Defining a Continuous Marketplace for the Trading and Distribution of Energy in the Smart Grid

Jesús Cerquides[1], Gauthier Picard[2], and Juan A. Rodríguez-Aguilar[1]

[1] IIIA-CSIC, Campus UAB, 08193 Cerdanyola, Catalonia, Spain
{cerquide,jar}@iiia.csic.es
[2] Laboratoire Hubert Curien UMR CNRS 5516
Institut Henri Fayol, Mines Saint-Etienne, France
picard@emse.fr

Abstract. Decentralized energy production is meant to reduce generation and distribution inefficiencies, leading to major economic and environmental benefits. This new model is meant to be supported by smart grids, electricity networks that can intelligently integrate the actions of all users connected to them —generators, consumers, and prosumers (those that do both)— to efficiently deliver sustainable, economic and secure electricity supplies. A major research challenge is the design of markets for prosumers in smart grids that consider distribution grid constraints. Recently, a discrete market model has been presented that allows prosumers to trade electricity while satisfying the constraints of the grid. However, most of the times energy flow problems possess a continuous nature, and that discrete market model can only provide approximate solutions. In this paper we extend the market model to deal with continuous (piecewise linear) utility functions. We also provide a mapping that shows that the clearing of such a market can be done by means of integer linear programming.

Keywords: smart grid; energy market; prosumers; mixed integer programming

1 Introduction

Our centralized model of production and transmission wastes enormous amounts of energy. According to [6], "...an astonishing two-thirds of primary energy inputs". Since power stations are generally far from centers of demand, much of the produced heat is not used, but vented up chimneys or discharged to rivers. Additional losses come about as the electricity travels along the wires of the transmission and distribution systems [6,23]. As argued in [23], favoring the decentralized generation of energy over traditional centralized electricity generation will reduce generation and distribution inefficiencies and will facilitate increased contributions from renewables. This new model is meant to be supported by smart grids.

Following [3], a smart grid is an electricity network that can intelligently integrate the actions of all users connected to it —generators, consumers, and *prosumers* (those that do both)— to efficiently deliver sustainable, economic and secure electricity supplies. In the smart grid the consumer can be either an individual or a household, but also a community or an SME. In its more general form, a smart grid is populated by prosumers capable of both generating and consuming energy. Therefore, smart grids clearly play the central role in the integration of all these prosumers (electricity grid users) by means of the enactment of a system that satisfies a number of societal goals. Out of these goals, there is that of setting market-based prices for electricity taking into account grid system constraints. Thus, a major research challenge in the heart of several roadmaps for the Smart Grid [3,4] is the design of markets for prosumers in smart grids that consider distribution grid constraints. This vision will allow prosumers to directly trade over the smart grid [8]. Following [18], market operations will involve a large number of heterogeneous prosumers, distributed throughout the network (closer to the point of use of electricity), and trading much smaller amounts of energy that are nowadays traded. The distribution of electricity employs one of the three common types of network topologies: radial, ring main, and interconnected [5,7,21]. On the one hand, radial networks are acyclic. On the other hand, as observed in [13], though ring main and interconnected networks contain cycles, they are configured into acyclic networks by means of switches to supply power [7,21].

The smart grid vision has spurred a wealth of research on the design of markets and trading agents for the smart grid. The state-of-the-art has mainly considered to employ different types of auctions for this endeavor. Thus, the market-based trading of energy is typically addressed by the literature by having prosumers participate in a double auction where energy is traded on a day-ahead basis [8,9,10,14,16,20]. Submitted buy and sell orders for energy are matched either by means of either a continuous double auction [10,16,20] or a call market [8,9,14]. Exceptions to this common approach are represented by the tailored multi-unit auctions in [22] and the simultaneous combinatorial reverse auctions employed in [17] to match demand and supply.

In [1], the limitation of the market mechanisms employed in the literature are identified, noticing that up to then, no mechanism takes into account grid system constraints. Thus, the clearing of the market occurs disregarding, for instance, that the transmission of energy is carried out along capacity-constrained distribution networks (which is an actual-world constraint [21]). Therefore, trading and distribution are considered as decoupled activities. Furthermore, the bidding language offered to grid users is pointed out to be not expressive enough to express a prosumer's energy profile since with the exception of [17], which supports combinatorial bids, double auctions limit a grid user to submit a single price-quantity bid to either buy or sell. This does not allow a prosumer to express a full energy profile encompassing a combination of all her buy and sell offers.

