MRI SEQUENCE DENOISING USING GAUSSIAN PROCESSES

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ABSTRACT: This article presents a new method for denoising a sequence of MRI images. The method is based on a description of the pixels spatial and time correlations by conditional Gaussian processes. It allows not to assume a functional form for the pixel intensity variations. It is found that, contrarily to local filtering approaches, the spatial correlation correctly separates the components of the image. An application is given to denoise images of a carotid artery where the time periodicity of the cardiac cycle is accounted for.

1 INTRODUCTION

Medical imaging has often to deal with a sequence of pixelized images corrupted by an unknown experimental noise. For example, in this article, we consider carotid artery cross-sections obtained from phase contrast magnetic resonance imaging (MRI) at a rate of 50 Hz and with a spatial resolution of a pixel every 0.1 mm. Even if an accurate knowledge of the artery morphology is necessary for diagnosis and treatement monitoring [1], the obtained images are often affected by random noise [2]. A basic method for smoothing data is to use local averaging [3]. However this approach can induce an important loss of contrast if the shape of the smoother is not adapted to the data. In our case study, the interface between the artery and the blood may not be represented by a sharp intensity gap after smoothing.
In this context, we propose a method for denoising which is made of two steps: The first step is a spatial smoothing which takes advantage of the repeated images to learn the spatial structure of the signal and filter the images based on this spatial structure; In a second time smoothing step, the eventual periodicity of the signal (as is the case with the cardiac cycle loading the arteries) is accounted for.

The mathematical background underlying the proposed denoising method assumes that pixel intensity, $I(x,t)$, is a random Gaussian process (GP) indexed by space, $x \in \mathbb{R}^2$, and time, $t \in \mathbb{R}$. The MRI measures are used to condition this process, i.e., to make instances of the process approach them (which has been popularized as kriging, [4]). Because the process is Gaussian, it is fully described by its mean and covariance (both are functions of space and time). As it will be seen shortly, there is a natural way to learn the mean and covariance from the image sequence. This allows denoising without assuming a functional form for $I(x,t)$ nor assuming a neighborhood in time and space for local averaging.
We further assume that the observed intensity I can be decomposed into the sum of 2 GPs:

$$I(x,t) = Z_s(x,t) + Z_n(x,t) \tag{1}$$

where Z_s represents the signal we want to extract and Z_n the observation noise. As the data is composed of a 60×60 pixel grid and 50 time steps, the covariance matrix of the observation is a $18.10^4 \times 18.10^4$ matrix ($18.10^4 = 60 \times 60 \times 50$). Because this size is too large to directly handle the whole covariance matrix, we will treat the spatial and the time components separately.

2 SPATIAL SMOOTHING

From the $N = 50$ time realizations of the intensity map, we can derive a spatial covariance matrix C of size $60^2 \times 60^2$,

$$C \;=\; \frac{1}{N}\sum_{i=1}^{N}\left(I(.,t_i) - \bar{I}(.)\right)\left(I(.,t_i) - \bar{I}(.)\right)^T \;. \tag{2}$$

where $I(.,t_i)$ is the vector of length 60^2 of the observed intensity at time i and $\bar{I}(.)$ the time average of $I(.,t_i)$. The eigenvalue decomposition of C shows that the first three eigenvectors, $P_{1\ldots3}$ seem to be associated to signal (large eigenvalues, $v_{1\ldots3} \gg v_{4\ldots3600}$, physical significance of the eigenvectors) whereas the others correspond to noise. The matrix C can thus be split as $C = C_s + C_n$ where C_s is the spatial covariance of Z_s, $C_s = \sum_{i=1}^{3} v_i P_i P_i^T$, and C_n is the complement for Z_n. Under the assumption we made, the optimal filtering of the image at t_i, $I_F(.,t_i)$, is given by the conditional expectation (any other random intensity field will, on the average, be further to the observed intensities):

$$I_F(.,t_i) = E[Z_s(.,t_i)|I(.,t_i)] = \bar{I}(.) + C_s C^{-1}\left(I(.,t_i) - \bar{I}(.)\right) \tag{3}$$

This approach is equivalent to proper orthogonal decomposition (i.e., orthogonal projection on the first three eigenvector) and it can be seen as a smoother based on the neighborhood given by the empirical covariance. This is illustrated in Fig. 1, left panel, where it should be noted that the neighborhood defined by the spatial covariance follows the contours of the artery. The right panel shows $I(\{43,25\},.)$ as dots and its smoothed version $I_F(\{43,25\},.)$ as a dotted line.

