Defining a Continuous Marketplace for the Trading and Distribution of Energy in the Smart Grid

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Smart grids: promises & expected outcomes

- New distribution rationale: **decentralized production**
  - Democratization of decentralized production: local balancing and reducing energy loss
- New context information: **energy awareness**
  - Frequently sensed data (consumption, production, pricing) impacts trading updates
- New trading rationale: **prosumption**
Smart grids: promises & expected outcomes

- New distribution rationale: decentralized production
  ▶ Democratization of decentralized production: local balancing and reducing energy loss

- New context information: energy awareness
  ▶ Frequently sensed data (consumption, production, pricing) impacts trading updates

- New trading rationale: prosumption

How to design a decentralized market for the trading and distribution of energy?
Example: energy trading scenario

- **Prosumers** ($j \in P$)
Example: energy trading scenario

- **Prosumers** ($j \in P$)
- **Offers** ($o_j : \mathbb{Z} \rightarrow \mathbb{R} \cup \{-\infty\}$)

<table>
<thead>
<tr>
<th>Bob</th>
<th>Carol</th>
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<tbody>
<tr>
<td>Units</td>
<td>Price</td>
</tr>
<tr>
<td>0</td>
<td>11.5</td>
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Example: energy trading scenario

- **Prosumers** \((j \in P)\)
- **Offers** \((o_j : \mathbb{Z} \rightarrow \mathbb{R} \cup \{-\infty\})\)
- **Links** \(\{i, j\}\) w/ some max capacity \((c_{ij})\)

Cerquides et al. Trading Energy in the Smart Grid
Example: energy trading scenario

How much energy to trade, and with whom, so that the overall benefit is maximized while the energy network’s capacity constraints are fulfilled?
Definition: energy allocation problem

The energy allocation problem (EAP) amounts to finding an allocation \( Y \) that maximizes the overall benefit \( \text{Value}(Y) \), with

\[
\text{Value}(Y) = \sum_{j \in P} v_j(Y_j)
\]

\[
v_j(Y_j) = o_j(\text{net}(Y_j))
\]

\[
\text{net}(Y_j) = \sum_{i \in \text{in}(j)} y_{ij} - \sum_{k \in \text{out}(j)} y_{jk}
\]

where \( y_{ij} \) stands for the number of units that prosumer \( i \) sells to prosumer \( j \) (bounded by \( c_{ij} \))
Example: energy trading scenario (solution)

\[ Value(Y) = o_{\text{Alice}}^{-2} + o_{\text{Bob}}^{5} + o_{\text{Carol}}^{0} + o_{\text{Dave}}^{-3} = -3.5 + 11.5 + 0 - 6 = 2 \]
Distributed allocation techniques

- **Market-based**
  - **Double auction** (call market or CDA) where energy is traded on a day-ahead basis
  - Matching between supply and demand computed by **central** authority
  - Current market mechanisms disregard grid constraints → **Trading and distribution as decoupled activities**

- **Message passing**
  - Dynamic programming (**Miller, 2014; Kumar et al., 2009**)
  - Belief-propagation (**Miller, 2014**)

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Our contribution

- Exploit the **tree** structure of energy networks \((\text{Gonen}, 2014)\)
- Solve EAP as a distributed constraint optimization problem \((\text{DCOP})\)
- Design an **exact message passing** algorithm based on dynamic programming
  - \textbf{ACYCLIC-SOLVING} \((\text{Dechter}, 2003)\)
- Assess efficiently messages by exploiting the algebraic structure of offers and messages: \textbf{valuations}
Message passing solution

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Carol
Message passing solution

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Message passing solution

Select and send valid offers

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Trading Energy in the Smart Grid
Message passing solution

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Message passing solution

select and send valid offers

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Trading Energy in the Smart Grid
Message passing solution

\[
\begin{array}{c|c}
q & v \\
--- & --- \\
2 & 1.75 \\
1 & 1.25 \\
0 & 0 \\
-2 & -6 \\
-3 & -11 \\
\end{array}
\]

Cerquides et al.

Trading Energy in the Smart Grid
Message passing solution

\[ y_{bc} = -3 \]
\[ y_{dc} = 3 \]

determine and send best assignment

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Trading Energy in the Smart Grid
Message passing solution

**Cerquides et al.** Trading Energy in the Smart Grid
Message passing solution

**send best assignment**

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\[ y_{ab} = 2 \]

Carol

Bob

Dave

Alice

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Trading Energy in the Smart Grid
Message passing solution

determine best assignment

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Trading Energy in the Smart Grid
Message passing solution

solution is found!