As a consequence of this analysis they introduce the Energy Allocation Problem (EAP) as the problem of deciding how much energy each prosumer trades as well as how energy must be distributed throughout the grid so that the overall benefit is maximized while complying with the grid constraints and the prosumers' preferences. On the one hand, they consider that the capacity of the distribution network is limited [21]. On the other hand, since a prosumer can both generate and consume energy, their formulation considers that each prosumer can encode her preferences as a combination of offers to both buy and sell energy. Solving the EAP amounts to clearing a prosumer-oriented market. However, in the EAP, prosumers are limited to bid for discrete amounts of energy. That is, a prosumer can offer to buy either 3 KW for 6c€ or 2 KW for 4c€, but it is not allowed to express that he will buy any amount of energy between 2 KW and 3 KW and that he will be willing to pay 2c€ per KW. In many energy settings, such offers make complete sense and provide a better representation of the prosumer interests when approaching the market. Thus, in this paper we make headway towards the application of these models by extending the EAP so that it allows prosumers to communicate continuous (piecewise linear) utility functions.

More precisely, we make the following contributions:

- We extend the Energy Allocation Problem (EAP) into the continuous energy allocation problem (CEAP). It turns out that the extension is not trivial and requires some mathematical development. We provide some of the results required to deal with piecewise linear functions to represent prosumer preferences.
- We show how to encode the CEAP as a mixed-integer program so that it can be optimally solved for any distribution network topology by means of off-the-shelf commercial solvers such as CPLEX or Gurobi.
- Finally, since the CEAP defines the allocation rule of our market, we also touch upon the design of payment rules that together with our allocation rule can help design a mechanism for our prosumer-oriented market.

The rest of the paper is organized as follows. Section 2 formally defines the allocation rule that we propose to clear prosumer-oriented electricity markets with piecewise linear valuation functions. Thereafter, section 3 shows how to implement the clearing of the market as a mixed-integer program (MIP). Next, section 4 touches upon how to cope with prosumers' strategic behavior, and section 5 concludes and sets paths to future research.

2 The energy allocation problem

The aim of this section is to provide a simple mathematical model for the energy market in a prosumer network, and the allocation rule proposed for that market. We start by providing an example of an energy trading scenario that illustrates the model of prosumers and the model of energy network that we will consider. Thereafter, we provide the allocation rule for that market as the solution to an optimization problem: the continuous energy allocation problem (CEAP).

2.1 Example: energy trading scenario

Figure 1 shows an example of an energy trading scenario involving four prosumers, each one represented by a circle. Each edge connecting two prosumers means that they are physically connected. Moreover, each link is labeled with its capacity, namely with the amount of energy it can transport. For instance, prosumer 1 is connected to prosumer 2, and their link can transport up to 2 energy units. Each prosumer can offer to either buy, sell or transmit energy. The offer of each prosumer is represented as a table next to each prosumer in Figure 1, where each entry in the table represents contains the range of

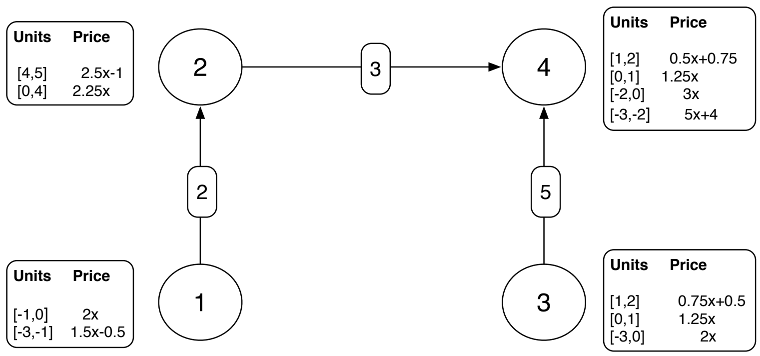

Fig. 1. Energy trading scenario.

energy units to which it applies and the linear function used to obtain the price provided that the prosumer is required to provide a number of energy units in that range. As a convention, a selling offer is expressed by means of a negative number of units, whereas a buying offer is encoded with a positive number of units. For instance, prosumer 4's first entry communicates that, if as a result of clearing the market, he is provided an amount of energy e between 1 KW and 2 KW, he will pay $(0.5 \cdot e + 0.75)$c€. That is if he is provided 1.5KW, he will pay 1.5c€. On the other hand, its last entry states that if he is requested to provide an amount of energy e between 2 and 3 KW, he will be paid $(5 \cdot e - 4)$ c€ (note that the sign is reversed from the expression in the table because we are encoding sell offers with negative numbers). Finally note that, by applying its third valuation, he shows his willingness to transmit energy for free (he will be happy to receive 0KW at price 0c€). In Figure 1, we observe that prosumer 1 only sells energy, and prosumer 2 only buys energy, while prosumers 3 and 4 can either buy or sell.