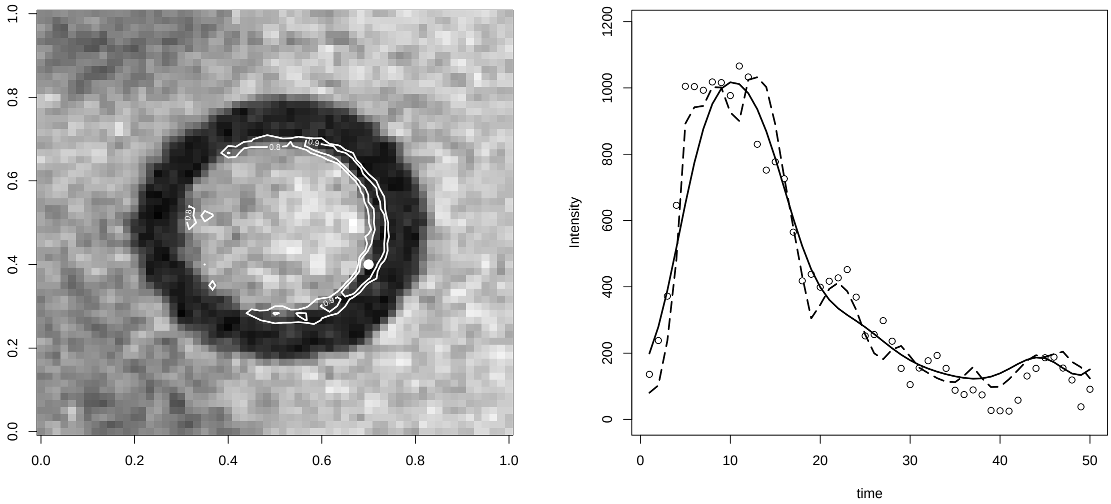

Figure 1: Left: an image of the MRI sequence and the contour lines of the correlation of the pixels intensities with the pixel $\{43, 25\}$ (white dot). Right: Intensity versus time for the pixel $\{43, 25\}$. Initial measures (dots), spatial smoothing (dotted line), spatial + time smoothing (continuous line).

3 TIME SMOOTHING

Contrarily to the previous part, we now consider the time dependency of the signal. We will treat the time smoothing independently of space: the denoising of the intensity curve associated to each pixel ignores the surrounding pixels.

Like many other organs, the artery displacements are periodical since they are created by the heart pulses. The periodicity of the phenomenon is described by the following time covariance structure:

$$K(t, t') = \sigma^2 \exp\left(-\lambda \sin\left(\frac{(t-t')\pi}{50}\right)^2\right). \tag{4}$$

Note that any periodical function does not provide a valid covariance function. The above expression is valid because it is the restriction of the classical Gaussian covariance kernel to the circle. In order to further smooth the signal, it is assumed that the observations are flawed by a white noise with variance τ^2. $K(t_i, t_j)$ are then assembled in a 50×50 matrix, τ^2 are added to the diagonal, and the resulting time covariance matrix plays a role equivalent to C in Eq. 3, but with t as variable.

In the test case, the values of σ^2 and λ were arbitrarily set to 5.10^4 and 5, and $\tau^2 = \sigma^2/3$. No trend was added. Note that, in general, σ^2, λ, τ and the trend can be estimated using maximum likelihood estimation. One cycle of a denoised pixel intensity is presented as a continuous line on the right of Fig. 1. The pixel is at the interface between the blood and the artery wall. The comparison with the raw data (dots) and the spatially smoothed signal (dotted line) shows that the combination of spatial and time smoothing efficiently cleans the image without bluring the artery contours.

4 CONCLUSION

We have proposed a computationally tractable approach based on Gaussian processes for denoising a sequence of images. The associated probabilistic framework could be further developped to describe the uncertainties associated to the measures (e.g., estimate the variance of $I(x, t)$). Another, more straightforward, perspective is to use the smoothed I to segment images.

References

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