Defining a Continuous Marketplace for the Trading and Distribution of Energy in the Smart Grid

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Smart grids: promises & expected outcomes

- New distribution rationale: **decentralized production**
  - Democratization of decentralized production: local balancing and reducing energy loss
- New context information: **energy awareness**
  - Frequently sensed data (consumption, production, pricing) impacts trading updates
- New trading rationale: **prosumption**

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How to design a decentralized market for the trading and distribution of energy?

Example: energy trading scenario

- *Prosumers* ($j \in P$)

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- *Offers* ($o_j : \mathbb{Z} \rightarrow \mathbb{R} \cup \{-\infty\}$)

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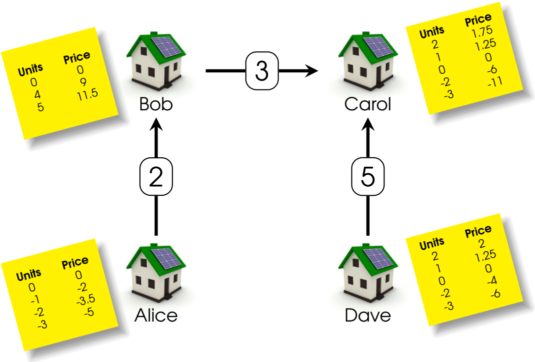
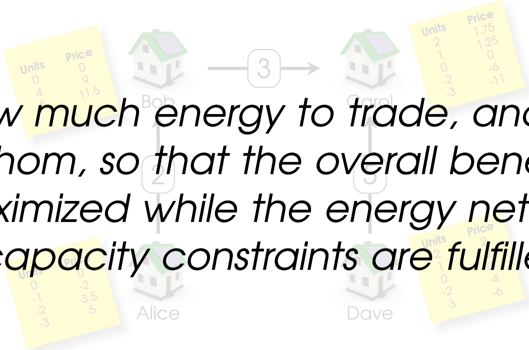


- *Prosumers* ($j \in P$)
- *Offers* ($o_j : \mathbb{Z} \rightarrow \mathbb{R} \cup \{-\infty\}$)
- *Links* ($\{i, j\}$) w/ some max capacity (c_{ij})

Example: energy trading scenario



How much energy to trade, and with whom, so that the overall benefit is maximized while the energy network's capacity constraints are fulfilled?

Definition: energy allocation problem

The energy allocation problem (*EAP*) amounts to finding an allocation $\mathbf{Y}$ that maximizes the overall benefit $Value(\mathbf{Y})$, with

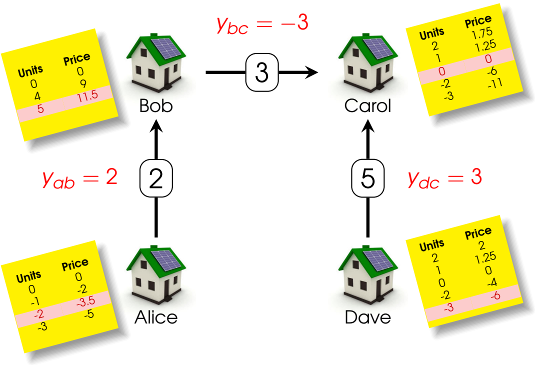
$$Value(\mathbf{Y}) = \sum_{j \in P} v_j(\mathbf{Y}_j)$$

$$v_j(\mathbf{Y}_j) = o_j(net(\mathbf{Y}_j))$$

$$net(\mathbf{Y}_j) = \sum_{i \in in(j)} \mathbf{y}_{ij} - \sum_{k \in out(j)} \mathbf{y}_{jk}$$

where $\mathbf{y}_{ij}$ stands for the number of units that prosumer i sells to prosumer j (bounded by c_{ij})

Example: energy trading scenario (solution)



$$Value(\mathbf{Y}) = o^{-2}_{Alice} + o^{5}_{Bob} + o^{0}_{Carol} + o^{-3}_{Dave} = -3.5 + 11.5 + 0 - 6 = 2$$

