A Two-Tier Estimation of Distribution Algorithm for Composite Laminate Optimization

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The paper proposes a new evolutionary algorithm for composite laminate optimization, named Double-Distribution Optimization Algorithm (DDOA). DDOA belongs to the family of estimation of distributions algorithms (EDA) that build a statistical model of promising regions of the design space based on sets of good points, and use it to guide the search. A generic framework for introducing statistical variable dependencies by making use of the physics of the problem is presented. The algorithm uses two distributions simultaneously: the marginal distributions of the design variables, complemented by the distribution of auxiliary variables. The combination of the two generates complex distributions at a low computational cost. The paper demonstrates the efficiency of DDOA for laminate strength maximization problem where the design variables are the fiber angles and the auxiliary variables are the lamination parameters. The results show that its reliability in finding the optima is greater than that of a simple EDA, the univariate marginal distribution algorithm. The paper specifically investigates how the compromise exploitation/exploration can be adjusted. It demonstrates that DDOA maintains a high level of exploration without sacrificing exploitation.

I. Introduction

Stacking sequence optimization of composite laminates is concerned with finding the optimal orientation of each ply. Because of availability of data on failure modes, fiber angles are usually restricted to a finite set of values. This constraint gives rise to combinatorial optimization problems. In the last decade, evolutionary algorithms (EA) have been used to address stacking sequence optimization problems.[1–3] Standard EAs are population-based algorithms that search the design space by combining portions of high quality solutions (crossover) and mixing it with local random exploration (mutation). While these algorithms have been used with success to solve a wide variety of problems, they typically require the user to tune many parameters by trial and error to maximize efficiency. A second limitation of standard EAs is the lack of theoretical support for their performance.

To address these shortcomings, a new class of evolutionary algorithms called estimation of distribution algorithms (EDAs) has been proposed in the last few years.[4–6] EDAs partly abandon the analogy to the Darwinian theory of evolution. Instead, they use a statistical framework to formalize the search mechanisms of EAs. This approach produces algorithms whose behavior is determined by statistically meaningful parameters. The present paper introduces an estimation of distribution algorithm for laminate optimization. The algorithm, called Double-Distribution Optimization Algorithm (DDOA), takes advantage of the physics of the problem to maximize efficiency: it combines statistical information on the fiber angles and auxiliary variables called the lamination parameters to guide the search.

The paper specifically investigates how the compromise exploitation/exploration can be adjusted. Indeed, two approaches can be adopted to guide a statistical optimizer: (1) construct a complex model of high fitness

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regions, i.e. exploit good solutions already found; or (2) use randomization techniques (therefore, explore) to escape local optima and correct erroneous predictions of these promising areas. In practice, all algorithms implement elements of the two strategies and include both exploitation and exploration components. The difficulty often consists in finding a satisfactory compromise between the two[a]. In this paper, two diversity preserving techniques are studied, and their influence on the algorithms' performance is analyzed.

The paper is organized as follows: Section II introduces the principles of estimation of distribution algorithms, Section III gives a review of lamination parameters, Section IV presents the Double-Distribution Optimization Algorithm, Section V compares DDOA to two other evolutionary algorithms, a genetic algorithm and a univariate marginal distribution algorithm, on three laminate optimization problems, and Section VI provides conclusions and perspectives.

II. Estimation of Distribution Algorithms

Let us consider the problem of maximizing an objective function[b] $F(\boldsymbol{\theta})$ with $\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_n)$, such that $\theta_k \in \mathbb{A}$, a discrete alphabet. Evolutionary algorithms search the design space by iteratively creating populations of points by recombining information from good points of the previous iteration. In standard EAs, a new population is created by applying crossover and mutation operators to a subset of points obtained by applying an objective-function-based selection procedure to the population. In estimation of distribution algorithms, the subset of selected points is viewed as a sample from a certain distribution $p^t(\theta_1, \ldots, \theta_n)$, the distribution of all good points at iteration t. The set of all good points at iteration t is determined by: (1) the distribution of the points before selection and (2) the selection procedure adopted. By iteratively sampling points from $p^t(\theta_1, \ldots, \theta_n)$, applying a selection operator and *explicitly* estimating $p^{t+1}(\theta_1, \ldots, \theta_n)$, the algorithm visits increasingly fit regions of the search space. The general EDA algorithm is the following:

1. initialize $t \leftarrow 0$ and $p^0(\theta_1, \ldots, \theta_n)$;
2. if $t \leq t_{\max}$, create a population of λ points according to $p^t(\theta_1, \ldots, \theta_n)$;
3. evaluate the function $F(\theta_1, \ldots, \theta_n)$ of the λ points;
4. select μ high objective function points among the λ individuals;
5. estimate the distribution $p^{t+1}(\theta_1, \ldots, \theta_n)$ of the μ selected individuals;
6. increment t and go to 2.