2.2 Problem definition

Now the problem faced by the prosumers in Figure 1 is to decide how much energy to trade and with whom so that the overall benefit (social welfare) is maximized while the energy network's capacity constraints are fulfilled. This means that: (i) each prosumer must select how much to trade; and (ii) each pair of prosumers connected by a link must agree on the amount of energy to be transferred by their link together with the direction of the transfer (with whom). In what follows we cast this problem as an optimization problem, and we put off the solution to this problem to sections 3.

Following example 1, we consider that the energy network connecting a set of prosumers P can be modeled as an undirected graph (P, E), where the vertexes stand for the prosumers and each edge in E connects a pair of prosumers. An edge $\{i, j\} \in E$ means that prosumer i and j are physically connected to trade energy. When $\{i, j\} \in E$, $i < j$ we say that i is an in-neighbor of j and that j is an out-neighbor of i. The set of in-neighbors (resp. out-neighbors) of j is $in(j)$ (resp. $out(j)$).

Each prosumer j expresses her offers to buy and sell energy by means of an *general valuation function* $o_j : \mathbb{R} \to \mathbb{R} \cup \{-\infty\}$. For instance, $o_j(3) = 2$ indicates that prosumer j is willing to buy 3 energy units at 2c€, while $o_j(-4) = -2$ indicates that she is willing to sell 4 energy units if paid 2c€. Notice that offer functions capture prosumers' constraints. To communicate her offer function, each prosumer sends a table like the ones in Figure 1 making explicit her feasible energy states and their values. Given a number of units x, if x does not belong to the interval of any of the entries in the table, it means that such energy state is unfeasible for the prosumer and thus its value $o_j(x)$ is $-\infty$. If x appears in more that one interval, then its $o_j(x)$ is the maximum among the values assigned for each of the entries in the table in which it is contained.

In the following we define formally the mathematical foundations that underlie piecewise linear valuations.

Definition 1. *A* general valuation *is any function* $\alpha : \mathbb{R} \to \mathbb{R} \cup \{-\infty\}$*. We use* F_α *to note the subset of* $\mathbb{R}$ *in which* α *takes finite values, that is* $F_\alpha = \alpha^{-1}(\mathbb{R})$*. We define the zero valuation* $\mathbf{0}$ *as the function that maps every real number to* $-\infty$*. That is, for all* $x \in mathbbR$ *we have that* $\mathbf{0}(x) = -\infty$*. We define the unit valuation* $\mathbf{1}$ *as the one that maps* 0 *to* 0 *and any other element to* $-\infty$*. That is,* $\mathbf{1}(x) = \begin{cases} 0 & \textit{if } x = 0 \\ -\infty & \textit{otherwise} \end{cases}$

Let $W = \{\omega_1, \ldots, \omega_n\}$ *be a finite set of general valuations. We define* F_W *as the set of values where at least one of the valuations in* W *takes a finite value. That is,* $F_W = \bigcup_{i=1}^n F_{\omega_i}$*.*

Furthermore, we can define the maximum valuation $\beta = \max W$ *as*

$$\beta(x) = (\max W)(x) = \begin{cases} \max_{1 \le i \le n} \{\omega_i(x) | x \in F_{\omega_i}\} & \textit{if } x \in F_W \\ -\infty & \textit{otherwise.} \end{cases} \tag{1}$$

Definition 2 (Point Valuation). *A general valuation* α *is a* point valuation *if and only if* F_α *contains a single element. We can always represent a point valuation by an ordered pair* $(p, q) \in \mathbb{R}^2$*, such that*

$$\alpha(x) = \begin{cases} q & \textit{if } x = p \\ -\infty & \textit{otherwise.} \end{cases} \tag{2}$$

Note that the unit valuation is a point valuation represented by the ordered pair $(0, 0)$*.*

Definition 3 (Linear Interval Valuation). *A real interval is a subset of real numbers* $[l, u] = \{x \in \mathbb{R} \mid l \le x \le u\}$*. A general valuation* α *is a* linear interval valuation *if and only if there is a real interval* $I_\alpha = [l_\alpha, u_\alpha]$*, and two real numbers* a_α*,* b_α*, such that for each* $x \in \mathbb{R}$

$$\alpha(x) = \begin{cases} a_\alpha \cdot x + b_\alpha & \textit{if } x \in I_\alpha \\ -\infty & \textit{otherwise} \end{cases} \tag{3}$$

We say that the ordered tuple $(l_\alpha, u_\alpha, a_\alpha, b_\alpha) \in \mathbb{R}^4$ *is a representation of* α*.*

Lemma 1. *Any point valuation is a linear interval valuation*

Proof. Let (p, q) be the representation of a point valuation α. Then, $(p, p, 0, q)$ is a representation of α as interval lineal valuation.