Distributed allocation techniques

- Market-based
  - **Double auction** (call market or CDA) where energy is traded on a day-ahead basis
  - Matching between supply and demand computed by **central** authority
  - Current market mechanisms disregard grid constraints → **Trading and distribution as decoupled activities**
- Message passing
  - Dynamic programming (MILLER, 2014; KUMAR et al., 2009)
  - Belief-propagation (MILLER, 2014)

Our contribution

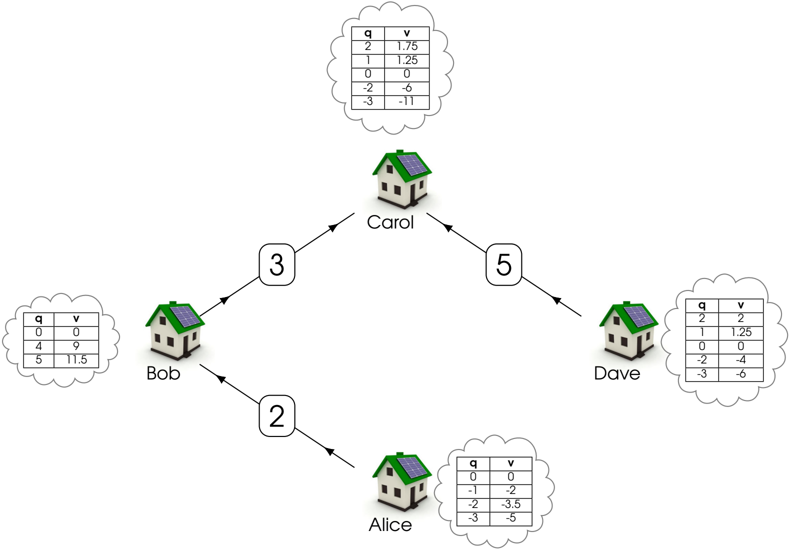
- Exploit the **tree** structure of energy networks (GONEN, 2014)
- Solve EAP as a distributed contraint optimization problem (**DCOP**)
- Design an **exact message passing** algorithm based on dynamic programming
  - ACYCLIC-SOLVING (DECHTER, 2003)
- Assess efficiently messages by exploiting the algebraic structure of offers and messages : **valuations**

Message passing solution

q	v
2	1.75
1	1.25
0	0
-2	-6
-3	-11

Carol

Message passing solution



Carol:

q	v
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Bob:

q	v
0	0
4	9
5	11.5

Dave:

q	v
2	2
1	1.25
0	0
-2	-4
-3	-6

Alice:

q	v
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-1	-2
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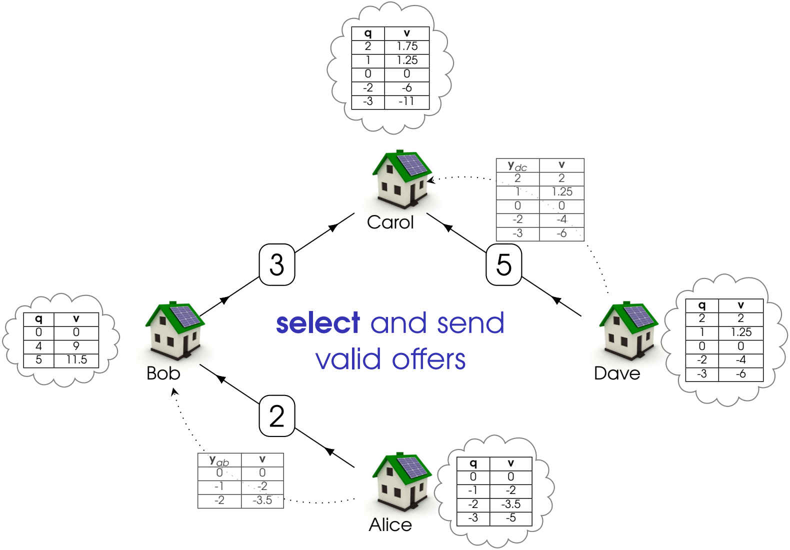
Message passing solution