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Message assessment

\[ \mu_{j \rightarrow p_j}(y_{jp_j}) = \]

\[ \max_{Y_{j-p_j}} \left( v_j(y_{jp_j}, Y_{j-p_j}) + \sum_{k \in \text{out}(j) \setminus \{p_j\}} \mu_{j \rightarrow k}(y_{jk}) + \sum_{i \in \text{in}(j)} \mu_{i \rightarrow j}(y_{ij}) \right) \]

- This is the computational hard point
- Computed in \( O((2C_j + 1)^{N_j}) \)
  - \( C_j \) is the capacity of the most powerful link
  - \( N_j \) is the number of neighbors of \( j \)

\( \Rightarrow \) Not applicable to dense networks
Message assessment

$$\mu_{j \rightarrow p_j}(y_{jp_j}) =$$

$$\max_{y_{j-p_j}} \left( v_j(y_{jp_j}, Y_{j-p_j}) + \sum_{k \in \text{out}(j) \setminus \{p_j\}} \mu_{j \rightarrow k}(y_{jk}) + \sum_{i \in \text{in}(j)} \mu_{i \rightarrow j}(y_{ij}) \right)$$

- This is the computational hard point
- Computed in $O((2C_j + 1)^{N_j})$
  - $C_j$ is the capacity of the most powerful link
  - $N_j$ is the number of neighbors of $j$

⇒ Not applicable to dense networks

⇒ Assess message more efficiently!
Algebra of valuations

- Take advantage of a particularity of the messages: **restricted capacity**
- Reformulate message assessment with 3 operations:
  - Restriction (linear):
    \[
    \alpha[D](k) = \begin{cases} 
    \alpha(k) & k \in D \\
    -\infty & \text{otherwise}
    \end{cases}
    \]
  - Complement (linear):
    \[
    \overline{\alpha}(k) = \alpha(-k)
    \]
  - Aggregation (polynomial):
    \[
    (\alpha \cdot \beta)(k) = \max_{i,j} \alpha(i) + \beta(j) \\
    \text{subject to } k = i + j
    \]

\[
\mu_{j\rightarrow p_j} = \left( \overline{O}_j \cdot \prod_{k \in \text{out}(j) \setminus \{p_j\}} \mu_{j\rightarrow k} \cdot \prod_{i \in \text{in}(j)} \mu_{i\rightarrow j} \right) [-D_{jp_j}]
\]
**RADPRO algorithm**

$= \text{ACYCLIC-SOLVING} + \text{efficient message assessment}$

- **Global complexity of message assessment:** *polynomial* in $\mathcal{O}(nN^2_{\text{max}}C^2_{\text{max}})$
  - Number of message assessments in $\mathcal{O}(n(2C_{\text{max}} + 1)^{N_{\text{max}}})$
  - Single message assessment in $\mathcal{O}(N_jC_jn_{o_j} + N_j^2C_j^2)$

- **Communication complexity:** *linear* in $\mathcal{O}(nC_{\text{max}})$
  - $2n$ messages of max size $2C_{\text{max}} + 1$

- **Easily distributable**

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<thead>
<tr>
<th></th>
<th>ACYCLIC-SOLVING</th>
<th>RADPRO</th>
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<tbody>
<tr>
<td>Communication</td>
<td>$\mathcal{O}(nC_{\text{max}})$</td>
<td>$\mathcal{O}(nC_{\text{max}})$</td>
</tr>
<tr>
<td>Computation</td>
<td>$\mathcal{O}(n(2C_{\text{max}} + 1)^{N_{\text{max}}})$</td>
<td>$\mathcal{O}(nN^2_{\text{max}}C^2_{\text{max}})$</td>
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Comparison with MIP solvers

RADPRO outperforms CPLEX & Gurobi
(more than one order of magnitude faster than CPLEX!)

Random networks (geometric distribution) with $C_j = \mathcal{N}(100, 50)$
Efficiency of message assessment

**RADPRO outperforms ACYCLIC-SOLVING**
(more than three orders of magnitude faster than ACYCLIC-SOLVING!)

![Graph showing speedup comparison between RADPRO and ACYCLIC-SOLVING](image)

Star-shaped networks (hubs) with $C_j \in \{10, 30, 50\}$ and $N_{max} \in [1..6]$
Scalability

**RADOPro** solves large-scale EAP with high branching factor (solving problems with capacity 100 and 100 neighbors in less than 1 min!)

Star-shaped networks (hubs) with $C_j \in \{10, 30, 50, 70, 100\}$ and $N_{max} \in [1..99]$
RadPro Limitations

- Offers are discrete: a prosumer can offer to buy either 3 KW for 6 EUR or 2 KW for 4 EUR, but not any amount of energy between 2 KW and 3 KW and pay 2 EUR per KW.
- Such offers provide a better representation of prosumers’ preferences.
- Our next goal is to extend the EAP to allow prosumers to communicate continuous (piecewise linear) utility functions.
Example: continuous energy trading scenario

- **Prosumers** $(j \in P)$
- **Offers** as *piecewise linear valuations*
- **Links** $(\{i, j\})$ w/ some max capacity $(c_{ij})$
Definition: continuous energy allocation problem

