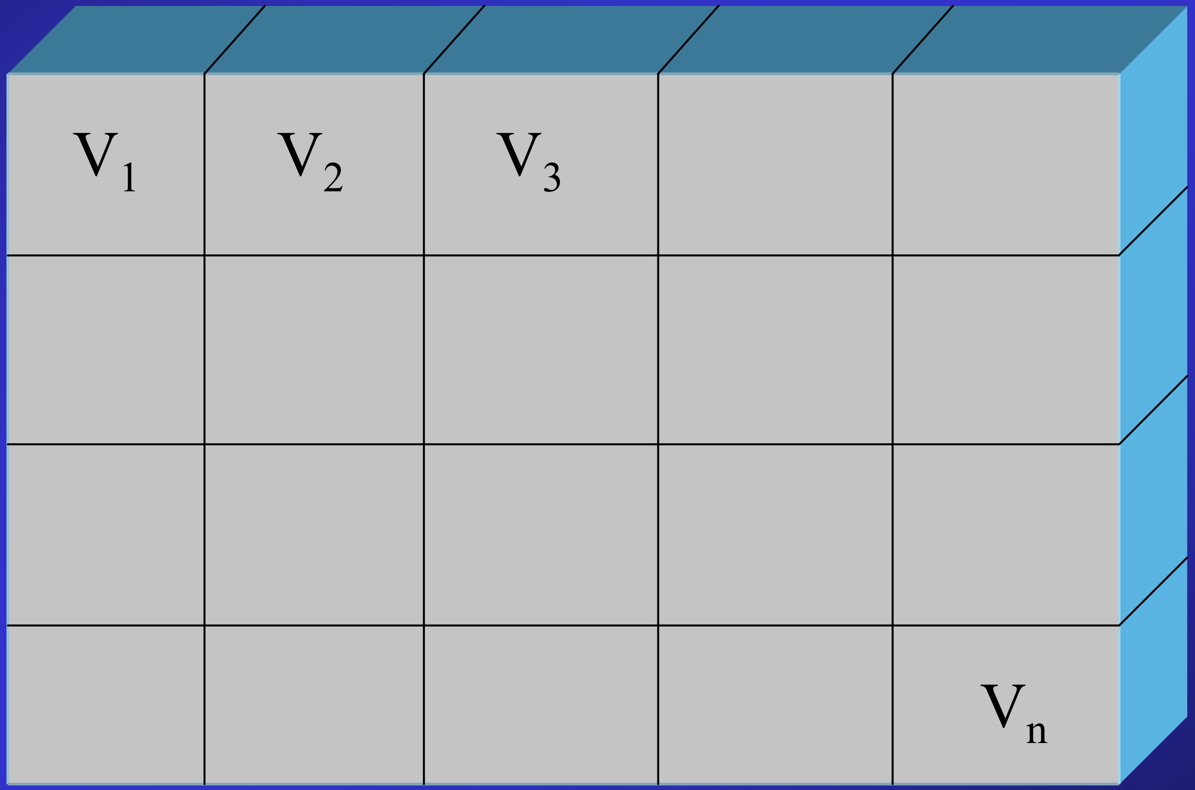
Heterogeneous modulus distribution

The virtual fields method for heterogeneous solids in linear elasticity

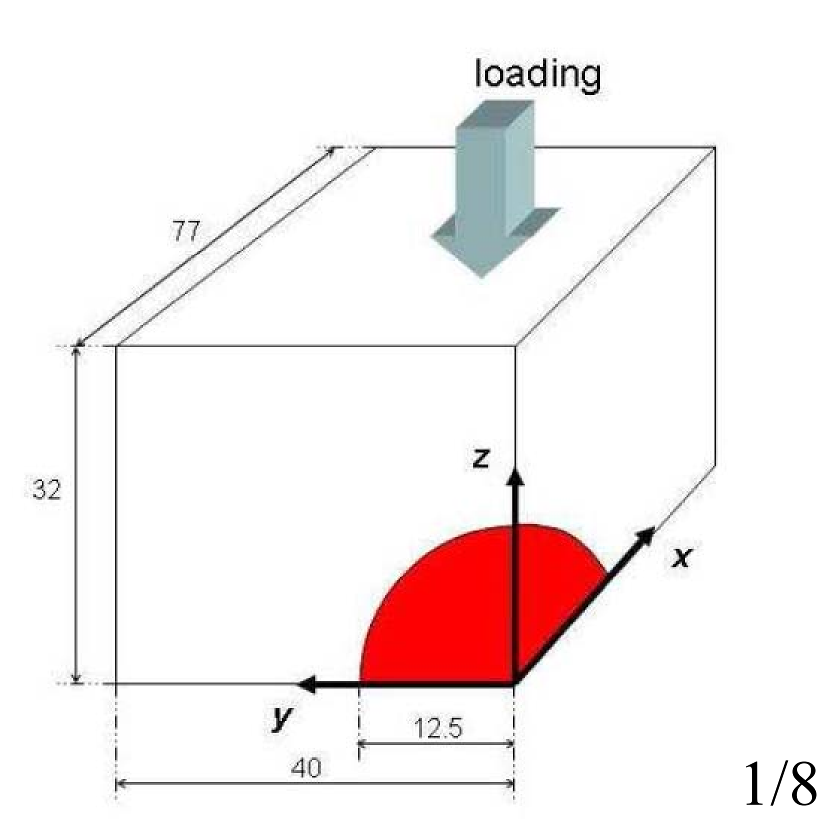
$$-\int_V C_{ijkl}\,\varepsilon_{kl}\,\varepsilon^*_{ij}\,dV + \int_{\partial V} T_i u^*_i\,dS = 0$$