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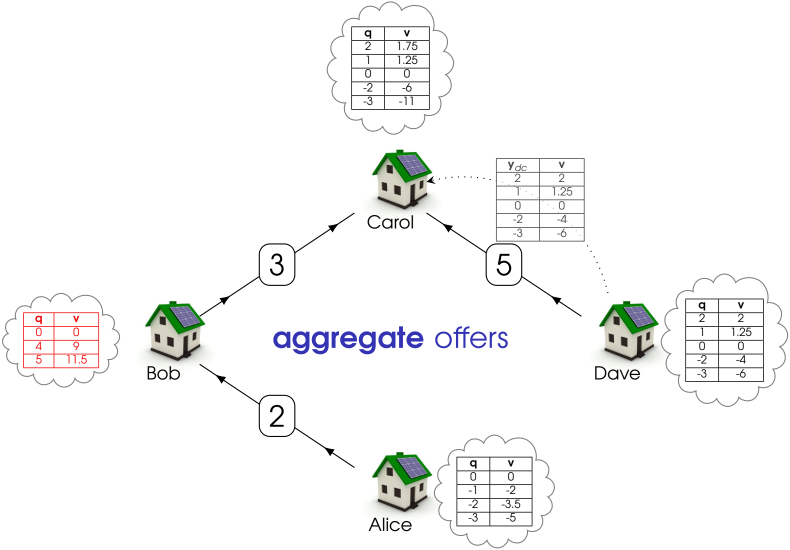

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Message 5:

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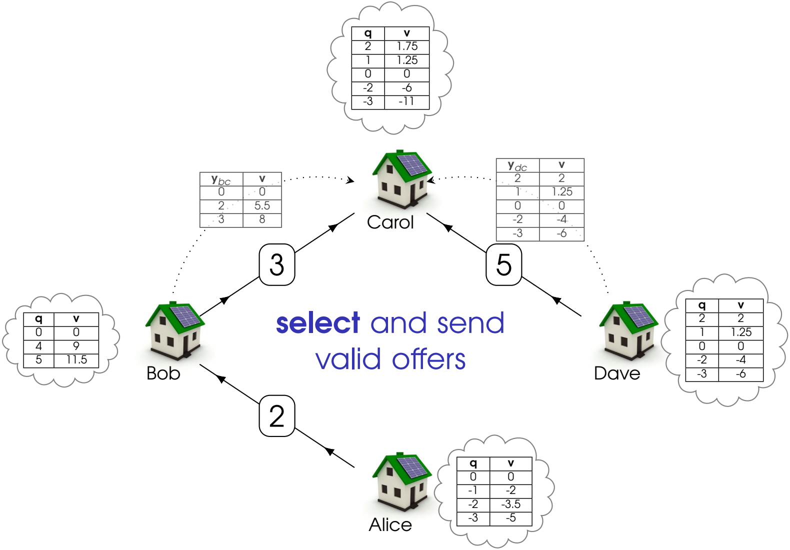

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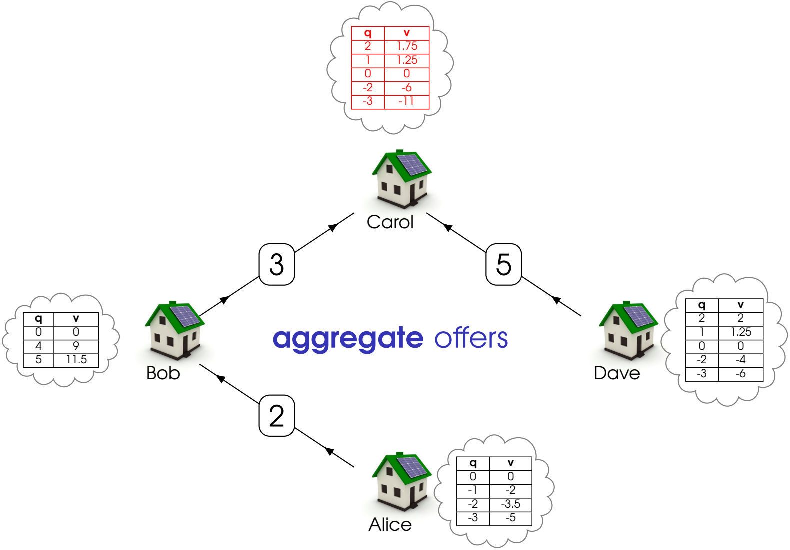