Given a set of prosumers $P$ whose offers are piecewise linear valuations, the **continuous energy allocation problem** (CEAP) amounts to finding an allocation $Y$ that maximizes the overall benefit $\text{Value}(Y)$, with

$$
\text{Value}(Y) = \sum_{i \in P} v_j(Y_j)
$$

$$
v_j(Y_j) = o_j(\text{net}(Y_j))
$$

$$
\text{net}(Y_j) = \sum_{i \in \text{in}(j)} y_{ij} - \sum_{k \in \text{out}(j)} y_{jk}
$$

where $y_{ij}$ stands for the number of units that prosumer $i$ sells to prosumer $j$ (bounded by $c_{ij}$)
Mapping the CEAP to a Linear Program

Decision variables per prosumer:

- interval valuation to select within a piecewise linear valuation
- amount of energy to trade within the chosen interval
Mapping the CEAP to a Linear Program

Decision variables per prosumer:
- interval valuation to select within a piecewise linear valuation
- amount of energy to trade within the chosen interval

Decision variable per link:
- amount of energy to trade between prosumers
Mapping the CEAP to a Linear Program

Decision variables per prosumer:

- interval valuation to select within a piecewise linear valuation
- amount of energy to trade within the chosen interval

Decision variable per link:

- amount of energy to trade between prosumers

Constraints:

- *Mutually exclusive intervals*: only an interval valuation per piecewise linear valuation
- *Energy balance*: amount of energy to trade per prosumer equals difference between input and output energy
- *Network capacity*: energy traded between prosumers respects links’ capacities
Example: continuous energy trading scenario

- **Prosumers** \((j \in P)\)
- **Offers** as \textbf{piecewise linear valuations}
- **Links** \((\{i, j\})\) w/ some max capacity \((C_{ij})\)

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Trading Energy in the Smart Grid
Mapping the CEAP to a Linear Program

LP that solves the continuous energy allocation problem:

\[
\begin{align*}
\text{maximize} & \quad \sum_{j=1}^{\left|P\right|} \sum_{k=1}^{\left|W_j\right|} a_{o_j}^k \cdot x_j^k + b_{o_j}^k \cdot z_j^k \\
\text{subject to} & \quad z_j^k \cdot l_{o_j}^k \leq x_j^k \leq z_j^k \cdot u_{o_j}^k \\
& \quad \sum_{k=1}^{\left|W_j\right|} z_j^k = 1 \\
& \quad \sum_{i<j} y_{ij} - \sum_{q>j} y_{jq} = \sum_{k=1}^{\left|W_j\right|} x_j^k \\
& \quad y_{ij} \in D_{ij} \\
& \quad z_j^k \in \{0, 1\} \\
& \quad x_j^k \in \mathbb{R} \\
& \quad \forall j \in P, 1 \leq k \leq |W_j|, \forall (i,j) \in E
\end{align*}
\]
CEAP: Current state

- Completed
  - Implementation of a MIP solver for the CEAP based on our LP mapping.
  - Extension of RadPro to provide a decentralised solver for the acyclic CEAP. This hinges on a valuation algebra whose valuations are piecewise linear functions.

- Ongoing
  - Empirical evaluation of both the centralised and decentralised solvers for CEAP.
Summary of contributions (I)

- **Energy Allocation Problem**
  - EAP is formulated as a **DCOP**

- **Valuation algebra**
  - Used to implement **messages** and **offers**
  - **Efficient** operations (aggregation, negation, selection)

- **RadPro algorithm**
  - Based on dynamic programming to solve **acyclic** EAP
  - **Outperforms** classical and DP-based solvers
  - Based on the valuation algebra for **polynomial message computation**
Summary of contributions (II)

Continuous Energy Allocation Problem

- CEAP offers more expressiveness to prosumers: offers as piecewise linear functions
- CEAP can be cast as a Linear Program and hence solved by commercial solvers like CPLEX or Gurobi.
- CEAP’s decentralised solver as an extension of RadPro.
Perspectives

- **RADPro’s next steps**
  1. Cope with **cyclic** networks
  2. Mechanism design issues (VCG payments are feasible!)
References (contd)


Solving EAP through MIP

maximize

\[ \sum_{j=1}^{\left| P \right|} \sum_{l=1}^{n_{oj}} x_{j}^{l} \cdot o_{j} \]

subject to

\[ \sum_{l=1}^{n_{oj}} x_{j}^{l} = 1 \quad \forall j \in P, 1 \leq l \leq n_{oj} \]

\[ \sum_{i<j} y_{ij} - \sum_{k>j} y_{jk} = \sum_{l=1}^{n_{oj}} x_{j}^{l} \cdot q_{j} \quad \forall j \in P \]

\[ y_{ij} \in D_{ij} \quad \forall (i, j) \in E \]

\[ x_{j}^{l} \in \{0, 1\} \quad \forall j \in P, 1 \leq l \leq n_{oj} \]

\[ x_{j}^{l} : j \text{ sells } l \text{ units} \]

\[ n_{oj} : \text{maximum number of units in the offer} \]

\[ y_{ij} : \text{number of units exchanged between } i \text{ and } j \]

\[ D_{ij} = [-c_{ij}, c_{ij}] : \text{domain of } c_{ij} \]