Definition 4 (Discrete Valuation). *A general valuation α is a* discrete valuation *if and only there exists a finite set of point valuations $W = \{\omega_1, \dots, \omega_n\}$, such that $\alpha = \max W$. That is, for each $x \in \mathbb{R}$, we have that $\alpha(x) = (\max W)(x)$.*

Definition 5. *A general valuation α is* piecewise linear *if and only there exists a finite set of linear valuations $W = \{\omega_1, \dots, \omega_n\}$, such that $\alpha = \max W$.*

In that case we say that W is a piecewise linear representation *of α of size n. Note that $F_\alpha = \bigcup_{i=1}^{n} I_{\omega_i}$.*

Lemma 2. *Any discrete valuation is piecewise linear.*

Proof. Directly from the definitions of discrete and piecewise linear valuation and Lemma 1.

Note that this means that piecewise linear valuations are a more general framework than that used in [1]. Thus, any algorithm or problem definition that assumes piecewise linear valuations will in particular be capable of working with discrete valuations. Next, we provi

Lemma 3. *Each piecewise linear valuation admits a representation $W = \{\omega_1, \dots, \omega_n\}$ in which*

1. *For each two linear interval valuations ω_i and ω_j, we have that $|F_{\omega_i} \cap F_{\omega_j}| \leq 1$. That is, the finite domains of ω_i and ω_j are either disjoint or share a single point.*
2. *There is no point shared by more than 3 linear interval valuations.*
3. *For each $1 \leq i < n$ we have that $u_{\omega_i} \leq l_{\omega_{i+1}}$.*

We call such a representation a canonical representation*.*

Proof. The proof proceeds constructively. It is relatively simple to build an algorithm that builds the canonical representation of the maximum of two valuations given their canonical representations. On the other hand, for any interval lineal valuation, its canonical representation is direct. Thus given a representation which is not canonical, the canonical representation can always be built by taking the canonical representations of the interval lineal valuations in the representation and then successively taking maximums between them until we have assessed the maximum of all the interval linear valuations in the representation.

Our fundamental assumption in this work is that prosumers' offers are piecewise linear valuations. Hence, in the remaining of the paper when we refer to a *valuation* we will always mean a piecewise linear valuation.

Besides prosumers' offers, we also consider that the energy network is physically constrained by the capacity of the connections between prosumers. We will note as c_{ij} the capacity limit of edge $\{i, j\}$, namely the maximum number of energy units that the link between prosumers i and j can transmit. An allocation specifies the number of units that each prosumer trades with each neighboring prosumer. We will encode an allocation by means of a set of variables $Y = \{y_{ij} \mid i \in P, j \in out(i)\}$, where $\mathbf{y}_{ij}$ stands for the number of units that prosumer i sells to prosumer j and is bounded by the capacity limit c_{ij}. That is, the domain of variable y_{ij} is $D_{ij} = [-c_{ij} .. c_{ij}]$. Thus, if $\mathbf{y}_{ij}$ takes on a value k greater than 0, it means that prosumer i sells k energy units to prosumer j. Otherwise, if $\mathbf{y}_{ij}$ takes on a negative value $-k$, we say that prosumer i buys k energy units from prosumer j. From this follows that $\mathbf{y}_{ij}$ represents a trade from prosumer i's perspective.

