Creating special effects by ray-tracing with non classical perspectives

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Abstract

Computer graphics generally use conic perspective with a fixed eye: it tries to reproduce the photographic principle, and therefore the photographic realism.

Some works have been done in order to integrate special effects used in art, photography or cinema. We have tried to generate new effects by only changing one part of the rendering process: the projection process. These effects, mainly inspired by art and photography, include anamorphosis, `OMNIMAX` and fish-eye projections, convergent and twisted projections. They have been integrated into a ray-tracing environment.

Keywords conical perspective - anamorphosis - `OMNIMAX`- fish-eye - art - Cubism - ray tracing.

1 Introduction

1.1 What is our problem?

Many perspectives have been and are still used in the world of representation. Choosing a kind of perspective reflects a way to understand the world. In western representative abstraction, the viewer is reduced to a single eye, itself reduced to a single point.

Questioning the realism of a picture is a necessity. Does a picture so-called *realistic* reflect a true reality, existing in the world and directly accessible without any transcoding, or is it just a cultural concept acquired as reality and which only reflects itself [Bou94]?

Our research is then not a strictly scientific research, for which the main goal would be the construction of models describing the behaviour of nature by means of absolute laws. We have to analyze models born in social sciences as well as sensorial interpretations. Our goal is the extraction of *concepts* which direct each style of representation. By conforming as far as possible these basic concepts, we may define methods for projection, and we will keep only pictures which have a definitely new esthetic.

1.2 Computer approach

The problem of perspective (or projection) is the transition between a three-dimensional world and a bi-dimensional picture.

It is rather classic in computer graphics to use a conical perspective. We have tested other perspectives which seem interesting, either by their historical aspects, or by their geometric properties. It is also important that such perspectives must not be too complicated, so we also focused on the easiness: a perspective must be defined by few parameters.

We already have worked in this direction. With a modeler based on particles system, we have computed pictures using a slightly modified z-buffer algorithm [BB92]. These pictures however were difficult to interpret, especially because of the lack of volumic aspect. We then decided to use a volumic modeler, based upon the Constructive Solid Geometry approach. The algorithm choosen for hidden surface elimination and rendering is a ray tracer [Roe93].

2 Previous works

2.1 Classical perspective

Let us remember what are the origins of what we now call classical perspective, ie conical perspective. Its main goal is to achieve realism: we want to build pictures as faithful to physical reality as possible.

This technique is not really new, since it comes from a method called *camera obscura* (darkroom). A wall of a dark room is pierced so that light can pass through the hole. The scene visible through the hole is then drawn (as a fully reversed picture) on the opposite wall.

A similar technique may be used by holding a mirror while turning one's back to the scene: a picture is created by reflexion on the mirror. In this case, the picture is left-right reversed.

During XVth century, Brunelleschi used the mirror technique, while Alberti [Alb35] used the *camera obscura* to formalize light travel and picture creation. We may consider indeed that this formalization was the first description of a ray tracer[1].

At the end of XVIth century, Giambattista della Porta improved the technique by inserting lenses into the hole. These lenses were improved throughout centuries. Then, during the XVIIIth century, the camera obscura became portable, and permitted much more scenes to be captured. Finally, in 1822, Nicéphore Niepce invented photography by inserting in the camera obscura a sensitive plate. Computer graphics has carried on with this concept of photographic realism.

From a computer point of view, a perspective may be defined by the following parameters:

- the eye's position
- the viewed point's position (or aim)
- the upward direction for the picture
- the half angle of width aperture α
- the half angle of height aperture β

[1] Some historians of art think that camera obscura was already used during antiquity.

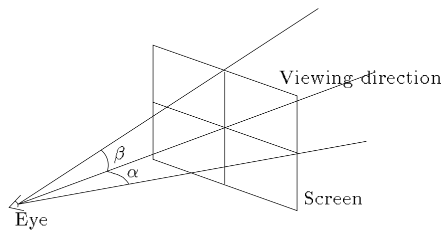

Figure 1: Classical perspective

The two angles must be non negative and less than 90 degrees. It is then possible to define a coordinate system associated with these definitions: the eye is the origin, the x-axis is along the upward direction, the z-axis is along the viewing direction, and the y-axis completes the system.