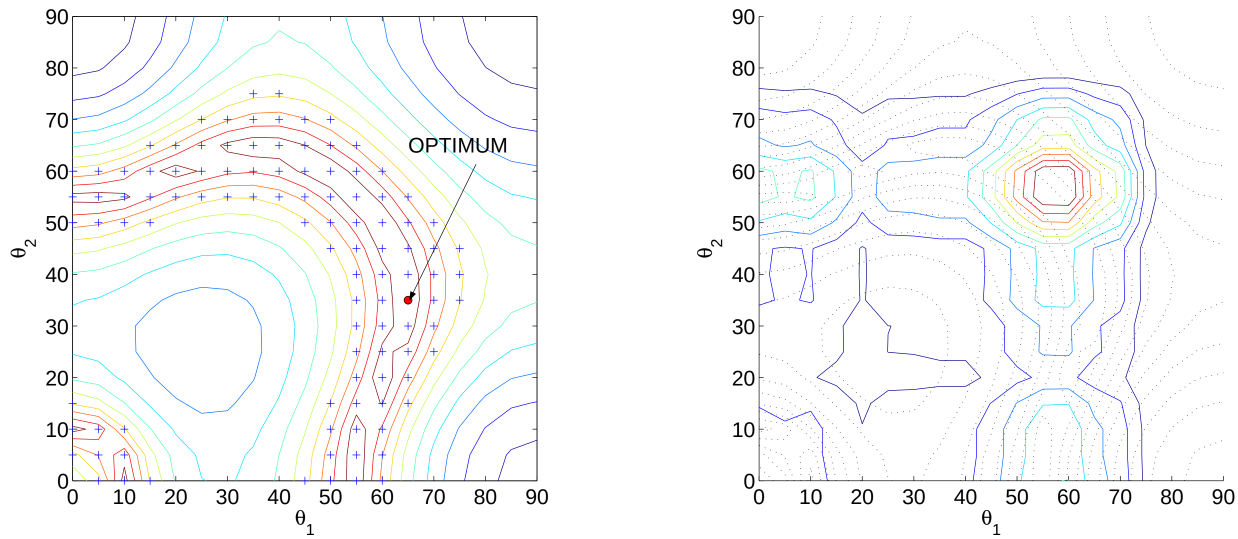
As the distribution of promising points $p^t(\theta_1, \ldots, \theta_n)$ has to be estimated from a finite number μ of selected points, it is replaced by its approximation $\hat{p}^t(\theta_1, \ldots, \theta_n)$, however we will drop the hat in the remainder of the paper to keep notations simple. Obtaining a good estimate of that distribution is critical for the algorithm's efficiency, as inaccurate probabilities result in wasteful function evaluations. Clearly, a trade-off has to be made between the flexibility of the statistical model p^t (its ability to represent complex variable interactions) and its robustness to the choice of selected individuals (the standard error on the model parameters).

Early works assumed simple probability structures. For example, the Probability-Based Incremental Learning algorithm[4] and the Univariate Marginal Distribution Algorithm[7] (UMDA) neglected all variable interactions. While this approach has been successful for problems with mild variable interactions, univariate distributions cannot capture dependencies between variables and can drive the optimization to a local optimum. Ref. 8 give an example of a deceptive problem where univariate distributions are misleading. A 2D example that is misleading for univariate distributions[c] is shown in Figure 1. After selection, the points follow the contours of the penalized objective function function, but the probability distribution obtained by neglecting variable interactions favors regions that are far from the optimum. For problems with strong variable interactions, one needs to use more complex distributions. Several researchers have proposed EDAs with higher-order distributions. In the MIMIC algorithm,[9] pairwise interactions between variables are incorporated through a chain model. In the Bivariate Marginal Distribution Algorithm, Ref. 10 generalize the

[a]Intensive exploitation tends to neglect large regions, while massive exploration often disrupts the search of high objective function areas identified in previous iterations.

[b]This function is commonly referred to as "fitness function" in the evolutionary computation community, however, we shall avoid the biologically inspired terminology in this work.

[c]Note that such variable dependencies are typically introduced by optimization constraints.



(a) Contours of the objective function and selected points.

(b) Univariate distribution.

Figure 1. Even with a large population of selected points in the high objective function regions (a), the high probability areas of the univariate distribution (b) do not coincide with high evaluation regions.

UMDA algorithm by using a tree model. Even higher-order models have been proposed: the Factorized Distribution Algorithm[11] uses the structure of the objective function to simplify the distribution representation. The Bayesian Optimization Algorithm, proposed by Ref. 12, represents the distribution of good selected individuals by Bayesian networks, thereby allowing any degree of complexity for the distribution. In practice, the order of the interactions incorporated in the model has to be limited to reduce the overhead associated with the estimation of the model parameters, and to avoid overfitting the selected points. In the present paper, we propose a physics-based strategy for incorporating variable dependencies into the probabilistic model at a low computational cost.