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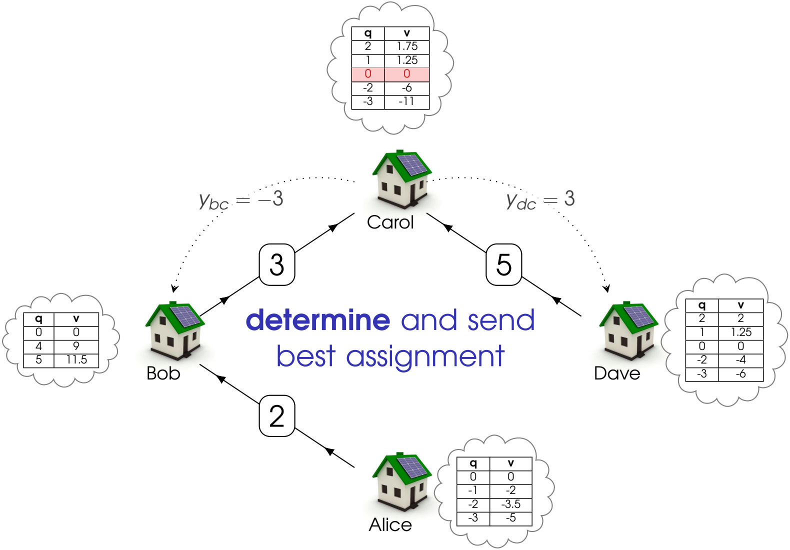

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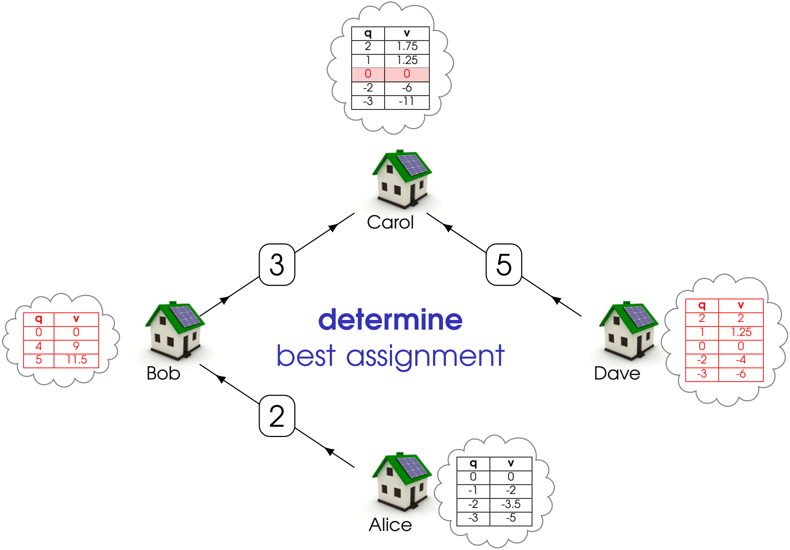

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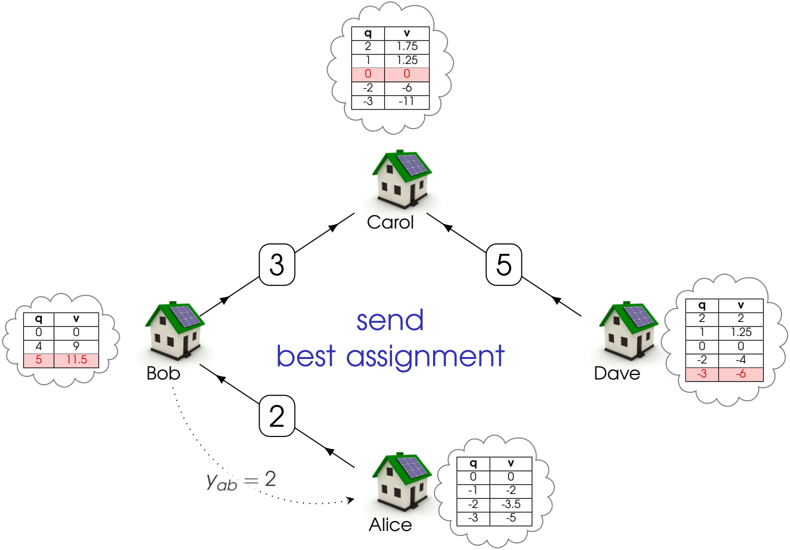