Figure 1 shows how these parameters are used. The screen is perpendicular to the viewing direction (the direction that joins the eye and the viewed point), the upward direction tells where the top of the screen is, and the two angles limit the projection plane to a rectangular portion. To avoid artefacts (spheres looking like ellipsoids), the screen resolution must be compatible with the aperture angles.

Important features in classical perspective are vanishing lines: it expresses a method for pictorial construction of a perspective. When realizing a perspective drawing, we begin by drawing the horizon. On this horizon, we place what we

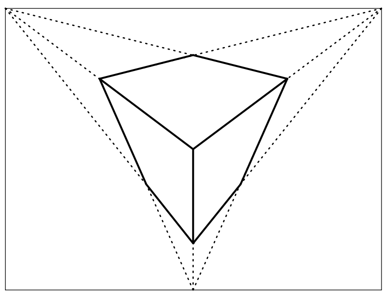

Figure 2: Vanishing lines

called a vanishing point, toward which all directions perpendicular to the projection screen will converge. These lines are called vanishing lines. Figure 2 shows how we may use vanishing lines to construct a perspective: three vanishing points are used, which represent projections of the three main axes at an infinite distance. We may notice here that vanishing lines are sometimes used to reconstruct the projection from a picture, and therefore to add some artificial objects in a real scene.

2.2 Computer approach

Classical perspective has a trumendous advantage: by using a convenient projection plane, planarity of objects is conserved, ie planes in real world are transformed into planes in projected space. This property has been extensively used in the traditional graphic pipeline: objects are modeled as planar facets, which are projected, clipped and z-buffered. This advantage has a drawback: other projections method (except orthogonal projection) are not allowed in z-buffer.

Ray-tracing does place far less restrictions on the projection method: it can however only trace *rays*, ie points of real space that are projected onto the same point of the screen have to belong to the same straight-line. Some improvements of ray-tracing even use nonlinear rays [Ina91][Gro95], which then permit other effects.

2.3 Lens effects

Some improvements have been made in order to increase the photographic realism. In fact, for each pixel of a screen or a picture, the color seen is an integral of what is seen on the small area covered by the pixel. If we use the traditional projection, the effect of simulating this area is just filtering, and then antialiasing.

A camera model has been developped first by Potmesil and Chakraverty [PC81]. Cook [CPC84] improved this model by simulating a real lens, as a camera would do. If the object seen at a given pixel is not exactly in the conjugate plane, a blurring effect might be seen: this effect is known as *depth of field blurring*.

Chen [Che88] also used a lens model to compute pictures for highly defocussed scenes: effects such as diffusion and concentration generated by an object partially blocking the scene were generated.

Kolb et al. [KMH95] improved the model by using real models for lenses. Complex effects such as vignetting were also handled, and they also propose a thick lens approximation, that requires less computations.

Max [Max83] proposed a method for handling distortion that occured in the `IMAX` projection method, which uses a cylindric screen.

2.4 Surrounding surfaces

Inakage [Ina92] also proposed a model to create deformations of three-dimensional scenes. In his model, synthetic objects which are to be rendered are surrounded by closed reflective surfaces, which have an infinite reflection coefficient. The closed mirror surfaces then create a "feedback" effect which deforms objects inside the reflective space.

He also proposed lens effects, also surrounding the eye by infinitely reflective curved surfaces (non-closed).

3 Perspectives with a moving eye

3.1 Pictorial analysis

This kind of perspective follows from the analysis of fish-bone perspective, which is an antic perspective also called *perspectiva naturalis* (by opposition to the *perspectiva artificialis*, which is the name given to conical perspective). This analysis was done on paintings in which space was regularly and measurabily subdivided, like with beams or tiles, perpendicular to the projection screen.

In some of these paintings, vanishing lines do not converge toward a unique vanishing point, but rather converge toward an upright vanishing axis at the center of the picture (see figure 3). Depending on paintings, vanishing lines are sometimes regularly spaced and parallel, and sometimes unequally spaced or even unordered. The *fish-bone* (vanishing axis and vanishing lines) is however always visible [Pan87].