III. Lamination Parameters

In laminate stacking sequence optimization, the overall stiffness properties of a laminate are completely captured by the "lamination parameters".[13] The utility of lamination parameters for composite optimization has been demonstrated by several researchers.[14,15] Performing the optimization in the lamination parameter space has several advantages: in some cases, it forces convexity of the objective function,[16] it reduces dimensionality, it renders the problem amenable to continuous optimization methods.

The only non-zero extensional lamination parameters V_i^* of a balanced symmetric laminate $[\pm\theta_1, \pm\theta_2, \cdots, \pm\theta_n]_s$ are the following:

$$\begin{aligned} V^*_{\{1,3\}} &= \frac{2}{h}\int_0^{h/2} \{\cos 2\theta, \cos 4\theta\} dz \\ &= \frac{1}{n}\sum_{k=1}^{n} \{\cos 2\theta_k, \cos 4\theta_k\} \ , \end{aligned} \tag{1}$$

where h designates the total laminate thickness. Note that Equation (1) defines a feasible domain $\mathcal{L}$ in (V_1^*, V_3^*), which is the set of the images of all possible laminates $[\pm\theta_1/\ldots/\pm\theta_n]_s$.

The stiffness matrix components of a balanced symmetric laminate can be expressed as linear functions of the lamination parameters as follows:[17]

$$\begin{Bmatrix} A_{11} \\ A_{22} \\ A_{12} \\ A_{66} \end{Bmatrix} = h \begin{bmatrix} U_1 & V_1^* & V_3^* \\ U_1 & -V_1^* & V_3^* \\ U_4 & 0 & -V_3^* \\ U_5 & 0 & -V_3^* \end{bmatrix} \begin{Bmatrix} 1 \\ U_2 \\ U_3 \end{Bmatrix} , \tag{2}$$

where U_1, U_2, U_3, U_4, and U_5 are the material invariants of the laminate.

IV. The Double-Distribution Optimization Algorithm

In the present paper, we propose a physics-based strategy for incorporating variable dependencies: instead of using a complex model to represent the interactions between the θ_k's, we observe that variable dependencies among selected points often reflect the fact that the overall response of the system is really a function of integral quantities, so that many combinations of the design variables can produce the same response. For example, the dimensions of the section of a beam determine its flexural behavior through the moment of inertia, the aerodynamic properties of a vehicle are captured by the drag coefficient, etc. These quantities are often inexpensive to calculate, and their number is limited and insensitive to the number of design variables. In the case of laminates, the lamination parameters are such quantities.

The proposed algorithm, named Double-Distribution Optimization Algorithm (DDOA) uses a simple univariate model to represent the angle distribution $p(\theta_1, \dots, \theta_n)$, but introduces variable dependencies by biasing the search based on the continuous distribution of selected points in the lamination parameter space[d]. Let $f(\mathbf{V})$ designate that distribution ($\mathbf{V}$ is the set of lamination parameters considered and depends on the problem at hand: $\mathbf{V} = (V_1^*, V_3^*)$ for in-plane problems). The DDOA algorithm is presented in Figure 2.

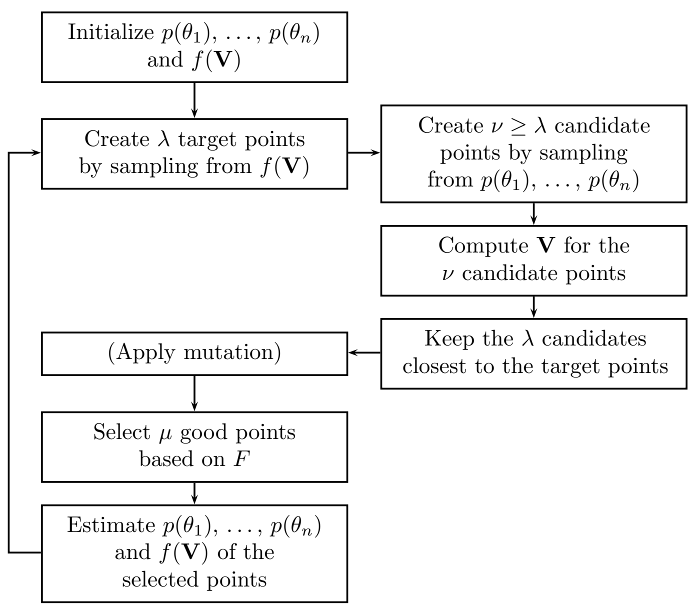


Figure 2. The DDOA algorithm

The two search distributions cooperate in the following manner: in problems that can be expressed as functions of the lamination parameters only, the distribution $f(\mathbf{V})$ accurately describes the promising regions. It can therefore be regarded as an "ideal" search distribution. To approximate that desirable distribution in the $\mathbf{V}$-space, while generating points in the θ-space, a three-step strategy is proposed, where the angle distribution provides a set of points that have correct univariate distributions, while the lamination parameter distribution introduces variable dependencies. Initially, $p(\theta_1, \dots, \theta_n)$ and $f(\mathbf{V})$ are initialized to uniform distributions (the actual form of $f(\mathbf{V})$ will be given in the next paragraph). The search then proceeds