Now we want to assess the value of a given allocation. Before that, we will define the local value of a given allocation for a single prosumer. We need to assess the amount of energy that a prosumer acquires and sells according to an allocation $\mathbf{Y}$. Prosumer j will only consider its local view of the allocation, represented by $\mathbf{Y}_j = \mathbf{y}_{\cdot j} \cup \mathbf{y}_{j\cdot}$. We can assess the *net energy balance* for prosumer j as

$$net(\mathbf{Y}_j) = \sum_{i \in in(j)} \mathbf{y}_{ij} - \sum_{k \in out(j)} \mathbf{y}_{jk}, \tag{4}$$

where each $\mathbf{y}_{ij}$ and $\mathbf{y}_{jk}$ are added with different signs because j takes the role of buyer in $\mathbf{y}_{ij}$ and that of seller in $\mathbf{y}_{jk}$. And therefore, the local value v_j of an allocation $\mathbf{Y}$ for prosumer j can be assessed as the value of her net energy balance by means of her offer function

$$v_j(\mathbf{Y}_j) = o_j(net(\mathbf{Y}_j)). \tag{5}$$

Therefore, the value of an allocation $\mathbf{Y}$ can be obtained by adding up the local value of the allocations for each prosumer.

$$Value(\mathbf{Y}) = \sum_{i \in P} v_j(\mathbf{Y}_j). \tag{6}$$

Now, we are ready to define the energy trading allocation as that of finding the allocation of maximum value that satisfies the capacity of the energy network.

Problem 1. Given a set of prosumers P, a canonical representation of their offers $\{o_j | j \in P\}$, and an undirected graph E where each edge is labeled with its capacity c_{ij}, the continuous energy allocation problem (CEAP) amounts to finding an allocation $\mathbf{Y}$ that maximizes $Value(\mathbf{Y})$. Whenever the graph E is acyclic we say that the CEAP is acyclic.

At this point we can consider again the example in Figure 1. When solving the CEAP defined by Problem 1, we obtain the variable assignment shown in Figure 2. The solution indicates that prosumer 1 transfers 2 energy units to prosumer 2 ($y_{12} = 2$), prosumer 2 also receives 3 energy units from prosumer 4 ($y_{24} = -3$), and prosumer 3 transfers 3 energy units to prosumer 4. Next to each offer table, we show the amount of energy $\mathbf{x_i}$ that each prosumer is provided (if $\mathbf{x_i}$ is positive) or requested (when $\mathbf{x_i}$ is negative). This corresponds to the net energy balance (Equation 4). The value of the offer of each prosumer in its energy balance state is added to assess the net value of the allocation (see Equation 5). Thus, the allocation that maximizes Equation 6 has a value of 2.

Notice that prosumer 2 obtains 5 energy units by *aggregating* the energy units received from prosumers 1 and 4. However, prosumer 4 does not sell anything to prosumer 2. The role of prosumer 4 is to *relay* to prosumer 2 the energy transferred from

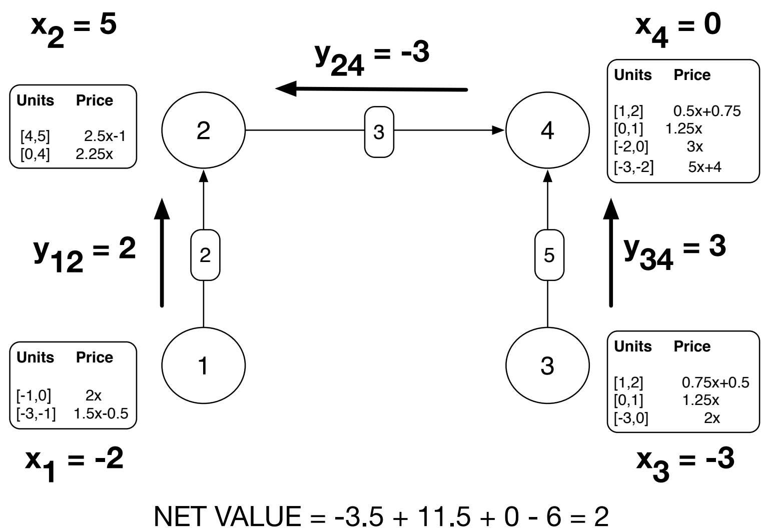

Fig. 2. Solution to the CEAP represented by the energy trading scenario.

prosumer 3, which is the one that does sell energy. In general, our model supports that each prosumer either: (i) aggregates energy received from its neighbors when buying energy; (ii) splits and distributes energy to its neighbors when selling energy; or (iii) relays energy so that other prosumers can satisfy their demand.

3 Solving the CEAP through MIP

Solving optimization problems by mapping them to linear programs has become a standard practice whenever such a mapping can be found. Through the advance of software capabilities (including CPLEX and Gurobi), this practice turns out to be difficult to beat even for problems, such as combinatorial auctions, that have attracted a stream of research in specific algorithms [11]. Along this line, in this section we show how the CEAP can be encoded as a linear program (LP).