Figure 3: Boscoreale painting

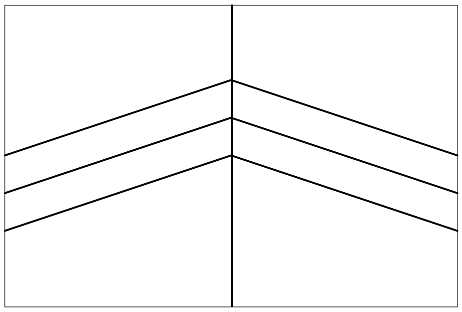

Figure 4: Vertical lines in a fish-bone projection

If we closely look at this projection, we may deduce the principle used: a conical projection is used, but for each point of space, a different vanishing point (or different eye and aim)is used. This principle was easy to use in a particle system, but is impossible in a ray-tracing system since the ray restriction is not satisfied: points that are projected onto the same pixel describe an algebraic or even a transcendental curve. We then retained as a principle the upright mobility of the eye in order to mimic this fish-bone. The displacement of the eye Δx is a function of the lateral position of the projection point (ie all points on a same column of the picture use the same position for the eye).

3.2 Fish-bone perspective

Upright displacement is here just proportional to the distance between the projection point and the vanishing axis:

$$\Delta x = k|y|$$

In this kind of projection, a vertical line (x and y constant) is projected as a chevron, composed by two segments which are symmetric with respect to the vanishing axis (see figure 4). The coefficient to be used for the proportionality may be choosen arbitrarily. In most cases, we choose a coefficient such that an horizontal line drawn in conical perspective at the center of the screen is drawn as a chevron going through bottom corners.

Another kind of perspective, called crossed-beams perspective, has a similar principle, except that points projected on a particular point of screen describe two straight lines, symmetrical with respect to an horizontal plane placed at the center of screen. We did not implement such a perspective.

3.3 Hyperbolic perspective

In that case, displacement is proportional to the inverse of the distance between the vanishing axis and the projection point.

$$\Delta x = \frac{k}{|y|}$$

This is a simple way to reproduce the inversion of vanishing lines that may be observed on some pictures: the vanishing point for vanishing lines that are nearer from the center of the picture is under the vanishing point for vanishing lines that are nearer from edges of the picture (figure 5). In this kind of perspective, an horizontal line is projected as two branches of an hyperbol.

In order to avoid computer overflows, we use the following technique: in a zone around the vanishing axis, the two branches of the hyperbol are joined by a piece of parabol. This junction is done as a continuous derivative junction to avoid projection artefacts due to discontinuity. The correct displacement is then:

$$\Delta x = \frac{k}{2S}\left(3 - \frac{y^2}{S^2}\right) \text{ if } |y| \leq S$$

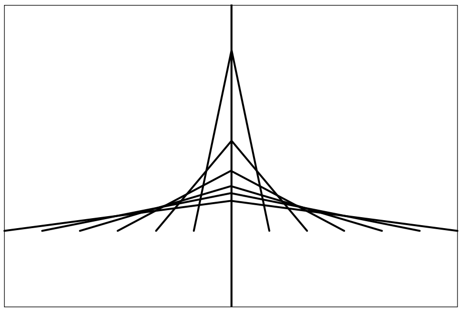

Figure 5: Vertical lines in a hyperbolic projection

$$= \frac{k}{|y|}\text{otherwise}$$

The user has then two parameters (threshold S and coefficient k) to adjust the perspective.

3.4 Mixed perspective

We have decided to create a mixed perspective, ie a combination between hyperbolic and fish-bone perspective. This combination has two major advantages:

- it gives a better control, since more parameters (and then more projections) are allowed.
- it makes it possible to create interpolations between the two basic perspectives by simply interpolating parameters: this is important for animation.

The displacement used is then:

$$\begin{aligned} \Delta x &= k_{fb}|y| + \frac{k_{hy}}{2S}\left(3 - \frac{y^2}{S^2}\right) \text{if} |y| \leq S \\ &= k_{fb}|y| + \frac{k}{|y|}\text{otherwise} \end{aligned}$$

where k_{fb} if the fish-bone parameter, k_{hy} is the hyperbolic parameter and S the hyperbolic threshold: if k_{fb} is zero, it is an hyperbolic perspective, if k_{hy} is zero, it is a fish-bone perspective.

4 Wide angle perspectives

As this has been pointed out for conical perspective (and this also applies to moving eye perspective), viewing field is limited to an half-space. In fact, for aperture angles near from 90 degrees, the picture is seriously distorted: objects at the limits of the viewing pyramid have a greater size than objects at the center of picture.

So it seems to us that it would be interesting to create perspectives that allows to see more than an half-space on a single picture, and avoiding distortions.