[d]Strictly speaking, the lamination parameters can only take discrete values, however, since these values are not known in advance, their distribution is best described by a continuous model.

as follows: first, a set a set of λ points is created in the $\mathbf{V}$-space by sampling from $f(\mathbf{V})$. Then, a larger set of ν candidate points is sampled from $p(\theta_1, \ldots, \theta_n)$. Finally, the λ candidate points that are closest to the target points in the $\mathbf{V}$-space are accepted as the new population (Euclidean distances between all possible target-candidate pairs are calculated, then for each target point, the closest candidate point is accepted for the next generation. Any pair that has been used is then discarded for the rest of the acceptance procedure). Depending on the degree of confidence placed in the distributions $p(\theta_1, \ldots, \theta_n)$ and $f(\mathbf{V})$, their relative influence in the creation of new points can be adjusted through the ratio ν/λ: when $\nu/\lambda = 1$, the lamination parameter distribution $f(\mathbf{V})$ plays no role in the search; when $\nu/\lambda \to \infty$, the optimization is based primarily on information in the lamination parameter space. Objective-function-based selection is then applied to the population, and the distributions $p(\theta_k)$, $k = 1, \ldots, n$ and $f(\mathbf{V})$ of the selected points are estimated.

The use of unbounded-support density functions, such as normal kernels, for the representation of $f(\mathbf{V})$ complicates the sampling of points in the bounded lamination parameter space, because infeasible target points (points that do not lie in the feasible domain $\mathcal{L}$, see Section III) can be generated. Tests revealed that forcing target points to stay in the feasible domain by using a rejection method biases the search toward inner regions of $\mathcal{L}$, when the optima often lie close to the boundaries. Consequently, we accepted infeasible target points, considering that all the points resulting from the two-distribution creation procedure are feasible by construction.

Two aspects of the algorithm deserve special consideration: the representation and the estimation of $f(\mathbf{V})$ and diversity preservation mechanism. Most of the works on continuous EDAs to date use univariate normal distributions to represent the distribution of selected points.[18,19] The disadvantages of that model is that it does not model dependencies between variables, and assumes a unimodal symmetric distribution. Ref. 20 propose to use a non-parametric kernel density estimation (KDE) method to achieve a more accurate approximation. Since the number of lamination parameters that we consider is small (two to four) and independent of the problem dimension, KDE is appropriate for modeling the distribution $f(\mathbf{V})$. In the kernel density estimation method, a kernel $K(\mathbf{u})$ is placed at each sample point. The distribution $f(\mathbf{V})$ is obtained as

$$f(\mathbf{V}) = \frac{1}{\mu} \sum_{i=1}^{\mu} K\left(\mathbf{V} - \mathbf{V}_i\right) \quad . \tag{3}$$

In this work, we used Gaussian kernels:

$$K(\mathbf{u}) = \frac{1}{(2\pi)^{d/2} \sigma^d} \exp\left(\frac{\mathbf{u}^T \mathbf{u}}{\sigma^2}\right) \tag{4}$$

where d is the dimension of $\mathbf{u}$ and the variance σ^2 is the bandwidth that needs to be adjusted: a small value of σ increases the resolution but also increases the variance of the estimate when few data points are available. Several methods for adjusting the value of σ exist, such as trial-and-error, maximum likelihood, or adaptive strategy. In this work, a maximum likelihood method was used (see Ref. 21 for a description of the method).

An essential, but often ignored, aspect of estimation of distribution algorithms, when they are to be applied to practical problems, is the issue of the preservation of diversity. Indeed, when one runs an EDA as presented in II, one faces the problem of "premature convergence" well known in genetic algorithms: after a few iterations, the probability of whole regions of the search space goes to zero, consequently these areas fall out of the search domain. If the remaining regions contain the optimum, this phenomenon is beneficial, however, this is not generally the case. If, at some point of the optimization, the probability of the optimum vanishes, a basic EDA will not be able to find it. Two factors contribute to the loss of optimum from the distribution:

1. fitness-based selection aims at identifying good sub-solutions (variables or groups of variables that make up the optimum.The frequency of these sub-solutions depends on their mean fitness in the population. Due to the effect of the other variables, this mean fitness is a random variable. Depending on its variability, the discrimination of sub-solutions may be difficult, and there is a chance that no instance of the optimal values of the variables considered will be present in the selected points, which immediately leads to a zero probability for the optimum;

2. the effect of noisy discrimination is compounded by the fact that even in the absence of fitness-based selection, iteratively sampling from a discrete distribution and estimating the updated distribution

from that sample results in the degeneracy of the distribution to a single point. Even if optimal values of all the variables are represented in the selected population, there is a finite probability that they will be lost in the sampling process.