Before translating the CEAP as an LP, we consider that the offer of prosumer j is expressed as a piecewise linear valuation o_j. According to lemma 3, each offer o_j admits a canonical representation that hereafter we denote as $W_j = \{o_j^1, \ldots, o_j^{n_j}\}$, where $o_j^1, \ldots, o_j^{n_j}$ are linear interval valuations. Thus, each linear interval valuation $o_j^k \in W_j$ is defined as follows:

$$o_j^k(x) = \begin{cases} a_{o_j^k} \cdot x + b_{o_j^k} & \text{if } x \in I_{o_j^k} \\ -\infty & \text{otherwise} \end{cases} \tag{7}$$

where $I_{o_j^k} = [l_{o_j^k}, u_{o_j^k}]$ is a real interval, $a_{o_j^k}$ and $b_{o_j^k}$ are two real numbers, and $x \in \mathbb{R}$.

To encode our optimization problem, we will consider two types of decision variables: network decision variables and prosumer decision variables. On the one hand, as to the network, as described in section 2, for each edge (i, j) in the trading energy network an integer variable y_{ij} will take on as a value the number of units that prosumer i sells to prosumer j (when $\mathbf{y}_{ij} > 0$), or that she buys from prosumer j (when $\mathbf{y}_{ij} < 0$). Notice that $\mathbf{y}_{ij}$ may also be zero if there is no trading between i and j. In general, the value of y_{ij} is within the domain D_{ij}.

On the prosumer side, since the prosumer value $v_j(\mathbf{Y}_j)$ of equation 5 cannot be encoded as a linear function in terms of these variables, for each prosumer j we introduce a set of auxiliary binary variables $\{z_j^k \mid j \in P, 1 \leq k \leq |W_j|\}$, where variable z_j^k indicates whether the k-th linear interval valuation in the offer is taken or not. Since the linear interval valuations within the offer of prosumer j are mutually exclusive, these variables are linked by a constraint that enforces that one and only one of them is active, namely $\sum_{k=1}^{|W_j|} z_j^k = 1$.

Besides choosing some linear interval valuation out of an offer, we must also decide the number of units that the prosumer is to trade. Thus, for each prosumer j we introduce a set of auxiliary real variables $\{x_j^k \mid j \in P, 1 \leq k \leq |W_j|\}$, where variable x_j^k indicates the number of units the prosumer decides to trade. Therefore, we can readily encode the value obtained from selecting x_j^k energy units to trade from the linear interval valuation o_j^k as $a_{o_j^k} \cdot x_j^k + b_{o_j^k} \cdot z_j^k$.

At this point, we can establish how to enable each z_j^k variable by means of the following constraint:

$$z_j^k = 1 \text{ if and only if } x_j^k \in I_{o_j^k} \tag{8}$$

This constraint ensures consistency between each prosumer's decisions. If variable x_j^k is set to a value within $I_{o_j^k}$, then variable z_j^k must be enabled to reflect that the k-th linear interval valuation of prosumer j is selected. Thus, each variable z_j^k acts as an indicator variable. Notice that equation 8 can be readily linearised by means of the following inequations: $z_j^k \cdot l_{o_j^k} \leq x_j^k \leq z_j^k \cdot u_{o_j^k}$.

Now we are ready to put together the network and prosumer decision variables. The net energy balance $net(\mathbf{Y}_j)$ from equation 4 provides a connection between the flows of energy in and out a prosumer and the offer selected. We can express equation 4 for prosumer j by means of the constraint

$$\sum_{i<j} y_{ij} - \sum_{q>j} y_{jq} = \sum_{k=1}^{|W_j|} x_j^k.$$

Finally, the prosumer value can be easily written as a linear expression in terms of these variables: $\sum_{l=1}^{|W_j|} v_j^k$, where $v_j^k = a_{o_j^k} \cdot x_j^k + b_{o_j^k} \cdot z_j^k$ is the value contributed by the k-th linear interval valuation.