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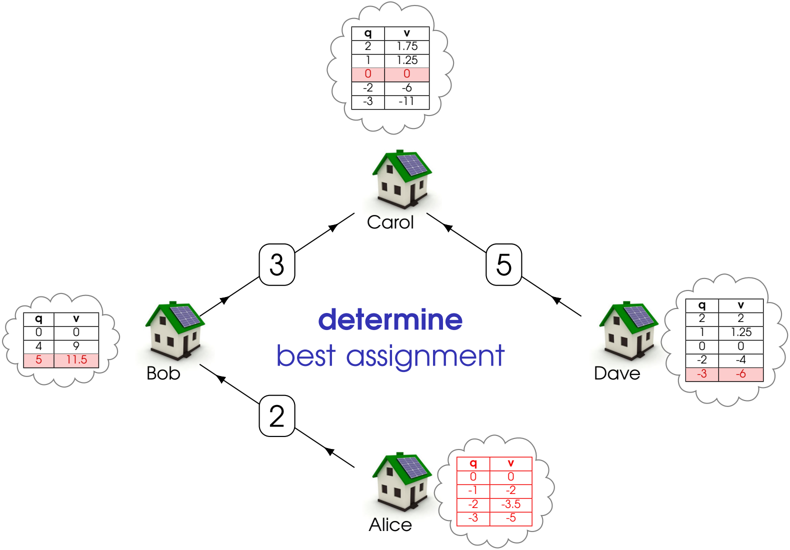

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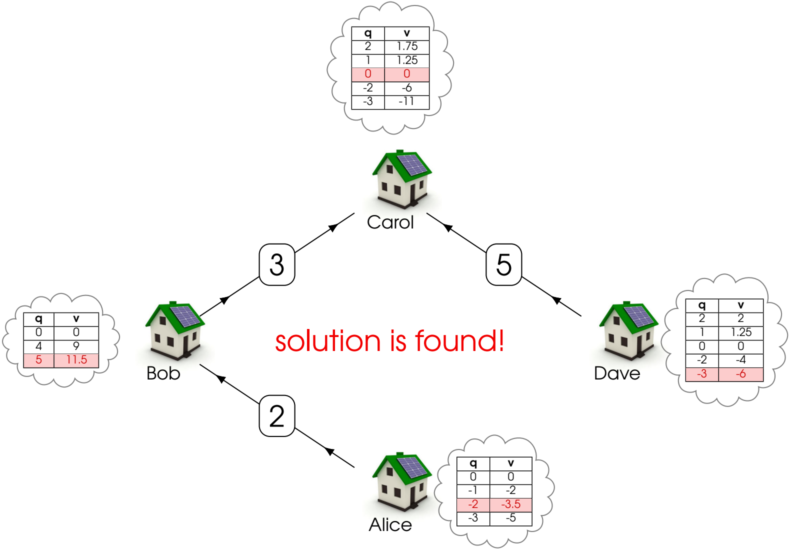

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Message assessment

$$\mu_{j\to p_j}(\mathbf{y}_{jp_j}) =$$
$$\max_{\mathbf{Y}_{j-p_j}} \left(v_j(\mathbf{y}_{jp_j}, \mathbf{Y}_{j-p_j}) + \sum_{k \in out(j) \setminus \{p_j\}} \mu_{j\to k}(\mathbf{y}_{jk}) + \sum_{i \in in(j)} \mu_{i\to j}(\mathbf{y}_{ij}) \right)$$

- This is the computational hard point
- Computed in $\mathcal{O}((2C_j + 1)^{N_j})$
  - C_j is the capacity of the most powerful link
  - N_j is the number of neighbors of j

$\Rightarrow$ Not applicable to dense networks

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$\Rightarrow$ Not applicable to dense networks

$\Rightarrow$ Assess message more efficiently!