In theoretical EDAs, no diversity preservation mechanism is implemented, because infinite populations are often assumed, so that optimal values are never lost, and the distribution is guaranteed to converge to the global optimum. However, practical implementations use finite populations, and it becomes essential to guard against loss of diversity. In this work, we provided two different diversity preserving mechanisms:

mutation: a perturbation is applied with probability p_m to each variable θ_k of each of the λ created points. The perturbation consists in changing the value of θ_k to one of the neighboring values with equal probability (e.g. 45° can be changed to 22.5° or 67.5°).

bounds on the marginal probabilities $p(\theta_k)$: when the probability $p(\theta_k = c_l)$ estimated from the μ selected points falls below a threshold ϵ, it is corrected as follows:

1. the probability $p(\theta_k = c_l)$ is set to ϵ;
2. the difference $\epsilon - p(\theta_k = c_l)$ is evenly subtracted to the other probabilities $p(\theta_k = c_m)$.

Mutation is a standard operator in evolutionary algorithms. It tends to smooth the probability distributions by performing a form of extrapolation between high-probability points. Limiting the marginal probabilities of the variables achieves a similar smoothing effect by forbidding too determinate distributions, which balance the extreme-value-amplification effect of the selection and sampling components of the algorithm. The difference is that mutation has a local effect while bounds affect the distribution globally.

So far, the merits of preserving diversity have been presented merely as a means to allow a pure EDA search procedure to converge when distributions are estimated from finite samples. This assumes that the EDA converges in the first place, which supposes that an accurate statistical model is used, and that judicious exploitation of the information summarized in the model leads to the optimum. Often, a simplified statistical model is used, and exploitation only cannot yield the optimum. In those situations, an exploration component has to be added in the form of a perturbation: mutation and bounds on the probabilities can play such a role. Even when an accurate model is used, a combination of exploitative and exploratory search components may turn out to be the most effective strategy. The next section investigates the benefits of mutation and limitation to the marginal probabilities for UMDA and DDOA for a laminate optimization problem.

V. Application to Laminate Optimization

We considered the problem of maximizing the load factor λ_s, using the first-ply-failure criterion based on the maximum strain, for a glass-epoxy laminate subjected to the in-plane loading $N_x = -1000 \times 10^3$ N/m, $N_y = 200 \times 10^3$ N/m, $N_{xy} = 400 \times 10^3$ N/m:

$$\textbf{maximize } \lambda_s = \min_{k=1}^{n} \left\{ \min \left[\max \left(\frac{\epsilon_1^{\mathrm{t}}}{\epsilon_1(k)}, -\frac{\epsilon_1^{\mathrm{c}}}{\epsilon_1(k)} \right), \max \left(\frac{\epsilon_2^{\mathrm{c}}}{\epsilon_2(k)}, -\frac{\epsilon_2^{\mathrm{t}}}{\epsilon_2(k)} \right), \frac{\gamma_{12}^{\mathrm{ult}}}{|\gamma_{12}(k)|} \right] \right\} \tag{5}$$

where the load factor λ_s is the coefficient by which the load has to be multiplied for the structure to fail. The material properties used for this problem are shown in Table 1. The total thickness of the laminate was $h = 0.02$ m. The particularity of this problem is that the lamination parameters provide only partial information about the objective function:

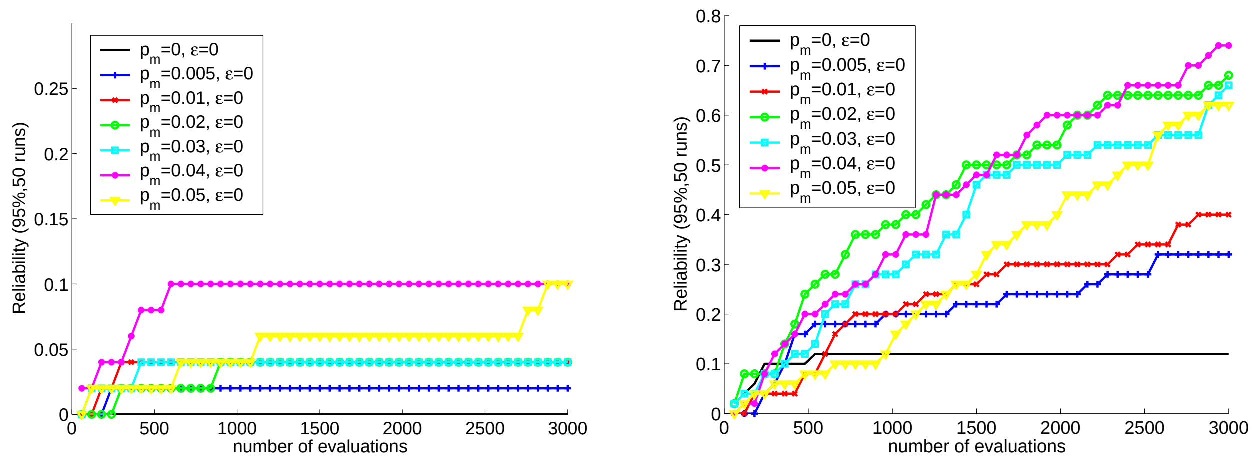
$$\lambda_s = \lambda_s(V_1^*, V_3^*, \theta_1, \ldots, \theta_n) \ .$$