Now we are ready to define the LP that solves the energy allocation problem introduced in the previous section.

$$\begin{array}{lrl}
\textbf{maximize} & \sum_{j=1}^{|P|}\sum_{k=1}^{|W_j|} a_{o_j^k}\cdot x_j^k + b_{o_j^k}\cdot z_j^k & \\
\textbf{subject to} & z_j^k\cdot l_{o_j^k} \le x_j^k \le z_j^k\cdot u_{o_j^k} & \forall j\in P, 1\le k\le |W_j| \\
 & \sum_{l=k}^{|W_j|} z_j^k = 1 & \forall j\in P, 1\le k\le |W_j| \\
 & \sum_{i<j} y_{ij} - \sum_{q>j} y_{jq} = \sum_{k=1}^{|W_j|} x_j^k & \forall j\in P, 1\le k\le |W_j| \\
 & & \\
 & y_{ij}\in D_{ij} & \forall (i,j)\in E \\
 & z_j^k\in\{0,1\} & \forall j\in P, 1\le k\le |W_j| \\
 & x_j^k\in\mathbb{R} & \forall j\in P, 1\le k\le |W_j|
\end{array}$$

Let us consider again the example in Figure 1, and its solution in Figure 2. The optimal allocation $\mathbf{Y}$ presented in the previous section is obtained by the MIP above by setting the network decision variables to the following values: $y_{12} = 2$, $y_{24} = -3$ and $y_{34} = 3$; and the prosumer decision variables to the following ones: $x_1^1 = -2$, $x_2^2 = 5$, $x_3^1 = -3$ and $z_1^1 = 1$, $z_2^2 = 1$, $z_3^1 = 1$, $z_4^3 = 1$ (otherwise $x_j^k = 0$ and $z_j^k = 0$). This leads to the following evaluation of the allocation (only those j, k sumands with $z_j^k = 1$ are shown, since all others are zero):

$$[1.5\cdot(-2) - 0.5] + [2.5\cdot 5 - 1] + [1.25\cdot 0] + [2\cdot(-3)] = -3.5 + 11.5 + 0 - 6 = 2$$

4 Mechanism Design

Up to now, we have concentrated on how to formalize and provide a solution to the CEAP through ILP, disregarding the strategic behavior of prosumers. Here we skim through some game-theoretic considerations.

Mechanisms are composed of both a choice rule and a payment rule [19]. From a mechanism design point of view, the CEAP can be understood as the choice rule that selects the energy trades in our network based on the valuations provided by the prosumers. The previous section shows that this choice rule can be assessed by means of ILP However, we have not proposed any payment rule that establishes how much should each agent pay/receive afterwards.

In their classical work from 1983, Myerson and Satterthwaite [15] proved the impossibility of having an efficient, individual-rational, incentive-compatible, and budget-balanced mechanism in a simple exchange environment in which a buyer and a seller trade a single unit of a given good. This very simple case is isomorph to an energy network with two connected participants where one has available an energy unit that the other one wants to buy. Thus, the impossibility result [15] extends to our setting.

On the other hand, the central result in mechanism design, on the incentive-compatibility of the Vickrey-Clarke-Groves (VCG) mechanism, carries over to our model. Recall that the VCG mechanism allocates goods in the most efficient manner and then determines the price to be paid by each bidder by subtracting from their offer the difference of the overall value of the winning bids and the overall value that would have been attainable without that bidder taking part. That is, this "discount" reflects the contribution to the overall production of value of the bidder in question. The VCG mechanism is strategy-proof: submitting their true valuation is a (weakly) dominant strategy for each bidder. As an inspection of standard proofs of this result reveals [12], this does not depend on the internal structure of the agreements that agents make. Hence, it also applies to our model.

Furthermore, assessing the VCG payment for each prosumer only requires solving a new CEAP problem where that particular prosumer is not present, which can also be done by means of generic ILP software such as CPLEX or Gurobi.

Further studying mechanism design properties of such markets (including alternative payment rules that could lead to asymptotic efficiency along the lines of [2]) remains as future work.

5 Conclusions and future work

In this paper we have investigated how to extend the work in [1] to enable energy trading in prosumer networks for prosumers with piecewise linear valuations, and taking into account grid system constraints. We propose to cast the energy trading problem as an optimization problem, the continuous energy allocation problem (CEAP). We then show that the CEAP can be formulated as an MIP so that it can be optimally solved for any network topology by means of commercial optimization solvers.

A solver for the CEAP by means of the mapping provided in this paper has effectively been implemented and is currently able to solve problems with hundreds of prosumers in the order of a tenth of a second. A detailed evaluation of the efficiency of that solver is ongoing.

In [1], an alternative distributed algorithm (RadPro) is provided for efficiently solving the discrete EAP when the graph is acyclic. Another promising line of future work is the extension of RadPro to provide a decentralized solver for the acyclic CEAP. Provided that this is successfully achieved, the next step will be to consider how to extend such a solver so that it is able to effectively solve problems which contain cycles.