4.1 Stereographic perspective

The stereographic projection is used to project a sphere (without one of its points) on a diametral plane perpendicular to a line joining the choosen point to the center of the sphere. By analogy with the earth, the choosen point is called the *north pole* of the projection and the projection plane is called the *equatorial plane*.

The principle used for the projection is as follow: S is the sphere, N is the north pole, P_E is the equatorial plane. Then any point P (different from N) of the sphere is projected onto the point obtained as the intersection of the straight line joining N to P and the equatorial plane P_E (see figure 6). In such a projection, the opposite pole (or *south pole*) is projected onto the origin of the equatorial plane, equatorial points are projected onto themselves, points of northern hemisphere (P_2 on the figure) are projected outside the equatorial circle (onto P_2') while points of southern hemisphere (like P_1) are projected inside the equatorial circle (onto P_1').

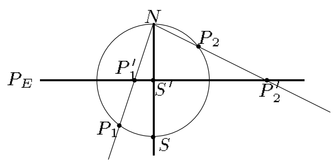

Figure 6: Stereographic projection

On another hand, parallels are projected as concentric circles, centered on the origin of the equatorial plane, while meridians are projected as straight lines going through the central point.

Another interesting property of the stereographic projection is the conservation of angles. If a curve drawn on the sphere intersects another curve also drawn on the sphere with a given angle, the two projected curves also intersect one another with the same angle. This is true for parallels and meridians which are perpendicular to each other. This property is used in geology as well as in geography.

The principle of stereographic perspective is then straightforward (see figure 7): the eye is at the center of the sphere, and the south pole is placed along the viewing direction; the north pole is then "behind" the eye. Points that lie along a direction non collinear with the north direction are then projected on a point which is the stereographic projection of the intersection between the direction of the sphere.

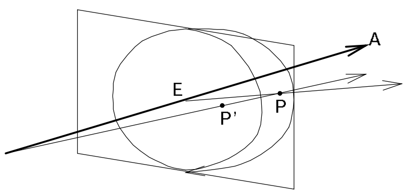

Figure 7: Stereographic perspective

Then visible space is the whole space, except points that lie along the northern axis. Practically, directions that are near this northern axis are not projected ("near" means that if the angle between the northern axis and the direction is less than a threshold value δ_{max}, the direction is not projected).

The picture computed in this way is a disc, the center of which is the projection of the southern axis. The boundary of that disc gathers projections of points along limit directions. As we compute rectangular pictures, we have choosen to center that disc in the picture, the disc being tangent to edges of the picture. Pixels which are outside the disc are just black pixels.

4.2 Spherical perspective

The stereographic perspective described above does not allow a really full space to be projected. Furthermore, for small values of limit angle, the projected disc becomes very large, and scaling necessary to integrate the disc into the picture enforces high compression ratio for directions that are near the southern axis. Properties of the stereographic projection are also rather geometric, and not really used in art, photography or cinema.

Another kind of wide-angle projection has been implemented that is much more interesting because it has limited compressions and it is widely used: this projection is known as *spherical perspective* [Bon85] or *curvilinear projection*. This perspective, mainly used in architecture (see figure 8, from [Bon85]) was also known to be used by Eskimos (to make pictures for their igloos), and is used by the OMNIMAX process, which allows projections of movies onto an hemispherical screen (like the *Géode* in Paris or the *Futuroscope* in Poitiers). The *fish-eye* lens [KMH95] uses a similar projection.