We applied the two algorithms to the case $n = 12$. The optimal laminate for that problem was $[0_{14}/\pm 67.5_5]_s$ (or any permutation of the same angles), which yielded a load factor $\lambda_s = 4.74$. The population sizes were $\lambda = 30$ and $\mu = 30$, and linear ranking selection was used for the two algorithms. A candidate pool size of $\nu = 150$ and a bandwidth of $\sigma = 0.15$ were used for DDOA. The two diversity preserving mechanisms were implemented in combination with the two algorithms.

The reliability (for a practical optimum of 95% of the optimum) of UMDA and DDOA for values of p_m ranging between 0 and 0.05 (without limitation on the probability distribution) is shown in Figure 3. When

Table 1. Material properties of glass-epoxy.

Longitudinal modulus E_1	69.0 GPa
Transverse modulus E_2	10.0 GPa
Shear modulus G_{12}	4.5 GPa
Poisson's ratio ν_{12}	0.31
Material invariant U_1	31.58 GPa
Material invariant U_2	27.91 GPa
Material invariant U_3	6.48 GPa
Material invariant U_4	9.63 GPa
Material invariant U_5	10.98 GPa
Ultimate strain, tension, ϵ_1^t	7.7×10^{-3}
Ultimate strain, compression, ϵ_1^c	6.3×10^{-3}
Ultimate strain, tension, ϵ_2^t	3.5×10^{-3}
Ultimate strain, compression, ϵ_2^c	11.0×10^{-3}
Ultimate strain, shear, γ_{12}^{ult}	15.6×10^{-3}

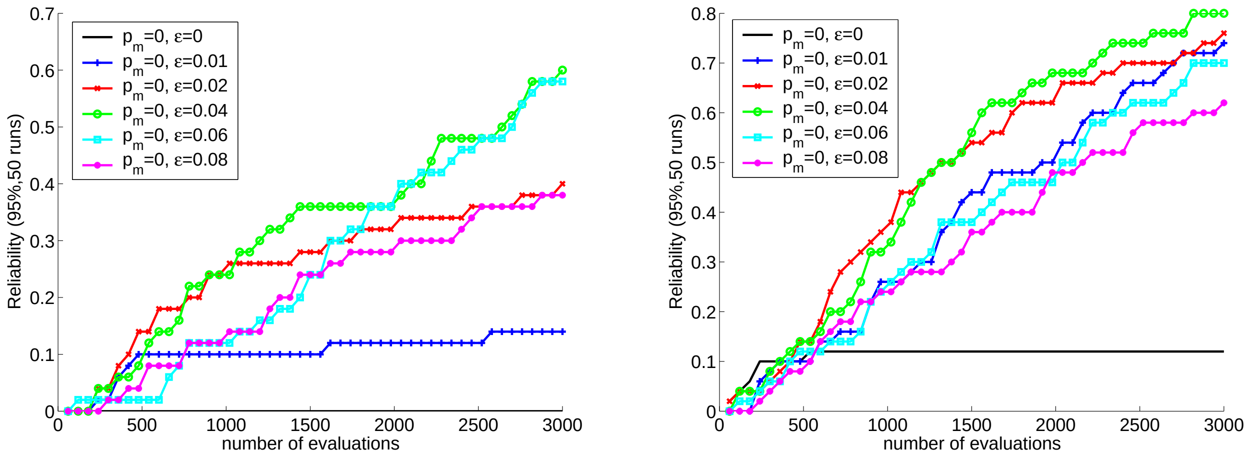


(a) (b)

Figure 3. Effect of mutation for UMDA (a) and DDOA (b).

no mutation is used ($p_m = 0$), UMDA never finds the optimum because the distribution quickly degenerates, which prevents any further progress. DDOA reaches a slightly higher reliability of 12%. An explanation for this higher efficiency will be given in the next paragraph. When the mutation rate is increased, the effectiveness of both algorithms improves, though UMDA's reliability never exceeds 10% for $p_m = 0.04$. With the same mutation rate, DDOA reaches a reliability of 74%. Further increasing the amount of random perturbation causes both algorithm's performance to deteriorate.