$$\Rightarrow \quad -\sum_n C^n_{ijkl} \int_{V_n} \varepsilon_{kl}\,\varepsilon^*_{ij}\,dV + \int_{\partial V} T_i u^*_i\,dS = 0$$

Discretization of the solid



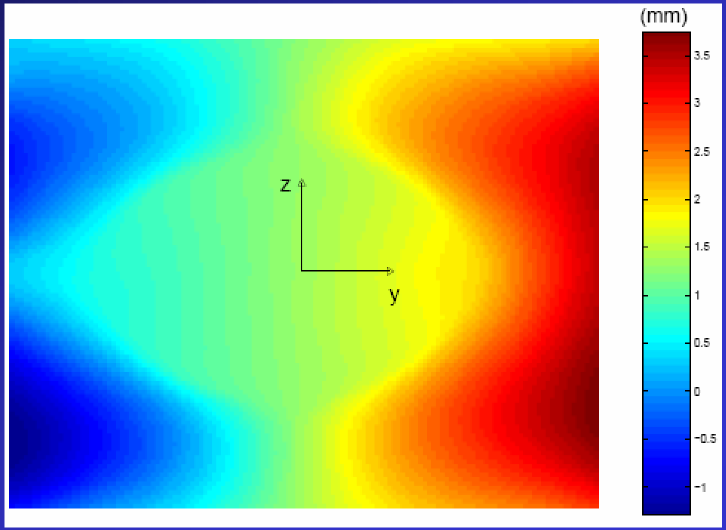
Experimental arrangements



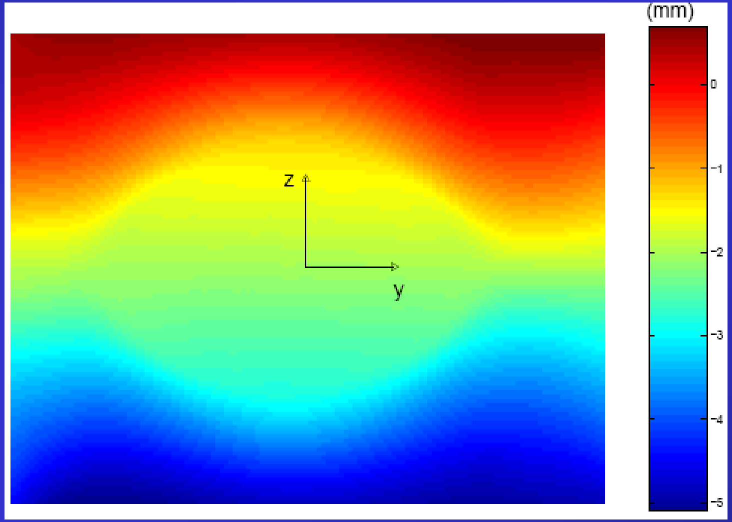
Cube with a stiff inclusion buried in it.