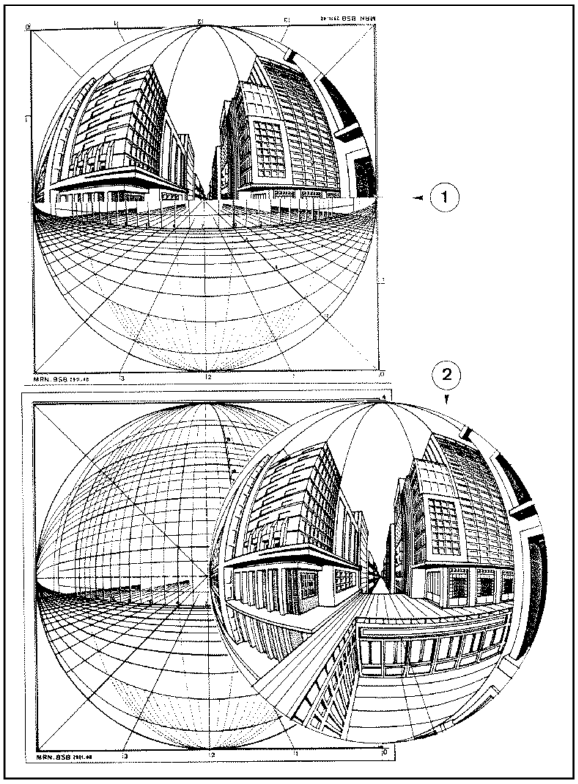

Figure 8: Spherical perspective in architecture

We may notice here that the OMNIMAX process uses an hemispherical screen, and the projector must be equipped with a lens with the same properties than the camera: the planar projection is just used to store pictures on a plane film. Photographs made with a fish-eye lens are used in their planar representation, while `OMNIMAX` restores a half-space projection. With our curvilinear ray-tracing, we are now able to compute real `OMNIMAX` synthetic movies: no piecewise reconstruction of the screen is done.

The principle is as follows (see figure 9): the eye E is once again at the center of a sphere, and the projection plane is tangent to that sphere, perpendicular to the viewing direction. The tangency point is the viewed point A. For any point P on the sphere, there is only one plane Π that contains both E, A and P. This plane intersect the sphere as an circular arc $\mathcal{C}$, and intersects the projection plane as a straight line Δ (in fact, just half a straight line, because we only keep points which are on the same side than P). Then points that lie along the direction joining E to P are projected onto the point p of Δ such that the distance between A and p is the length of the circular arc $\mathcal{C}$. The above principle is defined for an

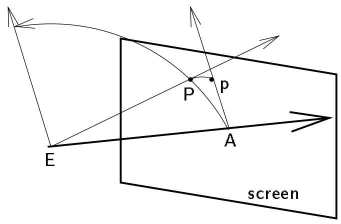

Figure 9: Spherical perspective

infinite screen, but we use screens with limited dimensions. In fact, we use an aperture angle δ_{max} (expressed in radians) and the screen is limited to have a radius equal to $\delta_{max} r$ (where r is the radius of the sphere). We may notice here that this aperture angle is not limited to 180 degrees. If it is greater, the picture includes concentric circles, centered on the center of the screen, at distances which are odd multiples of πr: these circles are composed with a unique color, which is the color of the object being seen at the opposite of the viewing direction. Similarly, concentric circles at distances which are even multiples of πr are composed with the color of the object being seen in the viewing direction.

5 Convergent perspectives

5.1 Why ?

The main idea of convergent perspectives resides in Cubism. Cubism was certainly one of the greatest break in history of art for representation models since the Quattrocento. Braque and Picasso have introduced in painting the relationship between space and time, giving to the picture a status of a demonstrative area for space and volume.

Experimental reality of volume, only perceptible by movement, is translated by gathering on a single picture a sum of differents points of view. The painter turns around his model and describes on his canvas the sum of his impressions [Pau90].

Remember here that Braque and Picasso refused to sign their paintings in order to automate that concept of space representation. The cubist vision should access a proper autonomy as the brunelleschian model escaped from Brunelleschi and Alberti to become a self evidence.

From cubism, we retained the following principles:

- the eye turns around the object being visualized,
- the picture gathers the sum of points of view.

We may notice that perspectives with a moving eye described before (fish-bone perspective and hyperbolic perspective) are not considered to be convergent perspectives: the eye is effectively moving but there is no convergence of viewing directions.

Although Cubism was our first application, convergent perspectives may be used in any application that requires multiple views of the same data: it is then possible to combine all the views on a single view, therefore reducing the complexity.

5.2 Bowlic perspective

In this kind of perspective, the viewed point A is at the center of a sphere S and the screen is developed onto the sphere.

We use a system based on latitude and longitude. More precisely, any point in a picture is given a latitude and a longitude. By convention, center of screen has zero latitude and longitude. Right edge has a maximal longitude (and left edge has the oppposite longitude), top edge has a maximal latitude (and bottom edge has the opposite latitude). These maximal latitude and longitude are parameters for the projection.