The influence of "repairing" degenerate probability distributions by imposing lower bounds on the $p(\theta_k)$'s is shown in Figure 4 for UMDA (a) and DDOA (b), which presents the algorithms' reliability for seven values of ϵ, ranging from 0 to 0.08 (which represents 2/5 of the uniform distribution $p(\theta_k = c_l) = 1/5 = 0.2$). This diversity preservation mechanisms is even more beneficial to the search performance than mutation: UMDA's reliability reaches 60% for $\epsilon = 0.04$, and DDOA's reliability at the end of the optimization increases to 80% for the same value of ϵ. As in the case of mutation, higher values of ϵ cause the performance of both algorithms to drop. The advantage of bounds on the probability over mutation can be explained by the fact that mutation is a costly way of reintroducing lost variable values: while repairing the distribution guarantees that all values have a chance of being generated, while inflicting only marginal perturbation to the convergence process, recovering a lost value through mutation may take a large number of iterations if



(a) (b)

Figure 4. Effect of bounds on the probability distributions $p(\theta_k)$ for UMDA (a) and DDOA (b).

p_m is not sufficiently high, or cause large perturbations if p_m is large.

While both mutation and the lower bound on marginal probabilities have an obvious positive influence on both algorithms, DDOA clearly benefits more than UMDA from this injection of randomness: even with high mutation rates, UMDA's reliability remains poor; and, when limits on the probabilities are imposed, DDOA performs better than UMDA on average, and its reliability is less sensitive to the value of ϵ than UMDA's. This can be explained by observing that the method used by DDOA to generate new points allows it to either compensate for a lack of diversity if the population becomes too uniform, or reduce its diversity when heavy perturbations have been introduced: the standard deviation of $f(\mathbf{V})$ cannot fall below the bandwidth σ. When the diversity of points sampled from the univariate distribution is low, the target points, sampled from $f(\mathbf{V})$, will be distributed over a wider range than the candidate points, created from $p(\theta_1, \ldots, \theta_n)$. The target-point-based acceptance procedure will therefore give proportionately more weight to extreme candidate points, so that the variance of the resulting population will be increased (see Figure 5). This explains why DDOA outperforms UMDA for small values of p_m and ϵ. The parameter σ can therefore been viewed as the minimum amount of diversity in the lamination parameter space.

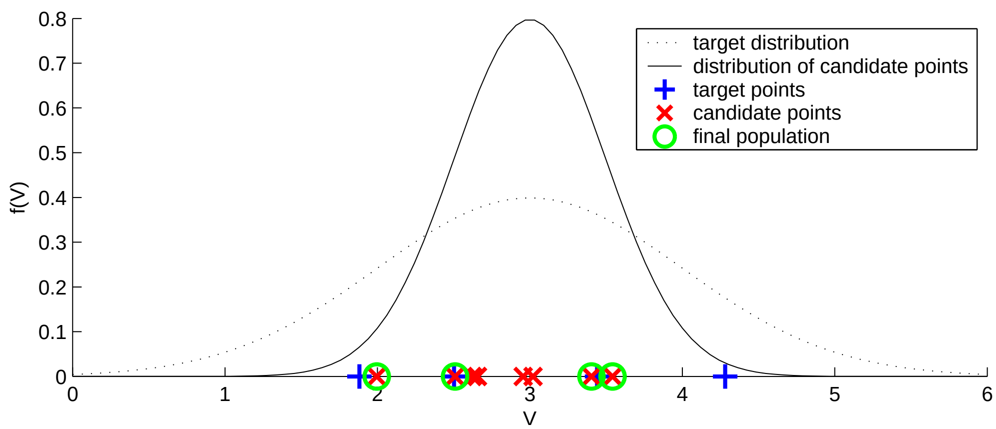


Figure 5. Increase of the population diversity through the two-distribution point creation procedure: the insufficient variance of candidate points sampled from $p(\theta_1, \ldots, \theta_n)$ is corrected because extreme points are proportionately more likely to get selected.

More important is what happens when large perturbations are applied either to the population (mutation) or to the probabilities (lower bounds). In those cases, DDOA's lamination-parameter-based point creation scheme reduces the excessive variability by preferentially keeping those points which likelihood in $f(\mathbf{V})$ is high (see Figure 6). It can be observed that DDOA benefits from high entropy, as it increases the weight of $f(\mathbf{V})$ in the creation of new points.