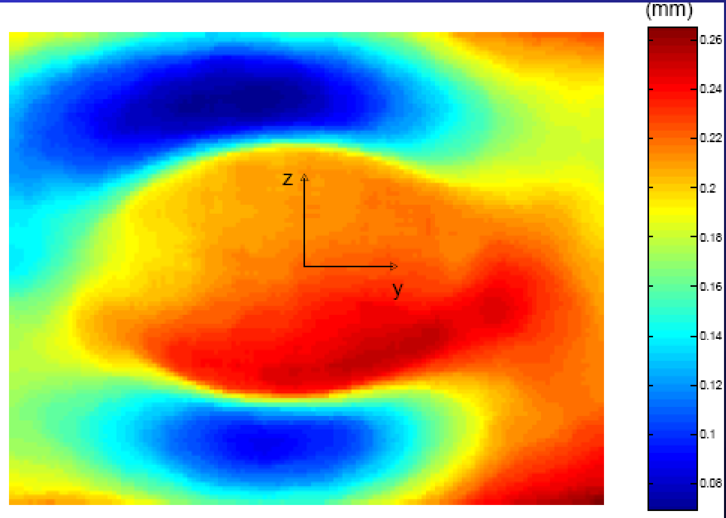
Silicone gel materials mimicking human tissue containing a tumour.

Displacement fields measured by MRI

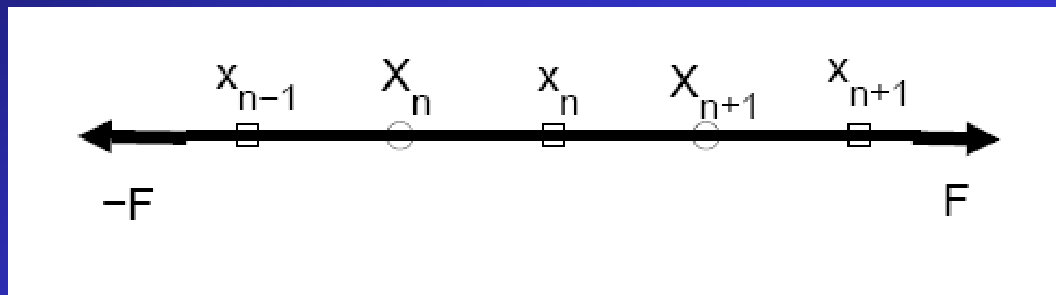


Scanning tomographic method:
→ 3D bulk measurements!!





Principle of the identification



$$F(x_n) = 0 \Rightarrow \frac{\sigma(X_{n+1}) - \sigma(X_n)}{X_{n+1} - X_n} = 0$$

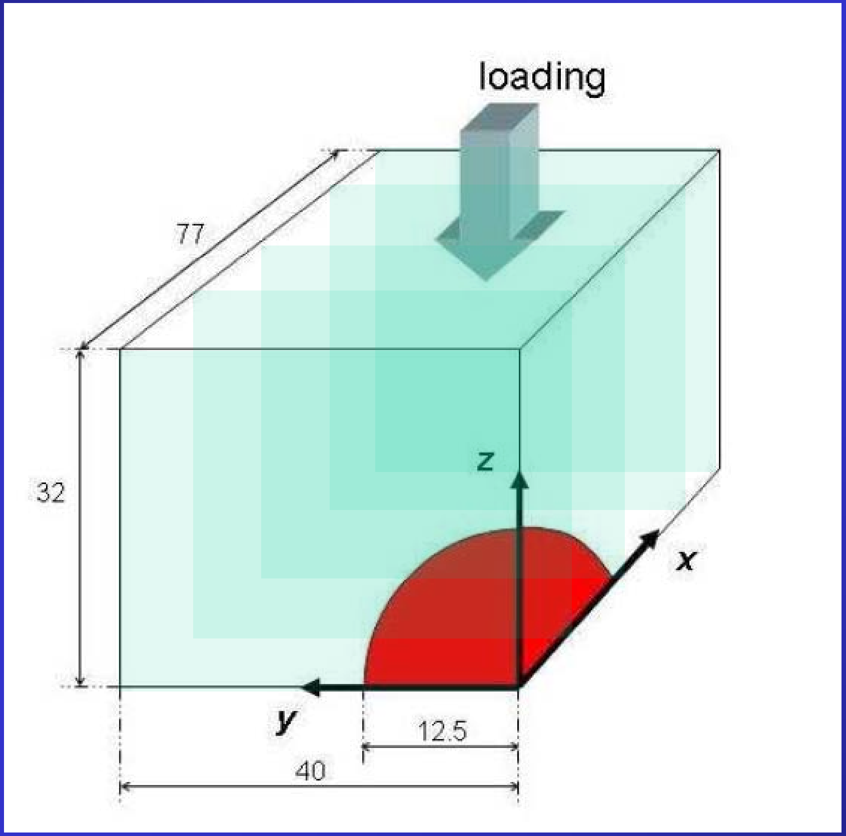
$$\Rightarrow \sigma(X_{n+1}) - \sigma(X_n) = 0$$