Then latitude and longitude for any point is simply computed as a linear interpolation between left edge and right edge for longitude and top edge and bottom edge for latitude. The eye is so placed at the point of the sphere given by latitude and longitude. Parallels are projected as equally spaced horizontal lines, while meridians are projected as equally spaced upright lines. Note that this is not the conventional Mercator projection used for planar maps of the world, in which the screen is developped onto a cylinder: on a Mercator map, meridians are equally spaced but parallels are not equally spaced (space between two parallels is proportional to the tangent of the latitude). In our bowlic projection, both parallels and meridians are equally spaced. An

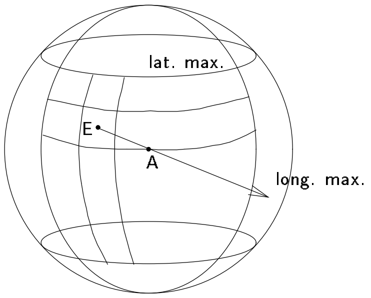

Figure 10: Bowlic projection

object may then be seen from all sides on a single vue. If the maximal latitude is greater than 90 degrees, we obtain horizontal lines with uniform color, corresponding to the color seen from the pole (northern or southern). This is due to the fact that longitude is not really defined for poles (any longitude gives the same point). For a maximal longitude greater than 180 degrees, we also obtain periodical patterns in the picture.

Another phenomenon is rather curious with this perspective: if an object is behind the viewed point (the center of the sphere), its projection is fully reversed. If this object is exactly at the center, it may be seen from any point of the picture. If there is only one object, then it is effectively seen from any point.

5.3 Circular perspective

In this case, the eye is moving on a circular arc, and screen is developed on a portion of a cylinder which has the same axis as the arc: this axis is called the *main axis* of the projection. Two

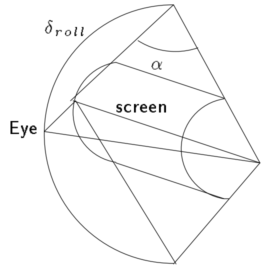

Figure 11: Circular perspective

angles are used to fully described the perspective: a width aperture angle α (same definition as in conical perspective) and a rolling angle δ_{roll}.

For rolling angles greater than 180 degrees, the picture is periodic. Similarly to bowlic perspective, an object placed behind the viewed point has a top-bottom reversed projection.

A simple extension of the circular perspective is the cylindric perspective: the eye is not moving on a circle, but on a cylinder, which is homothetic of the screen. The projection is then fully defined by an extra parameter: this parameter gives the ratio between the width of the eye-cylinder and the width of the screen. If this ratio is zero, we have a circular projection; if this ratio is one, all rays are perpendicular to the main axis of the projection; if greater than one, rays are converging behind the main axis.

It is also possible to define two kinds of cylindric projections: a vertical cylindric projection, for which the main axis is vertical (along the x axis) and an horizontal cylindric projection, for which the main axis is along the y axis. In the first case, a width aperture angle is needed, in the latter a height aperture angle is needed.

6 Twisted projections

Once again, Cubism was our inspirator. If we closely look at pictures painted by Picasso called "*Woman sat in a chair*", we may see that the woman's knees are seen from the right side, the trunk is seen from front of the woman and the face is seen from the left side: the same principle of the eye moving around the subject has been used, the movement consisting in a twisting around the aim.

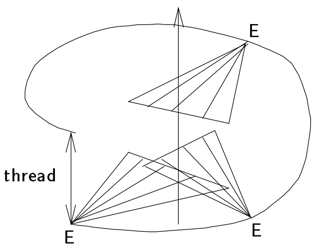

Figure 12: Twisted classical projection

This principle of twisted projection has been added to two main perspectives: the classical (conical) perspective and the cylindric perspective.

For the classical perspective (figure 12), we need three more parameters: the thread which gives the space between two points on the same vertical (it is expressed relatively to the height of the screen) and twist angles for the bottom and the top of the screen. If both the thread and twist angles are zero, we have a classical perspective; it is then very easy to create interpolations between classical and twisted-classical perspective by just interpolating these parameters.

The same modifications are applied to the vertical-cylindric perspective: we also use three extra parameters which have exactly the same definition than in the twisted-classical case. Interpolation is furthermore very easy.

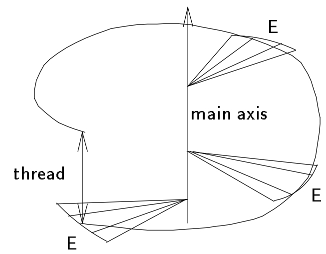

Figure 13: Twisted cylindric projection

7 Anamorphosis

7.1 What anamorphosis is?

Anamorphosis is a representation of objects that appeared deformed, but that might be seen with their original sizes if observed under a particular angle, or a particular viewing device (a curved mirror for example).