Algebra of valuations

- Take advantage of a particularity of the messages: **restricted capacity**
- Reformulate message assessment with 3 operations:
  - Restriction (linear): $\alpha[D](k) = \begin{cases} \alpha(k) & k \in D \\ -\infty & \text{otherwise} \end{cases}$
  - Complement (linear): $\overline{\alpha}(k) = \alpha(-k)$
  - Aggregation (polynomial): $(\alpha \cdot \beta)(k) = \max_{\substack{i,j \\ k=i+j}} \alpha(i) + \beta(j)$

$$\mu_{j \to p_j} = \left(\overline{o_j} \cdot \prod_{k \in out(j) \setminus \{p_j\}} \overline{\mu_{j \to k}} \cdot \prod_{i \in in(j)} \mu_{i \to j} \right) [-D_{jp_j}]$$

RadPro algorithm

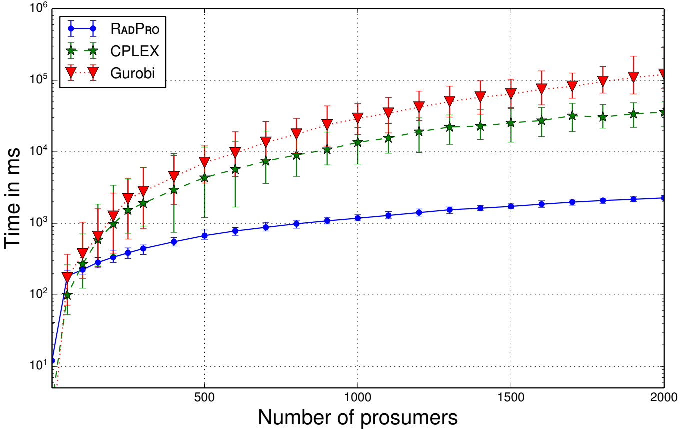
= Acyclic-Solving + efficient message assessment

- Global complexity of message assessment: **polynomial** in $\mathcal{O}(nN_{max}^2 C_{max}^2)$
  - Number of message assessments in $\mathcal{O}(n(2C_{max}+1)^{N_{max}})$
  - Single message assessment in $\mathcal{O}(N_j C_j n_{o_j} + N_j^2 C_j^2)$
- Communication complexity: **linear** in $\mathcal{O}(nC_{max})$
  - $2n$ messages of max size $2C_{max}+1$
- Easily distributable

	Acyclic-Solving	RadPro
Communication	$\mathcal{O}(nC_{max})$	$\mathcal{O}(nC_{max})$
Computation	$\mathcal{O}(n(2C_{max}+1)^{N_{max}})$	$\mathcal{O}(nN_{max}^2 C_{max}^2)$

Comparison with MIP solvers

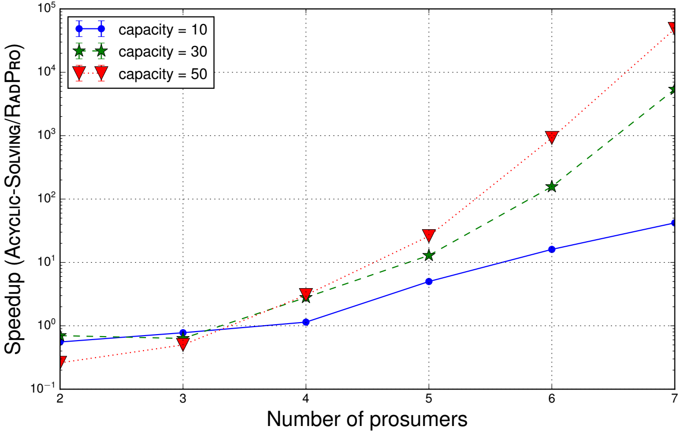
RadPro outperforms CPLEX & Gurobi
(more than one order of magnitude faster than CPLEX!)