$$\Rightarrow E(X_{n+1})\varepsilon(X_{n+1}) - E(X_n)\varepsilon(X_n) = 0$$

$$\Rightarrow E(X_{n+1})\frac{u(x_{n+1}) - u(x_n)}{x_{n+1} - x_n} - E(X_n)\frac{u(x_n) - u(x_{n-1})}{x_n - x_{n-1}} = 0$$

$$[u(x_{n+1}) - u(x_n)]E(X_{n+1}) - [u(x_n) - u(x_{n-1})]E(X_n) = 0$$

3D Results



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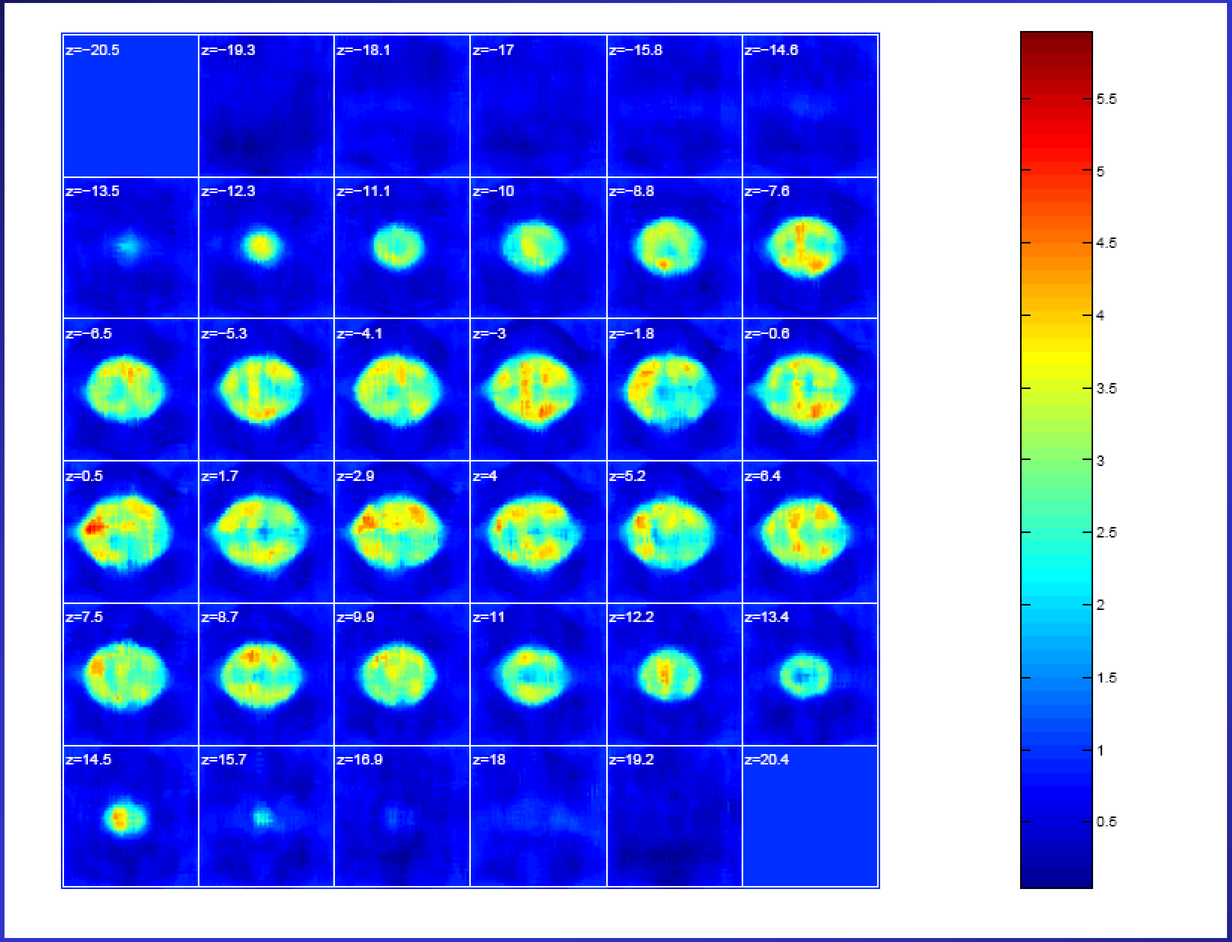