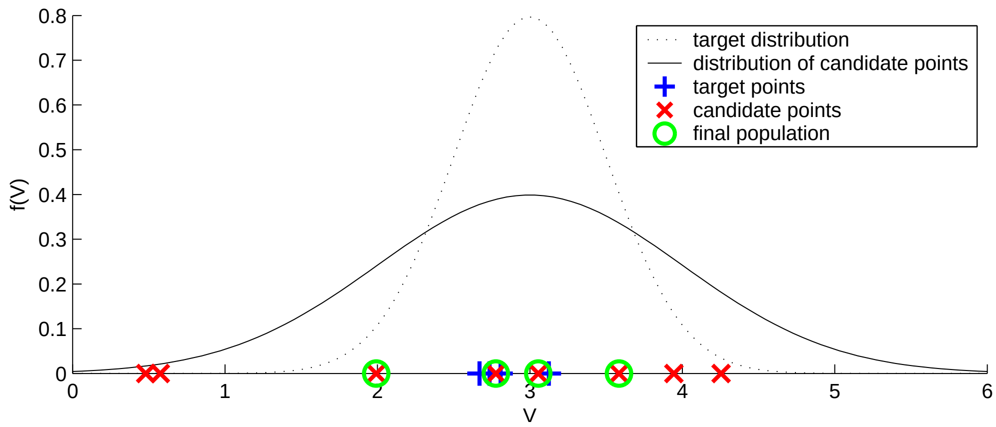


Figure 6. Decrease of the population diversity through the two-distribution point creation procedure: the excessive variance of candidate points sampled from $p(\theta_1, \ldots, \theta_n)$ is corrected because extreme points are unlikely to get selected.

Figure 7 isolates the best variants of the two algorithms. The algorithms' performance is directly influenced by the amount of variable interactions used to guide the search: a detailed analysis of the distribution of the best points visited reveals two basins of attraction, the first one, B_1, centered around $\lambda_s \approx 3.1$, the second one, B_2, around $\lambda_s \approx 4.2$. The relative importance of these two attractors evolves during the search, and is different for the two algorithms. UMDA generates more points in B_1, which corresponds to a uniform distribution of fiber angles, rendered very likely by the large value of ϵ. DDOA is able to filter out these points and to focus on B_2. Another effect of the high level of randomness on UMDA is the progression by successive jumps: the algorithm generates many points, until one good combination is found. This behavior is visible in the reliability, which displays plateaus without improvement, followed by sudden a rise. In contrast, DDOA shows a more gradual increase in the reliability, due to a more consistent sampling in high-fitness regions by using information about the lamination parameters distribution to filter out poor candidates.

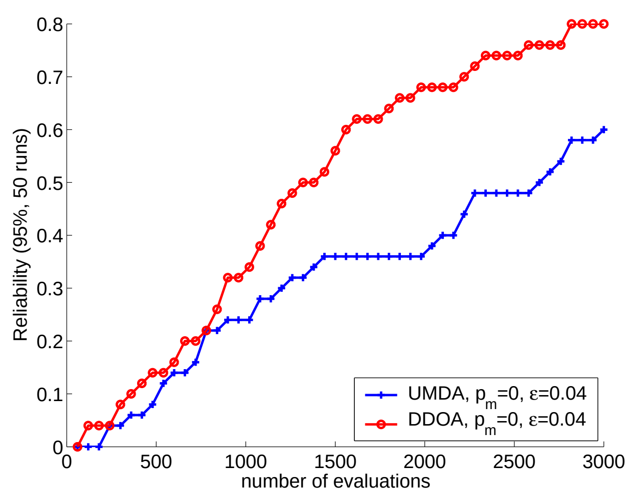


Figure 7. Reliability of the best variants of UMDA and DDOA.

A physical explanation for DDOA's advantage for this problem can be proposed: the variables of this problem are related with strong dependencies because changing the orientation of a particular ply causes a load redistribution, which directly affects the optimal orientation of other plies. DDOA provides a more explicit mechanism for separating overall response and local ply adjustment by allowing multiple laminates that have similar stiffness properties (hence similar strains) to be generated. UMDA, which extrapolates promising regions based on insufficient ply-level information, is not able to reliably produce high-strength laminates, hence its poor average best function evaluation, and the absence of progress.

VI. Concluding Remarks

A new approach for composite laminate optimization was proposed. The algorithm, called double-distribution optimization algorithm, is based on a simple version of estimation of distribution algorithm, the univariate marginal distribution algorithm, but corrects the distribution inaccuracy caused by the independent variable assumption. The new idea of the method rests on the use of the distribution of integral variables, the lamination parameters, to incorporate some dependencies, which obviates the need for complex probabilistic models that can be expensive to construct and overly sensitive to sampling errors.

The method was demonstrated on a laminate strength maximization problem, where some, but not all the information about the overall response is contained in the lamination parameters. The influence of two diversity preservation mechanisms, mutation, and bound on the marginal distributions, was investigated. The latter strategy proved more efficient than mutation. In the tested problem, DDOA clearly outperformed the basic UMDA: using the lamination parameter distribution led to dramatic improvement of the optimization reliability, even though they provided only partial information about the objective function, as it allowed separation of global stiffness and local angle adjustment. An additional advantage of DDOA is to provide a mechanism for adjusting the population diversity, making the algorithm less sensitive to the magnitude of the perturbation used for the search.

The authors would like to emphasize the fact that the double-distribution approach is not restricted to laminate optimization, but can be adapted to any problem that involves multiple-level variables, in particular all problems that introduce integral quantities, such as the moments of inertia in structural dynamics, the drag coefficient in aerodynamics, the permeability in the study of flows in porous media, etc. In all these situations, the distribution of higher-level quantities provides information about the lower-level variable dependencies and can be used to improve the efficiency of the search.

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