Random networks (geometric distribution) with $C_j = \mathcal{N}(100, 50)$

Efficiency of message assessment

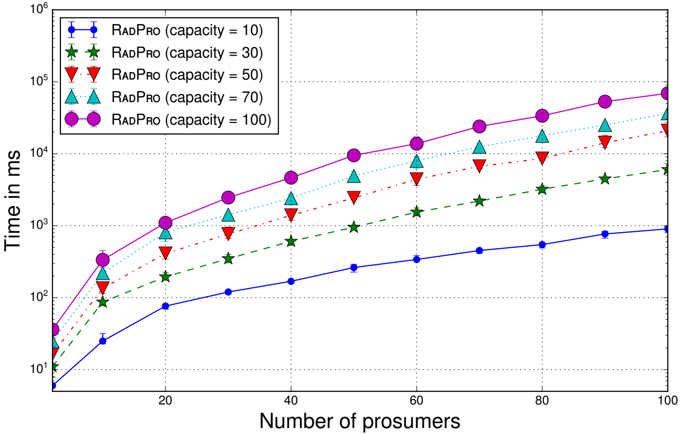
RADPRO outperforms ACYCLIC-SOLVING
(more than three orders of magnitude faster than ACYCLIC-SOLVING!)



Star-shaped networks (hubs) with $C_j \in \{10, 30, 50\}$ and $N_{max} \in [1..6]$

Scalabity

RadoPro solves large-scale EAP with high branching factor
(solving problems with capacity 100 and 100 neighbors in less than 1 min!)

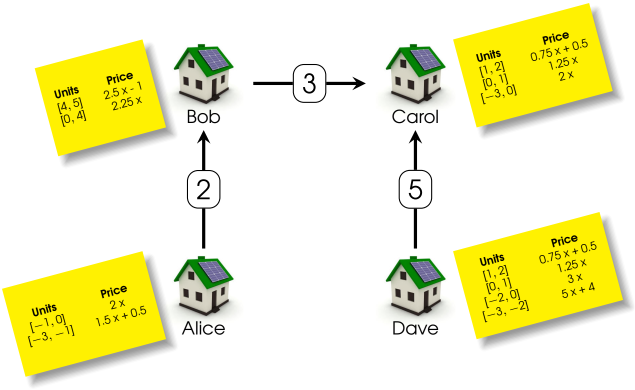


Star-shaped networks (hubs) with $C_j \in \{10, 30, 50, 70, 100\}$ and $N_{max} \in [1..99]$

RadPro Limitations

- Offers are discrete: a prosumer can offer to buy either 3 KW for 6 EUR or 2 KW for 4 EUR, but not any amount of energy between 2 KW and 3 KW and pay 2 EUR per KW.
- Such offers provide a better representation of prosumers' preferences.
- **Our next goal is to extend the EAP to allow prosumers to communicate continuous (piecewise linear) utility functions.**

Example: continuous energy trading scenario



- *Prosumers* ($j \in P$)
- *Offers* as **piecewise linear valuations**
- *Links* ($\{i, j\}$) w/ some max capacity (c_{ij})

Definition: continuous energy allocation problem

Given a set of prosumers P whose offers are piecewise linear valuations, the **continuous energy allocation problem** (*CEAP*) amounts to finding an allocation $\mathbf{Y}$ that maximizes the overall benefit $Value(\mathbf{Y})$, with

$$Value(\mathbf{Y}) = \sum_{i \in P} v_j(\mathbf{Y}_j)$$

$$v_j(\mathbf{Y}_j) = o_j(net(\mathbf{Y}_j))$$

$$net(\mathbf{Y}_j) = \sum_{i \in in(j)} \mathbf{y}_{ij} - \sum_{k \in out(j)} \mathbf{y}_{jk}$$

where $\mathbf{y}_{ij}$ stands for the number of units that prosumer i sells to prosumer j (bounded by c_{ij})

Mapping the CEAP to a Linear Program

Decision variables per prosumer:

- interval valuation to select within a piecewise linear valuation
- amount of energy to trade within the chosen interval