Avril S., Huntley J., Pierron F. and Steele D., 3D-Heterogeneous stiffness identification using 3D displacement field data and the virtual fields method, ***The Royal Society Interface***, in revision, 2007.

Conclusions

Prospects

- Huge potential for some difficult and important engineering issues (heterogeneous materials, high strain rate, micro-scale…);
- Training required on optical FFMT: not a black box, primary influence on identification results
- Full-field processing: noise filtering, displacements to strains…

Key point: field reconstruction

Model f is derived from:
- the constitutive equations,
- the conservation equations.

$F(A_i,M)=0$

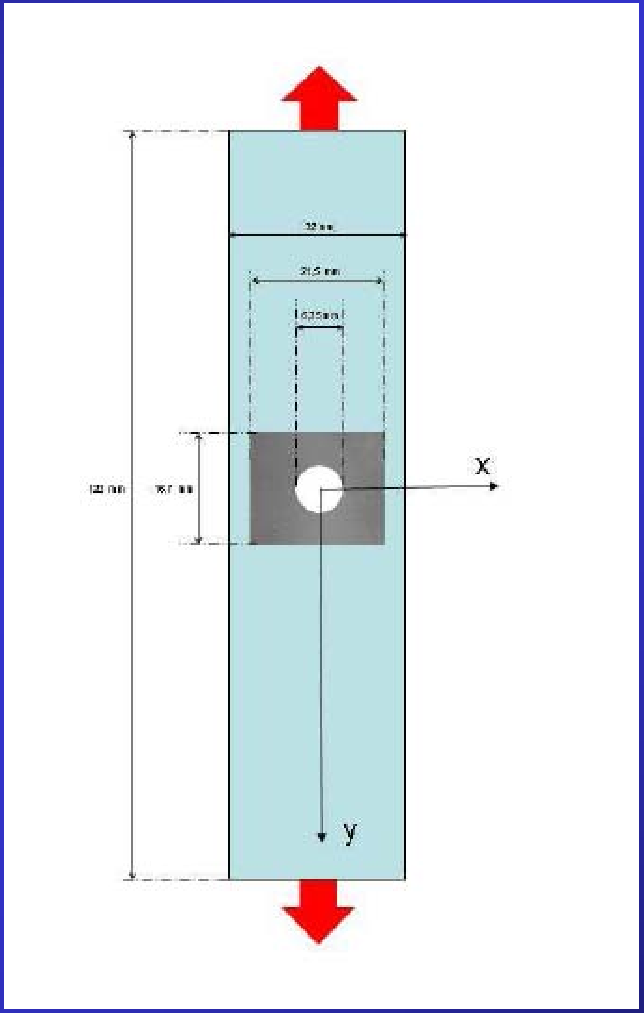
Measurements
$m(x,y,z)$

$F(A_i,m)=0$

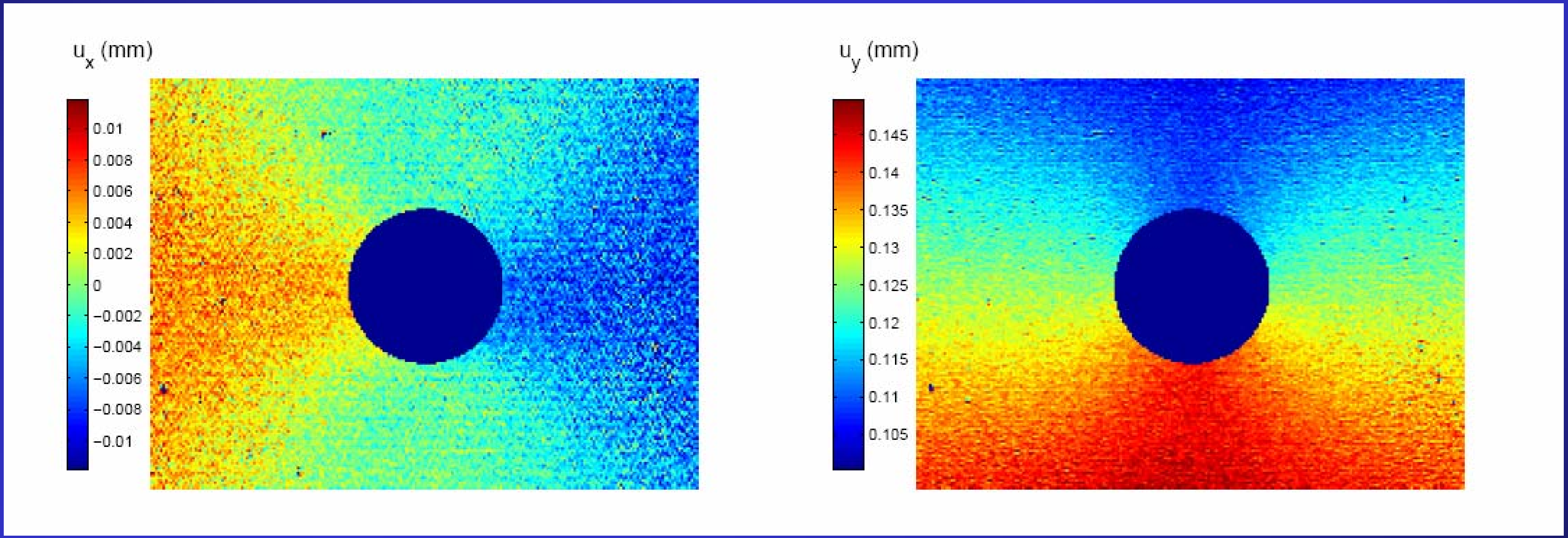
A_i

KEY POINT

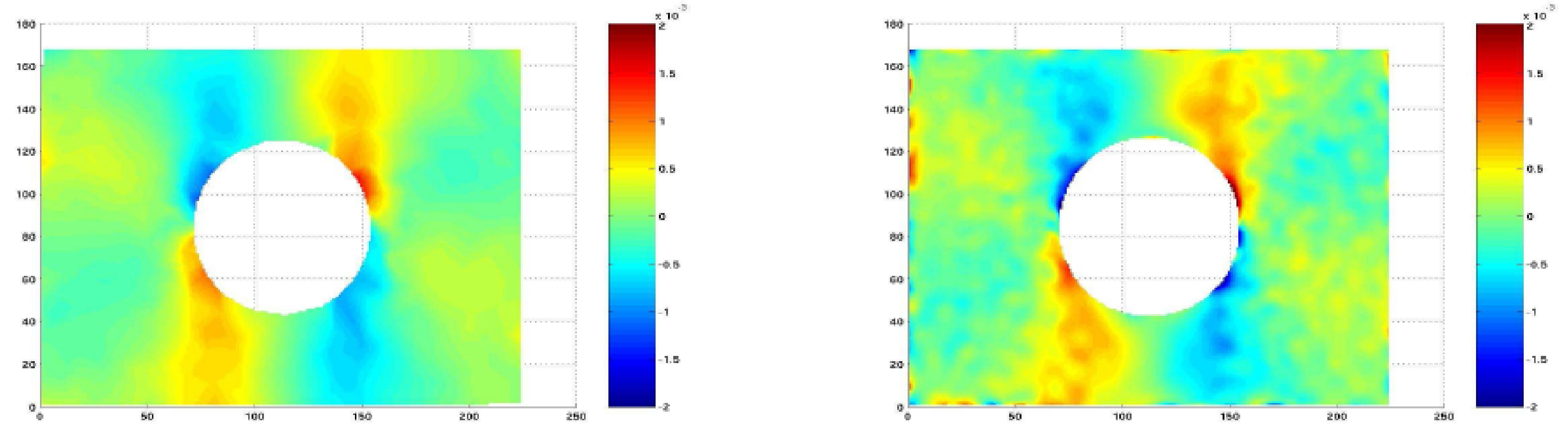
Comparison of two approaches



Comparison of two approaches



Comparison of two approaches



(a) Reconstruction par Éléments Finis

(b) Reconstructin par approximation diffuse

...

Thank you for attention

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