A Multi-Agent System Approach to Risk Allocation for Reliability Based Design Optimization

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1 Introduction

In probabilistic design, uncertainties (such as material properties or loads on the structure) are considered when calculating the reliability of a structure, and the structure is optimized such that the risk is distributed amongst different modes of failure. The changes in the design that protect against the failure in the different modes often conflict [1]. For example, we can consider a vertical column that experiences both a compressive load and thermal load at its top surface, where failure occurs if the column buckles or the temperature of the bottom face exceeds the allowable. If the column is made thin, the amount of heat transferred to the bottom surface is limited, and the probability of thermal failure is reduced. However, the risk of buckling failure is increased.

Reliability based design optimization (RBDO) methods are computationally expensive because of the uncertainties in the design. Typically, Monte Carlo methods that evaluate the reliability require a large number of simulations for high levels of accuracy. Computationally cheaper RBDO methods of evaluating the reliability do not provide all of the information needed to completely solve the problem. The first-order reliability method, for example, can efficiently provide RBDO candidate designs, but the accuracy of the probabilities of failure approximations depend on the linearity of the design constraints. To address this computational load problem, we make a step towards distributed RBDO approaches. We seek to distribute the reliability based optimization problem amongst agents, which form a multi-agent system (MAS). We will examine "formulation agents" where each agent is attached to solving a simplification of the global reliability based design problem. These agents exchange design points and characteristics of the optimization criteria (e.g., risk allocations) in an attempt to achieve agent autonomy, collective solving of the global reliability design problem with a global increased efficiency.

2 Methodology

2.1 Optimization Problem

The goal of the optimization problem is to minimize a function f, such as mass, subject to a constraint on the system probability of failure $P_{f,sys}(x)$ for design variables x. Failure in terms of a system can be defined as a failure in any single failure mode. We consider n failure modes,

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$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & P_{f,sys}(x) \leq P_{sys}^{allow} \end{aligned} \tag{1}$$

where P_{sys}^{allow} is a given allowable probability of system failure. However, it is generally expensive to solve for the system probability of failure since it is traditionally calculated using Monte Carlo methods. Instead, we decompose the problem into two levels. For the top level optimization problem, the objective is to minimize the function subject to a constraint on the system probability of failure, where the vector of risk allocations P_f^{allow}, or allowable probabilities of failure, serve as the design variables.

$$\begin{aligned} \underset{P_f^{allow}}{\text{minimize}} \quad & f(P_f^{allow}) \\ \text{subject to} \quad & P_{f,sys}(P_f^{allow}) \leq P_{sys}^{allow} \end{aligned} \tag{2}$$

The system probability of failure associated to a given risk vector, $P_{f,sys}(P_f^{allow})$, is calculated through two lower levels problems. First, the risk allocations from the top level define failure mode constraints in a problem where the design variables are the original design variables x.

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & P_{f,i}(x) \leq P_{f,i}^{allow} \quad i = 1 \ldots n \end{aligned} \tag{3}$$

In order to avoid calculating mode failure probabilities (which, recall, often imply costly Monte Carlo simulations), this problem is replaced by iterative resolutions of a deterministic optimization problem with safety factors S_i.

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & g_i(x) + S_i \leq 0 \qquad S_i \geq 0 \quad i = 1 \ldots n \end{aligned} \tag{4}$$

2.2 Agent Formulation

The optimization problems described in the last section are formulations that are varied and distributed amongst several agents. Agents autonomously solve subproblems that are spawned from each formulation and communicate with each other [2]. Specifically here, the agents are aggregations of the two lower level formulations, Eqs. (3) and (4). They vary in how they interpret the data (risk allocations and associated probabilities) used to solve the top level optimization problem shown in Eq.(2). To update their risk allocations (i.e., solve the top level problem), agents build different metamodels, such as polynomial response surfaces and Kriging surrogates, using their own existing sets of designs and risk allocations, and the shared information they receive.

At the lower levels, different implementations can also be used. For example, an agent can estimate the $P_{f,i}$'s in Eq.(3) (which is useful to update the lower level safety factors) by periodical calls to Monte Carlo simulations, while another agent can approximate them by a first order reliability method.

In summary, we define an agent by its interpretation of the current design space, which is shaped by its own history and any received information, and the optimization (sub)problem it solves.

Références

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