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References

1. J. Cerquides, G. Picard, and J. A. Rodriguez-Aguilar. Designing a Marketplace for the Trading and Distribution of Energy in the Smart Grid. In *AAMAS*, pages 1285–1293, 2015.
2. L. Chu and Z. Shen. Truthful double auction mechanisms. *Operations research*, 56(1):102–120, 2008.

3. European Technology Platform. SmartGrids SRA 2035. Strategic Research Agenda. Update of the SmartGrids SRA 2007 for the needs by the year 2035, March 2012.
4. Federation of German Industries (BDI). The Energy Industry on the Way to the Internet Age. BDI publication No.439, 2010.
5. T. Gonen. *Electric power distribution engineering*. CRC press, 2014.
6. Greenpeace. Decentralising power: An energy revolution for the 21st century. http://bit.ly/1xf1RCk, 2005.
7. L. L. Grigsby. *Electric Power Generation, Transmission, and Distribution*. CRC press, 2012.
8. D. Ilic, P. G. Da Silva, S. Karnouskos, and M. Griesemer. An energy market for trading electricity in smart grid neighbourhoods. In *6th IEEE International Conference on Digital Ecosystems Technologies (DEST), 2012*, pages 1–6. IEEE, 2012.
9. K. Kok, B. Roossien, P. MacDougall, O. van Pruissen, G. Venekamp, R. Kamphuis, J. Laarakkers, and C. Warmer. Dynamic pricing by scalable energy management systems—field experiences and simulation results using powermatcher. In *Power and Energy Society General Meeting, 2012 IEEE*, pages 1–8. IEEE, 2012.
10. S. Lamparter, S. Becher, and J.-G. Fischer. An agent-based market platform for smart grids. In *Proceedings of the 9th International Conference on Autonomous Agents and Multiagent Systems: Industry track*, pages 1689–1696. International Foundation for Autonomous Agents and Multiagent Systems, 2010.
11. K. Leyton-Brown, E. Nudelman, and Y. Shoham. Empirical hardness models. *Journal of the ACM*, 56:1–52, 2009.
12. A. Mas-Colell, M. D. Whinston, and J. R. Green. *Microeconomic theory*. Oxford University Press, 1995.
13. S. J. O. Miller. *Decentralised Coordination of Smart Distribution Networks using Message Passing*. PhD thesis, University of Southhampton, 2014.
14. J. Mockus. On simulation of the nash equilibrium in the stock exchange contest. *Informatica*, 23(1):77–104, 2012.
15. R. B. Myerson and M. A. Satterthwaite. Efficient mechanisms for bilateral trading. *Journal of Economic Theory*, 1(29):165–281, 1983.
16. M. A. Olson, S. J. Rassenti, V. L. Smith, M. L. Rigdon, and M. J. Ziegler. Market design and motivated human trading behavior in electricity markets. In *Proceedings of the 32nd Annual Hawaii International Conference on Systems Sciences, 1999. HICSS-32.*, pages 1–27. IEEE, 1999.
17. Y. K. Penya and N. R. Jennings. Optimal combinatorial electricity markets. *Web Intelligence and Agent Systems*, 6(2):123–135, 2008.
18. S. D. Ramchurn, P. Vytelingum, A. Rogers, and N. R. Jennings. Putting the 'smarts' into the smart grid: A grand challenge for artificial intelligence. *Commun. ACM*, 55(4):86–97, Apr. 2012.
19. Y. Shoham and K. Leyton-Brown. *Multiagent systems: Algorithmic, Game-Theoretic, and Logical Foundations*. Cambridge University Press, 2009.
20. P. Vytelingum, S. D. Ramchurn, T. D. Voice, A. Rogers, and N. R. Jennings. Trading agents for the smart electricity grid. In *Proceedings of the 9th International Conference on Autonomous Agents and Multiagent Systems: volume 1-Volume 1*, pages 897–904. International Foundation for Autonomous Agents and Multiagent Systems, 2010.
21. B. M. Weedy, B. J. Cory, N. Jenkins, J. Ekanayake, and G. Strbac. *Electric power systems*. John Wiley & Sons, 2012.
22. T. K. Wijaya, K. Larson, and K. Aberer. Matching demand with supply in the smart grid using agent-based multiunit auction. *2013 Fifth International Conference on Communication Systems and Networks (COMSNETS)*, pages 1–6, Jan. 2013.
23. P. Wolfe. The implications of an increasingly decentralised energy system. *Energy policy*, 36(12):4509–4513, 2008.