Anamorphosis has been studied from a long time [Bal80], since some legends about Phidias (fifth century BC) speak about what was called "slowed down perspective".

7.2 Applications of anamorphosis

Anamorphosis has been widely used in art, and for several purposes.

The first application is the possibility to do paintries that are not observed from the front of the picture. This is especially true for pictures that have been painted on domes, like the Sixtin Chapel in Rome. They are observed from the floor, and the surface of the dome is not perpendicular to the viewing direction. The same principle has been used in very long corridors, were paintries are observed from one end of the corridor: pictures must be painted deformed to be seen at a good size. This problem also arises in designing roadsigns: in tunnels for example, it is sometimes hard to place roadsigns orthogonally to the road; they have to be painted on the walls, and then anamorphosis occurs.

A second application is precisely the ability to "hide" some parts of a picture, and then to give full sense only to advised people. In the well-known paintry "The Ambassadors", from Holbein, we may see two characters, which seem rich and powerful. There is a white part just under the two characters, and if looked from the left side of the paintry, we may see a skull. The skull is in the picture to show that Death is more powerful than everything. Using the same idea, some anamorphotic pictures of French King Louis XVI, were distributed amongst the population during the French Revolution.

The third application is more ludic: anamorphosis was used just for fun. Chinese developed many technics to create anamorphotic pictures that might be correctly seen with a cylindric mirror.

7.3 Planar anamorphosis

Since anamorphosis covers many technics, we chose to implement only planar anamorphosis. It is the simpliest, but it is also the more widely used.

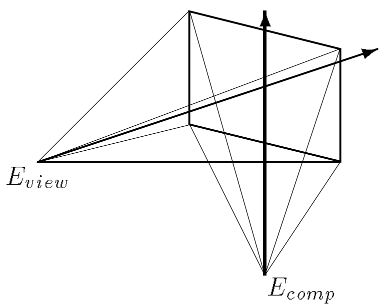

Figure 14: Planar anamorphosis

Planar anamorphosis may be simply implemented by the definition of two eyes: a *viewing eye* (E_{view}) which is used to view the picture, and a *computing eye* (E_{comp}) which is used to trace rays (see figure 14).

To simplify the implementation, we chose to define the position of the computing eye relatively to the viewing eye: the center of the screen is choosen as the origin, and the computing eye is defined in a spherical coordinate system. The user then has three parameters: a radius (expressed relatively to the distance between the viewing eye and the aim), an azimuthal angle and a zenithal angle. For a unit radius and null angles, we have a classical perspective. Then interpolation may be achieved by simply interpolating these three parameters.

8 Implementation

From an computer point of view, the various perspectives described in this paper have been integrated into the ray tracing software developed at the Ecole Nationale Supérieure des Mines de Saint-Etienne and called `yart` (for *Yet Another Ray Tracer*) [Roe93].

This software is part of the *Illumines* [PB89] testbed for computer graphics, running on Unix workstations. The basic concept of this testbed is a CSG modeler. We may then define primitives, boolean operations and affin transformations.

The `yart` software is widely configurable (various rendering models, various optimization techniques) and it is then possible to choose any perspective described in this paper. We may notice here that these new perspectives forbid us to use any optimization technique relying upon screen projection, since projections are not as simple as in conical perspective. We so use a bounding structure technique.

To create animation effects, we also defined methods to interpolate between perspectives. Both effects (new perspectives and interpolation) have been used to produce two short movies: the first one, called *Castorexion* [BR95], uses bowlic, circular, hyperbolic and fish-bone perspectives. The second one, called *Jonglerie sans paroles* [Far95], uses anamorphosis and twisted perspectives.

9 Conclusion and future works

Algorithms have been presented that permit to use new perspectives in computer graphics. These algorithm, which are well-suited to ray tracing, may be used for various purposes: 3D data visualization, artistical effects, `OMNIMAX` projection, architecture, geology, geography (GIS). Some other applications are certainly possible, since the only prerequisite for the ray-tracing algorithm is the ray conservation.

New algorithms are actually under investigations, especially concerning non-planar anamorphosis and nonlinear ray tracing. We also planned some applications for educational purposes. Eventually, we planned the integration of kaleidoscopic effects.

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