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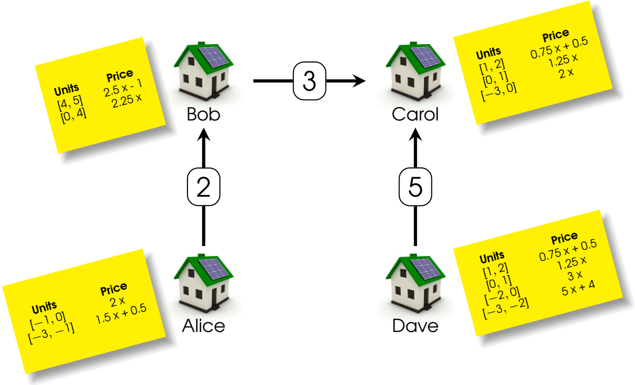
Decision variable per link:

- amount of energy to trade between prosumers

Constraints:

- *Mutually exclusive intervals:* only an interval valuation per piecewise linear valuation
- *Energy balance:* amount of energy to trade per prosumer equals difference between input and output energy
- *Network capacity:* energy traded between prosumers respects links' capacities

Example: continuous energy trading scenario



- *Prosumers* ($j \in P$)
- *Offers* as **piecewise linear valuations**
- *Links* ($\{i, j\}$) w/ some max capacity (c_{ij})

Mapping the CEAP to a Linear Program

LP that solves the continuous energy allocation problem:

$$\textbf{maximize} \quad \sum_{j=1}^{|P|} \sum_{k=1}^{|W_j|} a_{o_j^k} \cdot x_j^k + b_{o_j^k} \cdot z_j^k$$

$$\textbf{subject to} \quad z_j^k \cdot l_{o_j^k} \leq x_j^k \leq z_j^k \cdot u_{o_j^k}$$

$$\sum_{l=k}^{|W_j|} z_j^k = 1$$

$$\sum_{i<j} y_{ij} - \sum_{q>j} y_{jq} = \sum_{k=1}^{|W_j|} x_j^k$$

$$y_{ij} \in D_{ij}$$

$$z_j^k \in \{0, 1\}$$

$$x_j^k \in \mathbb{R}$$

$$\forall j \in P, 1 \leq k \leq |W_j|, \forall (i, j) \in E$$

CEAP: Current state

- Completed
  - Implementation of a MIP solver for the CEAP based on our LP mapping.
  - Extension of RadPro to provide a decentralised solver for the acyclic CEAP. This hinges on a valuation algebra whose valuations are piecewise linear functions.
- Ongoing
  - Empirical evaluation of both the centralised and decentralised solvers for CEAP.

Summary of contributions (I)

- **Energy Allocation Problem**
  - EAP is formulated as a **DCOP**
- **Valuation algebra**
  - Used to implement **messages** and **offers**
  - **Efficient** operations (agregation, negation, selection)
- **RADPRO algorithm**
  - Based on dynamic programming to solve **acyclic** EAP
  - **Outperforms** classical and DP-based solvers
  - Based on the valuation algebra for **polynomial message computation**

Summary of contributions (II)

- **Continuous Energy Allocation Problem**
  - CEAP offers more expressiveness to prosumers: **offers as piecewise linear functions**
  - **CEAP can be cast as a Linear Program** and hence solved by commercial solvers like CPLEX or Gurobi.
  - **CEAP's decentralised solver as an extension of RadPro.**

Perspectives

- **RADPRO's next steps**
  1. Cope with **cyclic** networks
  2. Mechanism design issues (VCG paymens are feasible!)

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Solving EAP through MIP

$$
\begin{array}{lll}
\textbf{maximize} & \sum_{j=1}^{|P|} \sum_{l=1}^{n_{o_j}} x_j^l \cdot o_j^l & \\
\textbf{subject to} & \sum_{l=1}^{n_{o_j}} x_j^l = 1 & \forall j \in P, 1 \leq l \leq n_{o_j} \\
& \sum_{i<j} y_{ij} - \sum_{k>j} y_{jk} = \sum_{l=1}^{n_{o_j}} x_j^l \cdot q_j^l & \forall j \in P \\
& y_{ij} \in D_{ij} & \forall (i,j) \in E \\
& x_j^l \in \{0,1\} & \forall j \in P, 1 \leq l \leq n_{o_j}
\end{array}
$$

x_j^l : j sells l units

n_{o_j} : maximum number of units in the offer

y_{ij} : number of units exchanged between i and j

$D_{ij} = [-c_{ij} .. c_{ij}]$: domain of c_{ij}